ON POSSIBLE STATIONARY EQUILIVRIUM OF DUSTY SELF-GRAVITATING SPACE PLASMAS

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INTRODUCTION

Cosmic dust is a common component of many astrophysical objects. Prime examples in the solar system are noctilucent clouds, cometary tails and comae, circumsolar and planetary dust rings. Other occurrences are in the asteroid belt and interstellar dust clouds. The dust grains in space come in sizes ranging from macromolecules to micron-sized grains and even to rock fragments and asteroids. When dust particles are immersed in plasmas and radiative environments, they inevitably become charged (usually negatively because of the greater mobility of electrons) and thus contribute to the plasma collective effects as a separate species. These particles have much larger masses than those of the plasma ions, but much smaller charge-to-mass ratios. The combination of charged dust and plasma is referred to as a dusty plasma.

For certain dusty plasmas containing rather heavy charged grains, it is assumed that the gravitational intergrain interactions could become important and the name a self-gravitating plasma is more appropriate. When self-gravitating effects are incorporated, the dusty plasma contains a highly diverse range of collective modes, unstable behavior, and linear and nonlinear waves [1]. Although self-gravitating dusty plasmas have received wide attention in recent years, for most of the considered collective phenomena, a homogeneous equilibrium state is assumed to exist. What is implied by a homogeneous equilibrium? In ordinary plasmas the model is an infinite system with vanishing electric fields in the zeroth order. The situation is essentially different when self-gravitational forces are included in the analysis. Since gravitation, contrary to the electromagnetic forces cannot be shielded, but is always attractive, gravitational collapse of extended regions with distributed masses is inevitable (Jeans instability), unless counteracted upon by pressure or other effects. In fact, there is no way to make the gravitational potential disappear as the mass densities do not satisfy any condition analogous to charge quasineutrality. This implies that a truly homogeneous equilibrium is impossible, and even more so an infinite one. Hence, it is physically relevant to investigate basic stationary nonuniform states in self-gravitating plasmas, and clarify the problem how heavy dust species influence the plasma equilibrium state, especially keeping in mind that the physics of self-gravitating dusty plasmas is becoming increasingly relevant in determination of the macroscopic behavior of extended systems in astrophysical scenarios.

Attempts to investigate a fundamental problem of stationary equilibria for dusty self-gravitating plasmas were recently made by several groups independently and almost simultaneously [2,3] . Physically, both approaches are considered the stationary state invoking the charge quasineuntallity of dusty plasmas. Although this condition is somewhat restrictive and limits the possible number of self-consistent equilibria, nonlinear equilibrium structures of hot quasineutral self-gravitating plasmas without and with uniform rotation of the dust were considered by Tsintsadze *et al*. [3]. The consequences of small perturbations were also studied. On the other hand, the equilibria by avoiding the usual Jeans swindle were considered for one-dimensional, cylindrical as well as spherical symmetrical cases [2] The stationary state was governed by a nonlinear differential equation for the inhomogeneous and stationary plasma flow, which has a singularity at the dust-acoustic velocity. The singularity is a manifestation of the equilibrium gravitational potential, which is inhomogeneous.

In the present paper we give a fully consistent analysis of the stationary state of a self-gravitating dusty plasma, rejecting the Jeans swindle assumption, as has been done by Rao [2]. But contrary to the paper [2], we consider the closed set of exact equations, including the Poisson equation for the electrostatic potential instead of using the charge quasineuntallity condition. In Cartesian geometry a mathematically correct nonlocal treatment can be performed in one dimension only, because there is no additional direction, along which the medium could be assumed strictly uniform. Moreover, in this model the computations can be done explicitly, for the stationary state. The considered equilibrium is obtained for a plasma with zero plasma flows.

GENERAL FORMALISM

We consider an infinite, unmagnetized and hot self-gravitating plasma. Then the plasma species are governed by the standard set of fluid equations

$$\frac{\partial v_\alpha}{\partial t} + v_\alpha \frac{\partial v_\alpha}{\partial x} + \frac{q_\alpha}{m_\alpha}\frac{\partial \psi_E}{\partial x} + \frac{\partial \psi_J}{\partial x} + \frac{v_{T\alpha}^2}{n_\alpha}\frac{\partial n_\alpha}{\partial x} = 0,$$

$$\frac{\partial n_\alpha}{\partial t} + \frac{\partial}{\partial x}\left(n_\alpha v_\alpha\right) = 0. \tag{1}$$

Two Poisson equations relate the electrostatic ψ_E and the gravitational ψ_J potentials to the particle densities n_α

$$\nabla^2 \psi_E = -4\pi \sum q_\alpha n_\alpha \ , \tag{2}$$

$$\nabla^2 \psi_J = 4\pi G \sum m_\alpha n_\alpha \; . \tag{3}$$

Here q_α and m_α denote the particle charges and masses, v_α and $v_{T\alpha} = \sqrt{T_\alpha / m_\alpha}$ the fluid and thermal velocities, for the particles of species α, G is the gravitational constant.

We are now looking for a stationary equilibrium, letting $\partial / \partial t = 0$. Then the continuity equations (2) give the conservation of the particle fluxes $n_\alpha v_\alpha = J_\alpha = const$. At the same time, the momentum equations (2) lead to the conservation of the total energy, consisting of the kinetic, electrostatic, thermal and gravitational components

$$\frac{v_\alpha^2}{2} + \frac{q_\alpha}{m_\alpha}\psi_E + \psi_J + v_{T\alpha}^2 \ln \frac{n_\alpha}{n_{0\alpha}} = 0 \,, \tag{4}$$

where J_α and $n_{0\alpha}$ are integration constants.

The simplest case that is compatible with the basic equations, assuming zero fluxes $v_{0\alpha} = J_\alpha = 0$. Solving (4), one gets the Boltzmann relation for the particle densities

$$n_\alpha = n_{0\alpha} \exp\left(- \frac{q_\alpha \psi_E + m_\alpha \psi_J}{T_\alpha} \right). \tag{5}$$

In order to have a reasonable model for some of the important dusty plasmas, we represent the charged dust by a limited number of discrete species, considering two species $\alpha = 1,2$ with different masses and other characteristics. Correspondingly, we introduce two dimensionless potentials as

$$\psi_\alpha = \frac{q_\alpha \psi_E + m_\alpha \psi_J}{T_\alpha} \,,$$

which obey two coupled nonlinear equations obtained by substitution of (5) into the Poisson equations (2) and (3):

$$\frac{\partial^2 \psi_1}{\partial x^2} = a n_{01} \exp(-\psi_1) + b n_{02} \exp(-\psi_2) \,,$$

$$\frac{\partial^2 \psi_2}{\partial x^2} = c n_{01} \exp(-\psi_1) + d n_{02} \exp(-\psi_2) \,. \tag{6}$$

Parameters a, b, c and d are completely specified by charges, masses and temperatures of the plasma particles

$$a = \frac{4\pi}{v_{T1}^2}\left(G m_1 - \frac{q_1^2}{m_1} \right), \; b = \frac{4\pi}{v_{T1}^2}\left(G m_2 - \frac{q_1 q_2}{m_1} \right),$$

$$c = \frac{4\pi}{v_{T2}^2}\left(G m_2 - \frac{q_2^2}{m_2} \right), \; d = \frac{4\pi}{v_{T2}^2}\left(G m_1 - \frac{q_1 q_2}{m_2} \right). \tag{7}$$

To solve the pair of equations (6), let us assume that

$$\psi_2(x) = \psi_1(x) - \ln(\gamma\, n_{01}/n_{02}) \,, \tag{8}$$

where γ is a positive constant to be determined. In view of (5), the latter relation signifies a similar spatial distribution of the two species

$$n_2(x) = \gamma\, n_1(x) \,. \tag{9}$$

Substituting (8) into (6) apparently gives two nonlinear equations for the potential $\psi_1(x)$, known as equations of Emden-type [4], *viz.*

$$\frac{\partial^2 \psi_1}{\partial x^2} = (a + \gamma b) n_{01} \exp(-\psi_1)$$

$$\frac{\partial^2 \psi_1}{\partial x^2} = (c + \gamma d b) n_{01} \exp(-\psi_1) \tag{10}$$

which become identical if the adjustable parameter γ is given by

$$\gamma = \frac{c - a}{b - d} \,. \tag{11}$$

Hereby, the pair of coupled nonlinear equations (6) is reduced to a single equation for determining the stationary equilibrium state

$$\frac{\partial^2 \psi_1}{\partial x^2} = n_{01} l^2 \exp(-\psi_1) \tag{12}$$

with $l^2 = (bc - ad)(b - d)^{-1}$. It can be easily shown that the solution of (12) posses the scaling and translation invariance.

In general, the Emden equation (12) can be solved analytically for different signs of coefficient l^2 [4]. But before discussing the qualitative properties of the equilibrium solution admissible for (12), we first analyze the possible values for l^2. The latter is completely determined by the parameters of the plasma particles through relations (7) (particle charges, masses and temperatures) and in principle, l^2 can be positive as well as negative. The same is also true for the parameter γ, however this should be positive by definition. Hence, a careful discussion of different parameter ranges is to be considered.

EQUILIBRIUM CONSIDERATIONS

It is readily verified with the help of definitions (7) that $(bc - ad)$ is always positive. At the same time the difference $(b - d)$ can be positive or negative. The only way to have both γ and l^2 positive requires $(c - a)$ and $(b - d)$ to be positive simultaneously. This restricts dusty plasma parameters such that

$$G - \frac{q_1^2}{m_1^2} < \frac{v_{T1}^2}{v_{T2}^2}\left(G - \frac{q_1 q_2}{m_1 m_2} \right)$$

$$G - \frac{q_2^2}{m_2^2} < \frac{v_{T2}^2}{v_{T1}^2}\left(G - \frac{q_1 q_2}{m_1 m_2} \right). \tag{13}$$

For definiteness, let us assume $m_1 << m_2$, and $q_1 > 0$ while $q_2 < 0$, bearing in mind for example a plasma consisting of the light positive ions and heavy dust grains. Then the first inequality in (13) can be satisfied automatically. So the second condition in (13) should be regarded as a main restriction on the parameters of self-gravitating plasmas, yielding

$$G q_2^2 / m_2^2 \geq 1 \,. \tag{14}$$

The left-hand side of (14) represents the ratio of electrostatic and gravitational forces between the heavier particles. Even two equal forces can still provide the validity of (13). Therefore, the condition $l^2 > 0$ actually implies such a dusty plasma, where gravitational interactions between heavier particles are about equally important as electrostatic ones. An alternative possibility, when $l^2 < 0$ in the Emden equation (12) can be expected for the limiting case where the prevailing force that controls particle dynamics is self-gravitation. Now, the pair of conditions $(c - a) < 0$ and $(b - d) < 0$ has to be satisfied simultaneously. Without going into details, it can be shown that these inequalities can be never satisfied for any reasonable parameters of plasma particles. This indicates that the possibility for negative l^2 should be rejected in the present context.

Summarizing, it can be concluded that when l^2 is negative, there is no stationary state with proportional densities as implied by (9) for a motionless dusty plasma with dominating self-gravitating interactions. Or, alternatively, the plasma species reach a stationary state, with inhomogeneous particle flows akin to those considered by Rao *et al.* [2].

Now we return to (12), to get the first integral admissible for $l^2 > 0$

$$\left(\frac{\partial \psi_1}{\partial x}\right)^2 = -2l^2 n_{01} \exp(-\psi_1) + \beta^2, \tag{15}$$

where β enters as an arbitrary integration constant. Assuming that the potential ψ_1 and its derivative $\partial \psi_1/\partial x$ vanish for large enough x, it follows immediately that β prescribes the normalization constant densities $n_{0\alpha}$ in (5) through $\beta^2 = 2l^2 n_{01}$ and $\beta^2 = 2l^2 n_{02}/\gamma$. It is now straightforward to obtain the general solution of the Emden equation (12) as

$$\psi_1 = \ln\left[\cosh^{-2}\left(\frac{\beta x}{2}\right)\right] \tag{16}$$

yielding the particle densities (5)

$$n_\alpha = n_{0\alpha} \ln\left[\cosh^{-2}\left(\frac{\beta x}{2}\right)\right]. \tag{17}$$

Therefore the basic stationary state of a dusty plasma with heavy charged particles cannot intrinsically be uniform when self-gravitational forces between grains become important, e.g. in the spirit of (14). Accordingly to (17), the characteristic length of inhomogeneity $\Delta \sim \beta^{-1}$ and maximum amplitudes of the density profiles $n_{0\alpha}$ are determined by the same constant β. The latter can be related either to the total number of particles N or to the total charge Q or to the total mass M per unit surface in the z, y-plane

$$N = \int_{-\infty}^{\infty} n_1(x)dx + \int_{-\infty}^{\infty} n_2(x)dx = \frac{2\beta}{l^2}(1+\gamma),$$

$$Q = q_1 \int_{-\infty}^{\infty} n_1(x)dx + q_2 \int_{-\infty}^{\infty} n_2(x)dx = \frac{2\beta}{l^2}(q_1 + \gamma q_2), \tag{18}$$

$$M = m_1 \int_{-\infty}^{\infty} n_1(x)dx + m_2 \int_{-\infty}^{\infty} n_2(x)dx = \frac{2\beta}{l^2}(m_1 + \gamma m_2).$$

Fixing one of these quantities determines β, and thus all other quantities and the lengthscale Δ.

Hence, from the standard basic equations (1)-(3) it follows as a mathematically rigorous result, that a dusty plasma with motionless gravitating grains has neither global ($Q \neq 0$) nor local charge neutrality ($\gamma = n_2/n_1 \neq -q_1/q_2$) in a stationary equilibrium. This point deserves special attention.

Recalling the definition of the parameter γ (11), and using the adopted model in the case $T_1 \sim T_2$, we get

$$\gamma \cong -\frac{q_1}{q_2}\left[1 - \frac{Gm_2^2}{q_2^2 + q_1|q_2|}\right]^{-1}.$$

This highlights the transition from a quasineutral dusty plasma $\gamma = n_2/n_1 = -q_1/q_2$ (consisting of such light particles, that the enormous domination of electric forces over gravitation allows to neglect gravitational interactions) to a charged self-gravitating plasma: as the mass of the heavier component increases, the parameter γ grows also. The physical consequence of this growth of γ is the existence of a global electrostatic charge of a self-gravitating plasma (18), which has the same sign as the heavier charged particles. This global charge of dusty plasmas in the equilibrium is easily understood on physical grounds. Indeed, for the stationary equilibrium of heavy gravitating particles it is necessary to balance their mutual gravitational attraction by electrostatic repulsion due to an excess of like charges.

CONCLUSIONS

In this paper we have considered the existence of self-consistent stationary equilibrium of a dusty plasma with heavy self-gravitating particles by avoiding the Jeans swindle assumption. Starting with the respective basic equations, we have shown that the possible equilibria are governed by nonlinear coupled differential equations for the plasma densities. The latter equations can be reduced to a single Emden-type equation by assumption of the similar spatial distribution for all the plasma species.

As it can be expected, the stationary equilibrium state of a dusty plasma with heavy charged particles cannot intrinsically be uniform when self-gravitational forces between grains become important. The spatial scale of inhomogeneity and maximum amplitudes of the density profiles are determined either by the total number or the total charge, or the total mass of plasma particles.

It is shown how that presence of heavy self-gravitating charged grains can modify the main property of plasmas, namely quasineutrality, at least in the context of adopted one dimensional model of dusty plasmas.

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