# ELECTROMAGNETIC WAVE SCATTERING ON A DIELECTRIC FIBER AND A SYSTEM OF PARALLEL FIBERS

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The scattering of the plane electromagnetic wave on the fiber-lake target is considered. The formula for the scattering cross section is used for building the kinetic equation that describes the propagation of the wave in the beam of parallel fibers. The evolution of intensity of the radiation propagating in the beam of straight and bent fibers is considered.

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## 1. INTRODUCTION

The interaction of electromagnetic (EM) waves with fiber-like structures is widely used for control of propagation of optical radiation using (see, e.g., recent review [1] and references therein). The present paper considers the interaction of radiation of arbitrary frequency with fiber-like target. The kinetic equation describing propagation of the radiation in the system of parallel fibers (rope) is constructed. It is demonstrated that not only optical but also hard (up to X-ray domain) radiation could propagate along the rope following its bending under some conditions.

## 2. CROSS SECTION FOR WAVE SCATTERING ON A FIBER

Scattering of EM wave by charged particle arises from oscillation of that particles under the action of the electric field of the incident wave (for incident wave of low intensity the influence of magnetic field would be negligibly small) that in its turn leads to emission of secondary EM waves. This emission is interpreted as the scattering of the incident wave; for the single charged particle the scattering cross section is described by well-known Thomson formula [2].

Let us consider the incidence of the plane monochromatic wave on the string of charges under the angle $\psi$ to the axis of the string (Fig. 1). The electric field in the incident wave is described by the formula

$$\mathbf{E}^{(in)}(\mathbf{r},t) = A\mathbf{e}^{(in)} \exp[i(\mathbf{k}^{(in)}\mathbf{r} - \omega t)], \qquad (1)$$

where $A$ is the wave amplitude, $\mathbf{e}^{(in)}$ is the polarization vector, $\mathbf{k}^{(in)}$ is the wave vector, $\omega = |\mathbf{k}^{(in)}|/c$ is the frequency. Scattering of the wave on the string of charges could be described as the coherent sum of Thomson scatterings on the charges of the string.

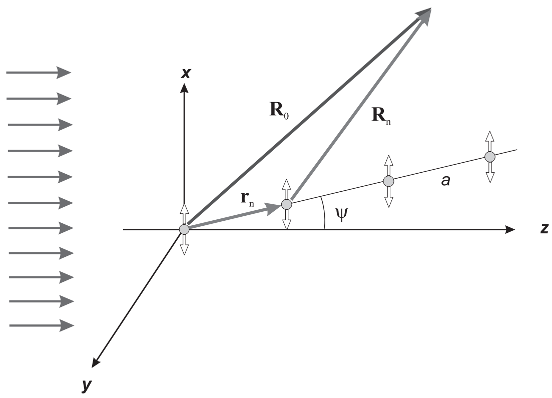


***Fig.1.*** *Incidence of the EM wave on the string of charges in the z direction causes their oscillations in the transverse plane*

The electric field of the wave scattered by $n$-th charge of the string could be presented on large distance from it, $R_0 \gg |\mathbf{r}_n|$, as [3]

$$\mathbf{E}_n = \frac{e^2 A}{m\omega^2 R_0} e^{-i\omega(t-R_0/c)} \left[\left[\mathbf{e}^{(in)}, \mathbf{k}^{(out)}\right], \mathbf{k}^{(out)}\right]$$

$$\times \exp\left[i(\mathbf{k}^{(in)} - \mathbf{k}^{(out)})\mathbf{r}_n\right], \qquad (2)$$

where $e$ and $m$ are charge and mass of the oscillating particle (electron), $\mathbf{k}^{(in)}$ is the wave vector of the scattered wave, $\mathbf{r}_n$ is the radius vector of the $n$-th charge. After summation of the contributions of large number of oscillating electrons forming the string, $N \gg 1$, and calculation the intensity of the scattered radiation in ratio to the incident energy flux density we obtain the cross section of the wave scattering by the

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string of charges:

$$\frac{d\sigma}{d\Omega} = 2\pi \left(\frac{e^2}{m}\right)^2 \frac{N}{a} \frac{\left[\mathbf{k}^{(out)}, \mathbf{e}^{(in)}\right]^2}{\omega^2 c^2}$$

$$\times \sum_{j=-\infty}^{\infty} \delta \left(k_{\parallel}^{(in)} - k_{\parallel}^{(out)} - \frac{2\pi}{a} j\right), \qquad (3)$$

where $a$ is the period of the string, $k_{\parallel}^{(in)}$ and $k_{\parallel}^{(out)}$ are the components of the wave vectors of the incident and scattered waves $\mathbf{k}^{(in)}$ and $\mathbf{k}^{(out)}$ parallel to the string axis.

In the limiting case of small $a$, $a < \pi/k^{(in)} = \lambda/2$, where $\lambda$ is the wavelength (that means the transition from the discrete string of charges to the uniform fiber) the only nonzero term in the sum over $j$ in (3) will be the $j = 0$ term. The cross section averaged over polarizations will be equal to

$$\frac{d\sigma}{d\Omega} = \pi r_0^2 L n_e^2 \left(1 + \left(\frac{c k_z^{(out)}}{\omega}\right)^2\right) \delta \left(k_{\parallel}^{(in)} - k_{\parallel}^{(out)}\right), \qquad (4)$$

where $n_e = 1/a$ is the number of electrons per unit length of the fiber, $r_0 = e^2/mc^2$ is the electron classical radius, $L = Na$ is the total length of the fiber.

Delta-function in (4) reflects the fact that the components of the wave vectors of the incident and scattered waves parallel to the fiber axis are equal to each other:

$$k_{\parallel}^{(in)} = k_{\parallel}^{(out)} = \frac{\omega}{c} \cos\psi. \qquad (5)$$

Since the absolute values of the wave vectors $\mathbf{k}^{(in)}$ and $\mathbf{k}^{(out)}$ are also equal, the angular distribution of the scattered radiation has the shape of empty cone with the half-opening angle equal to the incidence angle $\psi$ of the wave to the fiber. Such a behavior permits descriptive interpretation: the incident wave produces a perturbation in the fiber that moves along the fiber with the velocity $c/\cos\psi$. This superluminal motion generates the radiation analogous to Cherenkov one. Similar effect is described, particularly, in [4], where the radiation arising during the incidence a wave of dielectric permittivity of a substance (exited by the laser pulse) on the uniform charged fiber is considered.

The second term in the factor $1+c^2(k_z^{(out)})^2/\omega^2$ in (4) leads to the azimuthal asymmetry of the scattering cross section. This asymmetry is negligibly small for small angles of incidence $\psi \ll 1$, and the scattering becomes isotropic over azimuth angle in respect to the fiber axis. Hereafter we shall neglect the azimuthal asymmetry and put

$$1 + c^2 \left(k_z^{(out)}\right)^2/\omega^2 \approx 2. \qquad (6)$$

The account of the finite thickness of the fiber leads to the increase of the azimuthal asymmetry [5].

## 3. KINETICS OF THE RADIATION PROPAGATION THROUGH THE ROPE OF PARALLEL FIBERS

Consider now the scattering of the radiation incident on the system of parallel fibers (rope) randomly arranged in the plane transverse to the fiber axis (Fig. 2). Evolution of the radiation intensity angular distribution during the propagation along such fiber should be described by a kinetic equation analogous to one describing the multiple scattering of particles in amorphous medium [6]. However, our problem is more complicated because of dependence of the scattering cross section on the fiber not only on the scattering angle (in respect to the direction of incidence), but also on the direction of incidence itself (in respect to the fiber axis). So, the dependence of the radiation intensity $I$ on the $z$ coordinate would be described by the kinetic equation

$$\frac{\partial}{\partial z} I(\boldsymbol{\vartheta}, z) = \qquad (7)$$

$$= \frac{n_s}{L} \int [I(\boldsymbol{\vartheta} - \boldsymbol{\chi}, z) d\sigma(\boldsymbol{\vartheta} - \boldsymbol{\chi}, \boldsymbol{\chi}) - I(\boldsymbol{\vartheta}, z) d\sigma(\boldsymbol{\vartheta}, \boldsymbol{\chi})],$$

where $\boldsymbol{\vartheta}$ is the 2-dimensional angle that describes the radiation propagation direction in respect to the $z$ axis, $d\sigma(\boldsymbol{\vartheta}, \boldsymbol{\chi})$ cross section of the wave scattering to the 2-dimensional angle $\boldsymbol{\chi}$ from the initial direction $\boldsymbol{\vartheta}$, $n_s$ is the number of fibers per unit cross section of the rope. The first term in the right-hand side of (7) describes the income of the radiation intensity in the direction $\boldsymbol{\vartheta}$ because of scattering off the direction $\boldsymbol{\vartheta} - \boldsymbol{\chi}$ on the angle $\boldsymbol{\chi}$. The second term describes the outcome of the radiation intensity in the direction $\boldsymbol{\vartheta}$ because of the scattering off that direction.

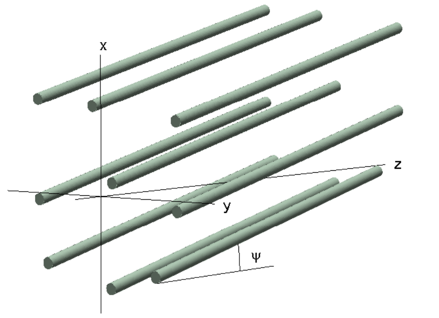


***Fig.2.*** *The rope of straight fibers; the initial wave propagates along the z axis*

However, it is convenient to re-write the kinetic equation (7) in slightly different form in order to use the cross section (4) specifics, when only the azimuthal scattering around the fiber axis takes the place. Let us re-write the cross section (4) with (6) in the form

$$\sigma(\boldsymbol{\theta}^{(in)}, \boldsymbol{\theta}^{(out)}) = \frac{1}{L} \frac{d\sigma}{d^2\theta^{(out)}} = \qquad (8)$$

$$= 2\pi r_0^2 n_e^2 \frac{c}{\omega} \delta \left(\cos\theta^{(in)} - \cos\theta^{(out)}\right),$$

where $\boldsymbol{\theta}$ is the 2-dimensional angle that describes the radiation propagation direction in respect to the fiber axis. Introducing the spherical coordinates $d^2\theta' = \sin\theta' d\theta' d\phi$ and integrating over the polar angle $\theta'$ using the delta-function in (8), we obtain the kinetic equation that takes into account the azimuthal character of the wave scattering on the fiber:

$$\frac{\partial}{\partial \xi} I(\theta, \varphi, \xi) = \tag{9}$$

$$= n_s \int_{-\pi}^{\pi} \left[ I(\theta, \varphi - \phi, \xi)\, \sigma_\phi(\theta) - I(\theta, \varphi, \xi)\, \sigma_\phi(\theta) \right] d\phi,$$

where the azimuthal angle $\phi$ is referenced from the incident wave azimuth, and the azimuthal scattering cross section

$$\sigma_\phi(\theta) \approx 2\pi r_0^2 n_e^2 c/\omega \tag{10}$$

in our approximation depends neither on the angle of incidence $\theta$ nor on the azimuthal scattering angle $\phi$. However, the equation (9) remains valid in general case.

Azimuthal isotropy of the cross section (10) permits the further simplification of the kinetic equation (9):

$$\frac{\partial}{\partial \xi} I(\theta, \varphi, \xi) = \tag{11}$$

$$= n_s \sigma_\phi \int_{-\pi}^{\pi} \left[ I(\theta, \varphi - \phi, \xi) - I(\theta, \varphi, \xi) \right] d\phi.$$

The integration of the second term in the right-hand side of the equation gives the factor $2\pi$, and the integration of the first term leads to azimuthal averaging of the radiation intensity for the given $\theta$. So,

$$\frac{\partial}{\partial \xi} I(\theta, \varphi, \xi) = -2\pi n_s \sigma_\phi \left\{ I(\theta, \varphi, \xi) - \langle I(\theta, \varphi, \xi) \rangle_\varphi \right\}, \tag{12}$$

where

$$\langle I(\theta, \varphi, \xi) \rangle_\varphi = \frac{1}{2\pi} \int_{-\pi}^{\pi} I(\theta, \varphi, \xi)\, d\varphi.$$

Let us rewrite the kinetic equation (12) in the finite-difference form:

$$I(\theta, \varphi, \xi + \Delta\xi) = I(\theta, \varphi, \xi) - \tag{13}$$

$$-\Delta\xi \cdot 2\pi n_s \sigma_\phi \left\{ I(\theta, \varphi, \xi) - \langle I(\theta, \varphi, \xi) \rangle_\varphi \right\}.$$

Equation(13) demonstrates that the evolution of the intensity distribution happens as follows: the initial intensity is weakened by the factor $(1 - \Delta\xi \cdot 2\pi n_s \sigma_\phi)$ at every step $\Delta\xi$ along the fiber, but at the same time for every value of the angle $\theta$ the azimuthal symmetrical "ring" (proportional to the average intensity for the given $\theta$) is added to the existing intensity. So, the initial distribution with the shape of a sharp peak directed under the angle $\psi$ to our fibers gradually transforms into the ring of the radius $\psi$ in the space of angles. This behavior is illustrated by Fig. 3 where the evolution of the initial intensity distribution of Gaussian form

$$I(\theta_x, \theta_y) = I_0 \exp[-((\theta_x - \psi)^2 + \theta_y^2)/2\sigma^2], \tag{14}$$

(where $\theta_x = \theta\cos\varphi$, $\theta_y = \theta\sin\varphi$, $\sigma$ is the dispersion) is presented. The evolution had been simulated numerically using the finite differences equation (13).

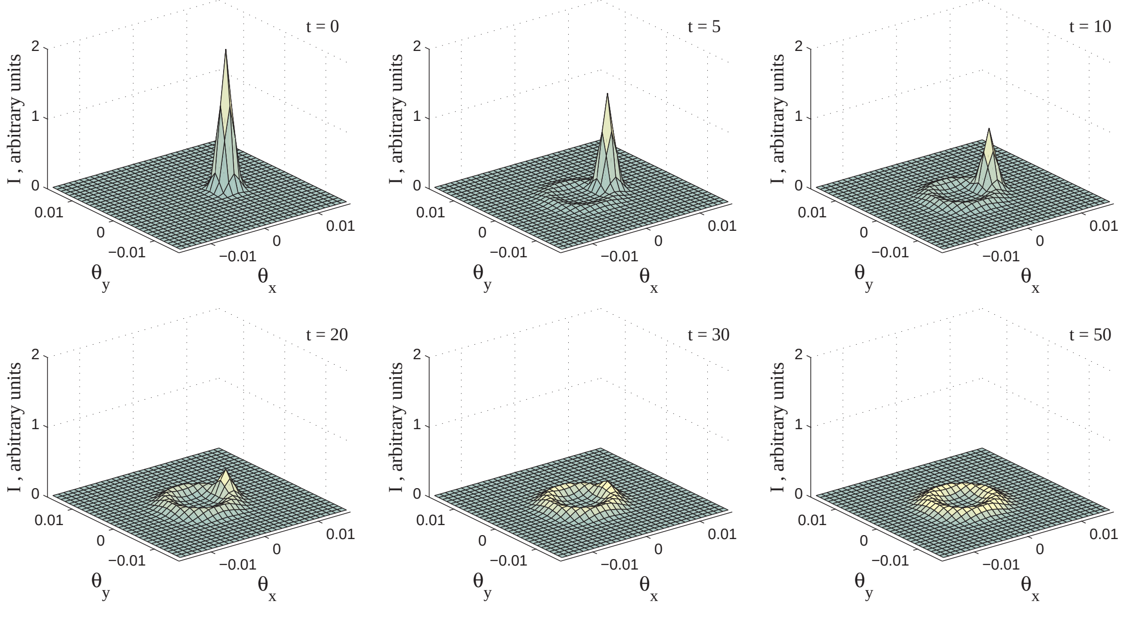


***Fig.3.*** *Evolution of the initial intensity distribution (14) with the dispersion* $\sigma = 0.001$ *and the angle of incidence* $\psi = 0.005$ *during propagation along the rope of straight fibers for the depths* $\Delta\xi \cdot t$*, where* $t = 0, 5, 10, 20, 30, 50$. *Coordinate step is* $\Delta\xi = 0.1/2\pi n_s \sigma_\phi$*, the direction of the fibers corresponds to the angles* $\theta_x = \theta_y = 0$

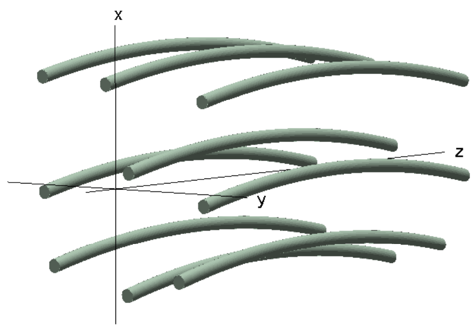


***Fig.4.*** *The rope of bent fibers; the initial wave propagates along the fibers*

The finite differences equation (13) permits to investigate the evolution of the radiation intensity under propagation in the smoothly bent rope. Our approach is similar to the so-called split operator method used for numerical solution of the time-dependent Schrödinger equation (see, e.g., [7]). Namely, the azimuthal scattering around the current direction of the fibers on the small part of the rope according to the finite-difference formula (13) is alternated with the changing the direction of the fibers for the given value.

Let us consider the rope bent in the plane $(x, z)$ by the constant angle $\alpha$ per each step $\Delta\xi$ along the rope (Fig. 4). The initial distribution $I(\vartheta_x, \vartheta_y)$ is described by the Gaussian function of the form (14) with the angles $\vartheta_x$ and $\vartheta_y$ referenced from the direction of the fixed axis $z$ as arguments, in contrast to the case of the straight rope considered above. Let the maximum of the initial distribution coincides with the local direction of the fibers.

The results of simulation of the evolution of that initial distribution are presented on Fig. 5; the rope bent per every step $\Delta\xi$ is $\alpha = 0.0002$. We see that along with the significant spreading of the angular distribution the drift (shift of the maximum of the distribution) following the rope bent. So the conditions under which the rope “leads” the radiation after its bent are possible.

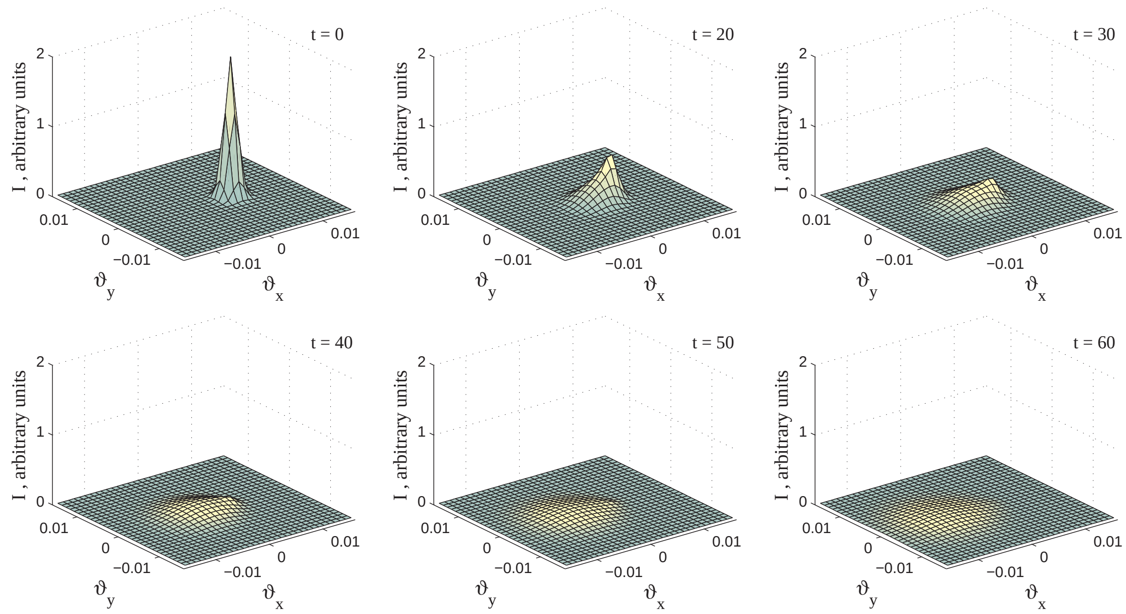


***Fig.5.*** *Intensity distribution for the radiation propagating in the bent rope (Fig. 4) for the distances* $\Delta\xi \cdot t$ *along the rope, where* $t = 0, 20, 30, 40, 50, 60$*; the coordinate step* $\Delta\xi$ *is the same as on Fig.3. The position of the maximum for* $t = 0$ *coincides with the local direction of the fibers axes*

## 4. CONCLUSIONS

The scattering of the electromagnetic wave under its oblique incidence on the linear string of charges is considered. The condition of validity of the approximation of the uniform fiber for that string is found. The scattering in the last case would happen only in the directions making the same angle with the fiber axis as the incidence angle. In the specific case of the small angle of incidence and the infinitely thin fiber the scattering cross section would possess the azimuthal symmetry around the fiber axis.

The cross section obtained is used for constructing the kinetic equation that describes the multiple scattering of the radiation in the system of parallel fibers (rope). The numerical procedure that simulates the wave propagation in the smoothly bent rope based on the finite difference form of the kinetic equation is developed. The existence of the conditions under which the maximum of the radiation angular distribution drifts following the rope bent is demonstrated.

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## РАССЕЯНИЕ ЭЛЕКТРОМАГНИТНОЙ ВОЛНЫ НА ДИЭЛЕКТРИЧЕСКОЙ НИТИ И СИСТЕМЕ ПАРАЛЛЕЛЬНЫХ НИТЕЙ

***Н. Ф. Шульга, В. В. Сыщенко***

Рассмотрено рассеяние плоской электромагнитной волны на нитевидной мишени. Полученное выражение для сечения рассеяния использовано при построении кинетического уравнения, описывающего распространение волны в пучке параллельных нитей. Рассмотрена эволюция интенсивности излучения, распространяющегося в пучке прямых и изогнутых нитей.

## РОЗСІЯННЯ ЕЛЕКТРОМАГНИТНОЇ ХВИЛІ НА ДІЕЛЕКТРИЧНІЙ НИТЦІ ТА СИСТЕМІ ПАРАЛЕЛЬНИХ НИТОК

***М. Ф. Шульга, В. В. Сищенко***

Розглянуто розсіяння плоскої електромагнитної хвилі на нитевидної мішені. Отриманий вираз для перерізу розсіяння використано для побудови кінетичного рівняння, що описує розповсюдження хвилі у пучку паралельних ниток. Розглянуто еволюцію інтенсивності випромінювання, що розповсюджується у пучку прямих та вигнутих ниток.