PARAMETRIZATION OF PROMPT NEUTRON YIELDS FROM PHOTOFISSION FRAGMENTS OF ACTINIDE NUCLEI FOR THE GIANT DIPOLE RESONANCE ENERGY RANGE

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The dependence of prompt neutron yield on fragment mass $\nu(A)$ of photofission of actinide nuclei ^{232}Th, ^{235}U and ^{238}U in the giant dipole resonance energy range has been parameterized. This allows us to describe the observed changes of saw-tooth behavior of neutron yield from the light and heavy fragments using few energy and nucleon composition dependent free parameters and to predict $\nu(A)$ for Pu isotopes.

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1. INTRODUCTION

Information about the dependence of prompt neutrons yield on fission fragments mass $\nu(A)$ plays an important role for both quantitative understanding of the dynamics of the fission process and a number of practical applications. Direct measurement of $\nu(A)$ is very difficult. Historically, the experimental data $\nu(A)$ and model descriptions exist for spontaneous fission of $^{244,248}Cm$, ^{252}Cf [1] and neutron fission of ^{233}U [2], ^{235}U [3], ^{237}Np [4], ^{239}Pu [5].

However, new challenges associated with the development of technologies for closed nuclear fuel cycle, hybrid reactors, simulation of radiation shielding of accelerators and nuclear material monitoring [6-9] require data on multiplicity of prompt neutrons after photofission of wide class of actinide nuclei for the giant dipole resonance energy range, up to $30\,MeV$.

The same data are also used to obtain the mass and charge distribution of actinide nuclei fission products and to convert the secondary fragments (products) yields into the primary ones. Typically, to estimate the number of neutrons $\nu(A)$, emitted by corresponding fission fragments of atomic mass A the phenomenological Wahl method [10] is applied, which is widely used till present for estimation of $\nu(A)$ during neutron- and gamma-induced fission [11, 12]. However, this method does not reflect the complex structure of sawtooth-like $\nu(A)$ dependence due to nuclear shells effect.

So far there is only information about the average number of neutrons emitted by two conjugated fragments and there is no information on photofission neutron emission curves. Primarily, this is due to experimental difficulties of time-span or direct neutron measurements in photofission experiments. However, in principle, neutron emission curves can be obtained combining the measurement of fragments mass distribution and post-fission neutrons.

Calculations of $\nu(A)$ for ^{238}U photofission [13] based on the parameters of asymmetric fragments mass distribution and the average total neutron yield $\langle\nu_{tot}\rangle$ provide only qualitative, approximate reflection of saw-tooth behavior (see dash-dot line in Figs.2 and 4).

There is, however, another, more advanced method to determine the neutron emission curves $\nu(A)$, based only on the mass distribution of fission fragments and products using the so-called Terrell technique [14]. This method was used to calculate prompt neutron yields $\nu(A)$ at photofission of ^{232}Th, ^{235}U and ^{238}U [15, 16], based on technique [14, 17] and data from the primary [18, 19] and secondary [20, 21] mass distributions of photofission fragments.

The question is whether it is possible to identify some of the basic laws using the results of calculations depending on such parameters of actinides photofission as charge, mass, photon energy, shell characteristics. According to the results, one can parameterize neutron yields depending on the mass of actinide photofission fragments and other parameters that would reproduce the complex structure of sawtooth behavior and allow to predict the dependence of $\nu(A)$ for the photofission of a wide class of actinide nuclei. This is the task of extreme interest.

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2. PARAMETRIZATION OF $\nu(A)$ AND RESULT OF CALCULATION

A recent analysis [22, 23] of experimental data on neutron yields from fragments of thermal neutron fission of ^{233}U, ^{235}U, ^{239}Pu and spontaneous fission of ^{252}Cf showed that for a detailed account of "saw-tooth" particularity of dependence of fission neutron yield on the mass the efficient tool is the value of $R(A)$, introduced by Wahl [24], which is defined as

$$R(A) = \frac{\nu_{L,H}(A)}{\nu_F(A)}\,, \qquad (1)$$

where $\nu_{L,H}(A)$ - prompt neutron yield of light and heavy fragment mass respectively, $\nu_F(A)$ - total neutron yield, A - fragment mass and consists of several segments to reflect the observed features, depending on the complexity of the experimental behavior of $R(A)$. Therefore, the whole range of fragments mass was divided for more then 2 x 4 segments (compare with solid broken line in Fig.4.2.39 [25]).

We have known the experimental values of $\nu_{L,H}(A)$ and $\nu_F(A)$, where A_H is the heavy fragment mass. Values $\nu_F(A_L)$, where A_L is the light fragment mass, are the reflected points to the values of $\nu_F(A_H)$. "Experimental" values of $R(A)$ (with errors) can be determined using formula (1) from experimental values of $\nu_{L,H}(A)$, $\nu_F(A_H)$ and $\nu_F(A_L)$.

Model function $R(A)$ is chosen as a linear function for each segment:

$$R^L{}_i(A) = a^L{}_i + b^L{}_i(A - A_{HN})\,, \qquad (2)$$

$$R^H{}_i(A) = 1 - R^L{}_i(A - A_{HN}) \qquad (3)$$

for light and heavy fragments, respectively, i - number of the segment, $a^L{}_i$, $b^L{}_i$, A_{LN} - parameters, $A_{HN} = A_f - A_{LN}$, A_f - mass of compound nucleus.

The total number of parameters can be significantly reduced taking into account the boundary conditions. Thus the phenomenological analysis of fission of ^{233}U, ^{235}U, ^{239}Pu and ^{252}Cf showed that the chosen parameterization and established regularities allow to describe observed change of saw-tooth behavior of neutron yield from light and heavy fission fragments [23]. Based on the established laws the possible neutron emission curves for $^{237}Np(n,f)$ fission were predicted.

To parameterize photo-neutron emission and identify general prediction patterns we will act in a similar way. For this we use the well-known results of $\nu(A)$ calculation for the photofission of ^{232}Th, ^{235}U and ^{238}U at bremsstrahlung boundary energies of 12...30 MeV [15, 16].

However, prompt neutrons emission curves obtained from the calculations in [15, 16] is not precise enough (Figs. 2-4 here) to describe all the observed features of R(A) functions built using these data. So there is no sense to try to describe them adequately using complicated function with more than 2×2 segments. Here, using the same methodology we simulate the behavior of neutrons from photofission of ^{232}Th, ^{235}U and ^{238}U actinides depending on the energy and nucleon composition in the giant dipole resonance energy range.

Let us simplify this task and divide the mass interval of light and heavy fragments for the function $R(A)$ into two segments. The number of unknown parameters is also reduced while keeping the model efficiency. As a result, we get the following picture (Fig.1).

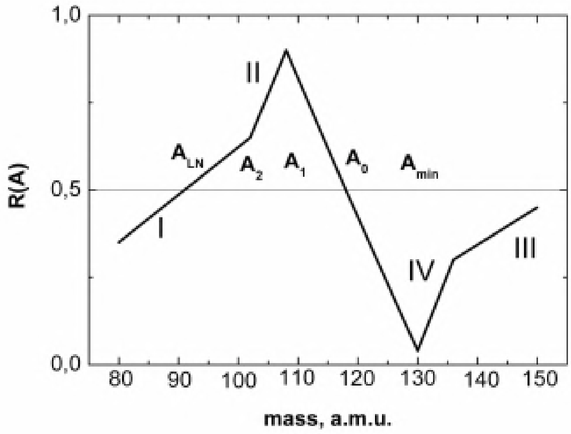


Fig.1. *An example of R(A) function. The segments I-II and III-IV correspond to our parameterization*

Let us consider some features of $R(A)$ function. At the point of symmetric fission $A_0 = A_F/2$; $R(A_0) = 0.5$ and $R(A_{LN}) = 0.5$ [23], where A_{LN} is determined from fitting.

Kink points A_{min}, A_1 and A_2 are chosen from physical considerations and experiment: $A_{min} = 130$ corresponds to the mass of nearly magic nucleus fragment associated with spherical shells $Z = 50$ and $N = 82$, where fission neutron yield is minimal. Then maximum neutron fission yield for light fragments will match point A_1, which is symmetrical to A_{min} relative to A_0, $A_1 = 2A_0 - A_{min}$.

The kink point A_2 corresponds to the average fragment mass of light fragments, $A_2 = \langle A_L\rangle = A_F - \langle A_H\rangle$, where $\langle A_H\rangle = 138$ [26-28].

The parameters a_i, b_i and A_{LN} are determined by calculating the function $R(A)$ for 4 segments I...IV (see Fig.1). The number of free parameters can be reduced using the conditions

$$a_1 = R(A_{LN}) = 0.5\,, \qquad (4)$$

$$a_2 = a_1 + (b_1 - b_2)(A_2 - A_{LN})\,. \qquad (5)$$

The dependence of b_i slopes on excitation energy is noticed on the figure for $R(A)$, so to study this dependence we have chosen $b_i = x_i + yE_\gamma$. Thus, we need to determine A_{LN}, b_i, x_i and y parameters for several hundred "experimental" points.

The value of A_{LN} varies significantly with changes of actinide mass, at least for neutron-induced actinide fission (Fig.4) [23]. Therefore, we chose a similar parameterization

$$A_{LN} = -80 + B(A_0)\,. \qquad (6)$$

To take even-even and even-odd effect into account we introduce the factor

$$1 - c(-1)^N \,. \tag{7}$$

As a result bi slopes will look as

$$b_j = (x_j + yE_\gamma)[1 - c(-1)^N]\,,$$
$$j = 1.2, \quad N = A_F - Z_F\,. \tag{8}$$

We calculate the function $R(A)$ for the photofission of actinides ^{232}Th, ^{235}U and ^{238}U according to (1)-(10) by fitting of 406 "experimental" values of $R(A)$.

Using the least squares method 5 parameters were defined to satisfactorily describe the characteristic "saw-tooth" behavior of prompt neutrons from the photofission of actinides with $A = 232 ... 238\,a.m.u.$

Calculated parameters of $R(A)$ function are:

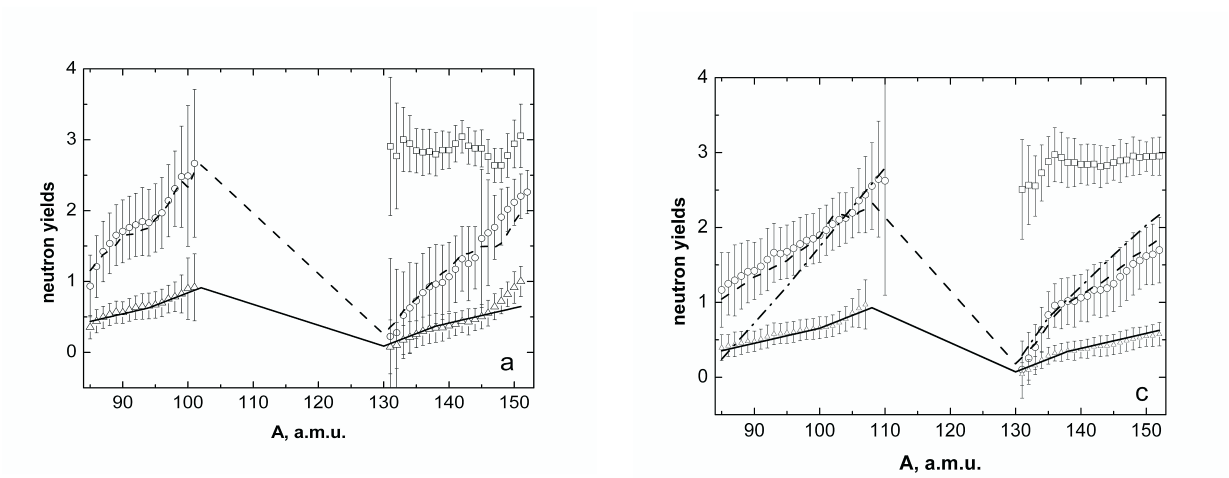
Parameter	Value	Error
x_1	$2.69 \cdot 10^{-2}$	$0.62 \cdot 10^{-2}$
x_2	$3.86 \cdot 10^{-2}$	$0.69 \cdot 10^{-2}$
y	$-0.108 \cdot 10^{-2}$	$0.047 \cdot 10^{-2}$
c	$0.215 \cdot 10^{-2}$	$0.061 \cdot 10^{-2}$
B	1.45	0.04

The results of $R(A)$ calculation are shown in Figs. 2-4 (solid curve).

The curves for prompt neutrons yield $\nu_{L,H}(A)$ can be calculated with help of formula (1).

The results of $\nu_{L,H}(A)$ calculation are shown in Figs. 2-4 (dashed curve). As can be seen from the figures the calculated values for prompt neutrons yield are everywhere within the errors.

These observations allow to estimate the possible values of fission neutrons yield from light and heavy fragments with known total yields just through the mass distributions of fission fragments using the modified Terrell method [14].



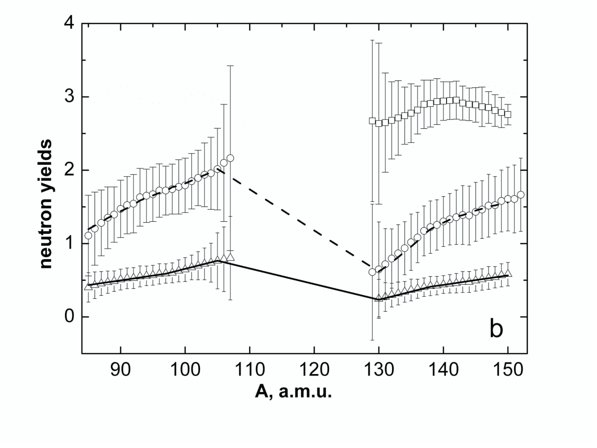
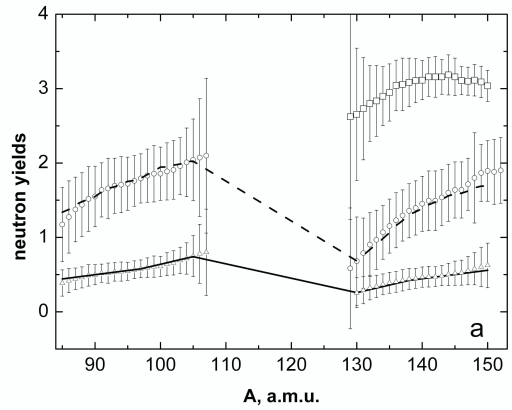
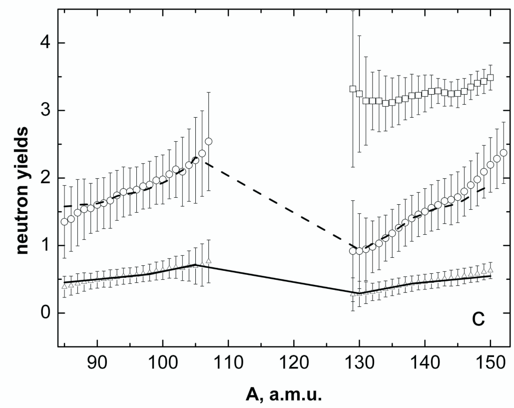


Fig.2. *Results of calculation of $\nu_{L,H}(A)$ (dashed line) and $R(A)$ function (solid curve) of a) ^{232}Th, b) ^{235}U and c) ^{238}U photofission with bremsstrahlung maximum energy $12\,MeV$. Dash-dot line - Brosa calculation [13]. Squares - $\nu_F(A)$, circles - $\nu_{L,H}(A)$, up triangles - $R(A)$*





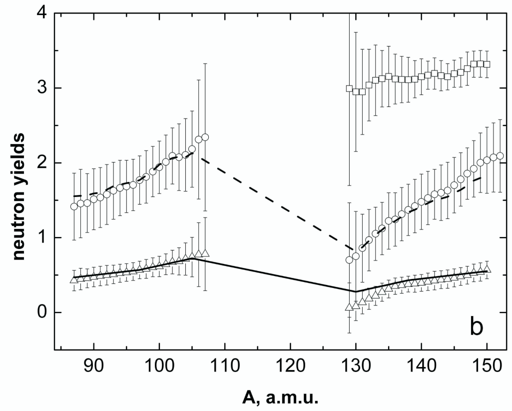
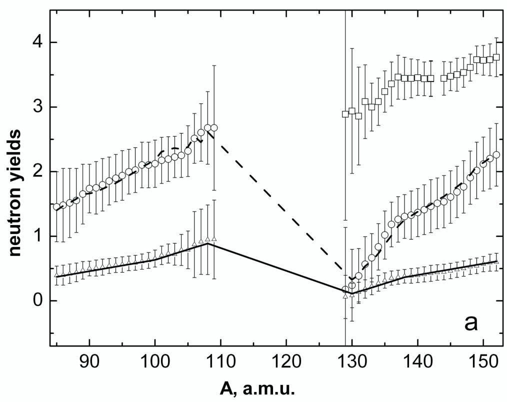
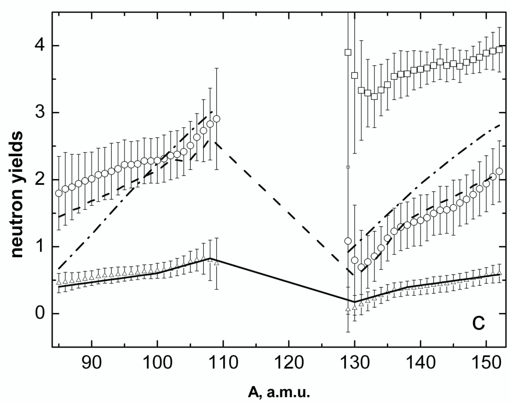


Fig.3. *Results of calculation of $\nu_{L,H}(A)$ (dash line) and $R(A)$ function (solid curve) of ^{235}U photofission with bremsstrahlung maximum energy of a) 15, b) 20 and c) $30\,MeV$. Squares - $\nu_F(A)$, circles - $\nu_{L,H}(A)$, up triangles - $R(A)$*





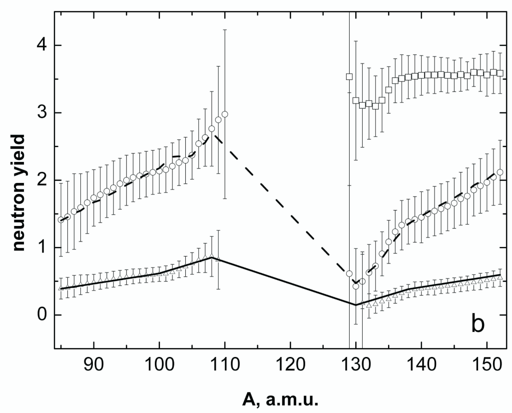


Fig.4. *Results of calculation of $\nu_{L,H}(A)$ (dash line) and $R(A)$ function (solid curve) of ^{238}U photofission with bremsstrahlung maximum energy of a) 15, b) 20 and c) $30\,MeV$. Dash-dot line - Brosa calculation [13]. Squares - $\nu_F(A)$, circles - $\nu_{L,H}(A)$, up triangles - $R(A)$*

With $R(A)$ function parameterization, which fairly well reproduces the characteristic features of its behavior, we can calculate the expected values of prompt neutrons yield for other actinides, such as ^{239}Pu or ^{240}Pu. Calculation results are shown in Fig.5.

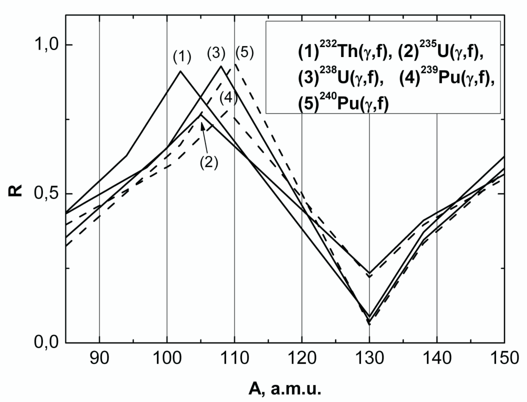


Fig.5. *The result of $R(A)$ functions calculation for ^{232}Th, ^{235}U, ^{238}U photofission (solid curve), and predictions for ^{239}Pu and ^{240}Pu photofission (dotted line) at the bremsstrahlung maximum energy* $12\,MeV$

3. CONCLUSIONS

Thus, to determine the photofission yield for these actinides we need to know the value $\nu_F(A)$, which is determined by the general empirical formula, for example [27], or with its experimental values, calculated on the basis of the mass distributions of fission products [30]. This technique allows fast and efficient calculation of photofission yield for not very heavy nuclei in the excitation energy range up to 2n threshold, avoiding cumbersome calculations by Terrell method [14].

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ПАРАМЕТРИЗАЦИЯ ВЫХОДОВ МГНОВЕННЫХ НЕЙТРОНОВ ИЗ ОСКОЛКОВ ФОТОДЕЛЕНИЯ АКТИНИДОВ ДЛЯ ОБЛАСТИ ЭНЕРГИЙ ГИГАНТСКОГО ДИПОЛЬНОГО РЕЗОНАНСА

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Выполнена параметризация зависимости выходов мгновенных нейтронов от массы осколков $\nu(A)$ фотоделения актинидных ядер ^{232}Th, ^{235}U и ^{238}U для области энергий гигантского дипольного резонанса. Это позволяет описать наблюдаемые изменения пилообразного поведения выхода нейтронов из легкого и тяжелого осколков небольшим числом свободных параметров в зависимости от энергии и нуклонного состава и предсказать $\nu(A)$ для изотопов Pu.

ПАРАМЕТРИЗАЦІЯ ВИХОДІВ МИТТЄВИХ НЕЙТРОНІВ З УЛАМКІВ ФОТОПОДІЛУ АКТИНІДІВ ДЛЯ ОБЛАСТІ ЕНЕРГІЙ ГІГАНТСЬКОГО ДИПОЛЬНОГО РЕЗОНАНСУ

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Виконана параметризація залежності виходів миттєвих нейтронів від маси уламків $\nu(A)$ фотоподілу актинідних ядер ^{232}Th, ^{235}U і ^{238}U для області енергій гігантського дипольного резонансу. Це дозволяє описати спостережувані зміни пилкоподібної поведінки виходів нейтронів з легкого і важкого уламків невеликим числом вільних параметрів у залежності від енергії та нуклонного складу, а також передбачити $\nu(A)$ для ізотопів Pu.