MODULATIONAL INSTABILITY OF THE ELECTRON CYCLOTRON WAVES IN AN ADIABATIC WAVE - PARTICLE INTERACTION

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Modulational instability of a circularly polarized electromagnetic wave, propagation along an external constant magnetic filed in plasma is investigated. The method is based on the derivation of a nonlinear Schrodinger equation, which contains nonlinear terms associated with trapped electrons and also relativistic electron quiver velocity. Among different mechanisms of trapping the collision-less process is considered. Since we are interested in characteristic spatial scales larger than the Debye length, the condition of quasi-neutrality is assumed. The maximum growth rate is calculated and the result shows that the trapped particles prevent the spatial localization of the electron cyclotron waves.
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INTRODUCTION

The development of ultra intense short pulse lasers allows exploration of fundamentally new parameter regimes for nonlinear laser plasma interaction. Now we have laser beams with intensities up to 10^{19} W/cm^2 in laboratories, at such a high intensities electrons quiver velocity becomes relativistic. In such a situation, consideration of the relativistic electron mass variation [1] as well as the relativistic ponderomotive force [2] is very essential in the study of nonlinear laser plasma interactions.

On the other hands, propagation of high frequency waves in plasma can generate the longitudinal (potential) waves in which some of the plasma particles can be trapped (electron here). Among different mechanisms of trapping, the mechanism resulted from nonstationarity of the field is considered [3, 4]. It means that τ the characteristic time of variation of the field is small compared with the electron mean free time, so that the plasma is collisionless. Besides, we restrict ourseleves to the adiabatic approximation. That means, τ should be much larger than the period of electronic oscillations in the field trough. By virtue of this approximation, the field may be regarded as stationary during the passage of an electron through it and to the same accuracy, the electron distribution function in the field can be expressed by an adiabatic invariant. Under this approximation the electrons density can be analytically calculated as follows:

$$\frac{\langle n_e \rangle}{n_0} = e^{-\frac{U_{eff}}{T_e}}\left(1 - erf\sqrt{-\frac{U_{eff}}{T_e}}\right) + \frac{2}{\sqrt{\pi}}\sqrt{-\frac{U_{eff}}{T_e}}, \qquad (1)$$

where U_{eff} describes the potential well and T_e is the temperature of electrons. We will later explain the meaning of brackets.

In this paper, we consider the influence of trapped electrons on the modulational instability of electron cyclotron waves. The organization of the paper is as follows. First, from basic equation, an envelope equation, including trapped electrons, will be obtained. Then, it is solved for the investigation of modulational instabilities. Finally, a summary of results will be presented.

BASIC EQUATIONS

Consider the propagation of a right-handed circularly polarized electromagnetic wave along the external magnetic field $\vec{B}_0 = B_0\hat{z}$ in the form of,

$$\vec{E} = \frac{1}{2}(\hat{x} + i\hat{y})E(z,t)\exp(-i\omega t) + c.c., \qquad (2)$$

where ω is the frequency of external electromagnetic field and $c.c.$ stands for the complex conjugate.

The complete set of equations is:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\partial_t \vec{E} + \frac{4\pi}{c}\vec{J}, \qquad (3)$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\partial_t \vec{B}, \qquad (4)$$

$$\nabla \cdot E = 4\pi\rho, \qquad (5)$$

$$\rho = e(n_i - n_e), \qquad (6)$$

$$J = e(n_i V_i - n_e V_e), \qquad (7)$$

where $e = |e|$. In order to close the above systems of equations, ρ and J should be properly defined. Since we are going to study the effect of trapped electrons we assume electron temperature, T_e, is much larger that ions one. Therefore, electrons are treated kinetically [eq. (1)] while the fluid treatment is used for the ions dynamics.

In our formalism there are two distinguishable time scales: one associated with the fast motion with the characteristic time of the order of the high frequency oscillation, $\tau_p = 2\pi/\omega$, and another with the slow motion resulting from the ions motion. Therefore, each quantity can be decomposed into the slowly and rapidly varying parts:

$$A = \langle A \rangle + \widetilde{A}, \qquad (8)$$

where the brackets denote averaging over the time interval τ_p.

We consider the one-dimensional case in which $\partial_x = \partial_y = 0$. Therefore, the nonlinear interaction of a pump field with the background plasma that will generate a slowly varying envelope for the amplitude of electromagnetic wave propagating along the external magnetic field involving the weak relativistic quiver velocity is governed by [5]:

$$2i\omega\,\partial_t E + c^2\partial_{zz}E - \left(\frac{\omega\,\omega_{p_e}^2}{\omega - \Omega_e} - \omega^2\right)E$$
$$-\frac{\omega\,\omega_{p_e}^2}{\omega - \Omega_e}\left[\frac{\delta n_e}{n_0} - \frac{\omega}{\omega - \Omega_e}\frac{e^2|E|^2}{2m_e^2c^2(\omega - \Omega_e)^2}\right]E = 0, \qquad (9)$$

where c is light velocity, $\omega_{p_e}^2 = 4\pi e^2 n_0/m_e$ the electron plasma frequency, n_0 the unperturbed electron density, $\Omega_e = eB_0/m_e c$ the electron cyclotron frequency, m_e the electron rest mass and $\delta n_e = \langle n_e\rangle - n_0$.

In order to define the slowly varying deviation of the density, δn_e, taking into account the electron trapping we have to proceed from eq. (1). In eq. (1) we need the definition of U_{eff}. For this purpose one should calculate the average of Vlasov equation over the time interval τ_p to find [6]:

$$\partial_t\langle f_e\rangle + v_z\,\partial_z\langle f_e\rangle - \partial_z\left[-e\varphi + \frac{e^2|E|^2}{2m_e\omega(\omega - \Omega_e)}\right]\partial_{p_z}\langle f_e\rangle = 0, \qquad (10)$$

where f_e denotes the distribution of electrons. From here we can define the effective potential energy as follow:

$$U_{eff} = -e\varphi + \frac{e^2|E|^2}{2m_e\omega(\omega - \Omega_e)} \qquad (11)$$

We assume the effective potential energy has the form of a single potential well, vanishing at infinity (soliton solution). Then, by substituting eq. (11) in eq. (1) the density of electrons is clearly defined. As it is mentioned, the ions will also be described by the hydrodynamic equations assuming their temperature is negligibly small. Then for the shallow well when $U_{eff} << T_e$ ($\delta n_e << n_0$) and for the case when quasi-neutrality condition is fulfilled ($\delta n_e \approx \delta n_i$) one can find for the δn_e the following expression:

$$\frac{\delta n_e}{n_0} = \frac{c_s^2}{u^2 - c_s^2}\frac{e^2|E|^2}{2m_eT_e\omega(\omega - \Omega_e)} + \frac{4}{3\sqrt{\pi}}\left(\frac{c_s^2}{u^2 - c_s^2}\right)^{5/2}\left[\frac{e^2|E|^2}{2m_eT_e\omega(\omega - \Omega_e)}\right]^{3/2}, \qquad (12)$$

where u is the velocity of stationary envelope, and $c_s^2 = T_e/m_i$ (m_i =mass of ion).

Equation (12) together with equation (9) construct a complete set of equations for the investigation of modulational instabilities of electron cyclotron waves.

MODULATIONAL INSTABILITY

Now we Consider the case of propagating wave along the constant magnetic field. Let define the slowly varying amplitude as $E = \psi(z,t)\,e^{ikz}$. Substituting E in eqs. (9) and (12), we have:

$$2i\omega\,\partial_t\psi + c^2\left(\partial_{zz}\psi + 2ik\partial_z\psi - k^2\psi\right) - \left(\frac{\omega\,\omega_p^2}{\omega - \Omega_e} - \omega^2\right)\psi$$
$$-\frac{\omega\,\omega_p^2}{\omega - \Omega_e}\left[\frac{c_s^2}{u^2 - c_s^2}\frac{e^2|\psi|^2}{2m_eT_e\omega(\omega - \Omega_e)} + \frac{4}{3\sqrt{\pi}}\left(\frac{c_s^2}{u^2 - c_s^2}\right)^{5/2}\left(\frac{e^2|\psi|^2}{2m_eT_e\omega(\omega - \Omega_e)}\right)^{3/2} - \frac{\omega}{\omega - \Omega_e}\frac{e^2|\psi|^2}{2m_e^2c^2(\omega - \Omega_e)^2}\right]\psi = 0. \qquad (13)$$

To consider the effect of trapping on the modulational instabilities, we perturbed eq. (13) (with a long time and wavelength perturbation) around its plane wave solution, as follows:

$$\psi = (\psi_0 + \delta\psi)\,Exp(-i\Delta_\omega t). \qquad (14)$$

Here ψ_0 is real and Δ_ω shows the nonlinear frequency shift. On substituting eq. (14) in eq. (13) and considering its zero order (respect to $\delta\psi$), one can find the following expression for the nonlinear frequency shift

$$\Delta_{\omega} = \frac{1}{2\omega}\left(\frac{\omega\omega_p^2}{\omega-\Omega_e} - \omega^2 + k^2c^2\right) + \frac{\omega_p^2}{2(\omega-\Omega_e)}\left[\frac{c_s^2}{u^2-c_s^2}\frac{e^2}{2m_eT_e\omega(\omega-\Omega_e)} - \frac{\omega}{\omega-\Omega_e}\frac{e^2}{2m_e^2c^2(\omega-\Omega_e)^2}\right]\psi_0^2 + \frac{\omega_p^2}{\omega-\Omega_e}\frac{4}{3\sqrt{\pi}}\left(\frac{c_s^2}{u^2-c_s^2}\right)^{5/2}\left(\frac{e^2}{2m_eT_e\omega(\omega-\Omega_e)}\right)^{3/2}\psi_0^3. \tag{15}$$

The first order of eq. (13) respect to $\delta\psi$ together with eq. (15) results to the following equation for the complex amplitude of the perturbation,

$$2i\left[\omega\partial_t + kc^2\partial_z\right]\delta\psi + c^2\partial_{zz}\delta\psi - \frac{\omega_p^2}{(\omega-\Omega_e)^2}\left[\frac{c_s^2}{u^2-c_s^2}\frac{c^2}{v_T^2} - \frac{\omega^2}{(\omega-\Omega_e)^2}\right]\frac{e^2\psi_0^2}{2m_e^2c^2}\delta\psi - \frac{2}{\sqrt{\pi}}\frac{\omega_p^2}{\sqrt{\omega}(\omega-\Omega_e)^{5/2}}\left(\frac{c_s^2}{u^2-c_s^2}\right)^{5/2}\left(\frac{e^2\psi_0^2}{2m_eT_e}\right)^{3/2}\delta\psi = 0. \tag{16}$$

By dividing the $\delta\psi$ into real and imaginary parts and considering the plane wave solutions, as follows,

$$\delta\psi = M + iN, \qquad \begin{bmatrix} M \\ N \end{bmatrix} = \begin{bmatrix} \tilde{M} \\ \tilde{N} \end{bmatrix} Exp(i\chi z - i\omega t), \tag{17}$$

we can find the nonlinear dispersion relation,

$$\left(\Omega - \frac{kc^2}{\omega}\chi\right)^2 - \frac{c^4\chi^4}{4\omega^2}\left[1 - \frac{\omega^2\omega_p^2}{c^2\chi^2(\omega-\Omega_e)}\frac{e^2\psi_0^2}{m_e c^2}\right] = \frac{\omega_p^2}{(\omega-\Omega_e)^2}\frac{\chi^2c^2}{4\omega^2}\frac{c_s^2\chi^2}{\Omega^2-c_s^2\chi^2}\frac{c^2}{v_T^2}\frac{e^2\psi_0^2}{m_e^2c^2} + \frac{\chi^2c^2\omega_p^2}{\sqrt{\pi}\,\omega^{3/2}(\omega-\Omega_e)^{5/2}}\left(\frac{c_s^2\chi^2}{\Omega^2-c_s^2\chi^2}\right)^{5/2}\left(\frac{e^2\psi_0^2}{2m_eT_e}\right)^{3/2}. \tag{18}$$

Among different possible cases, we consider the case when

$$\Omega = \frac{kc^2}{\omega}\chi + \delta\Omega, \qquad \left(\delta\Omega << \frac{kc^2}{\omega}\chi\right). \tag{19}$$

The maximum value of the growth rate is achieved at $\chi = \chi_m$, where,

$$\frac{c^2\chi_m^2}{\omega^2} = \frac{\omega_p^2}{2\omega(\omega-\Omega_e)}W^2\left[1 - \frac{1}{\omega(\omega-\Omega_e)}\frac{c^2}{v_T^2}\left(\frac{c_s^2}{\left(\frac{kc^2}{\omega}\right)-c_s^2}\right)\left\{1 + \frac{4}{\sqrt{\pi}}\sqrt{\frac{\omega}{\omega(\omega-\Omega_e)}\frac{v_T^2}{c^2}}\left(\frac{c_s^2}{\left(\frac{kc^2}{\omega}\right)-c_s^2}\right)^{3/2}W\right\}\right], \tag{20}$$

and it is equal to

$$\mathrm{Im}\,\delta\Omega_{Max} = \frac{1}{2}\frac{c^2\chi_m^2}{\omega^2}. \tag{21}$$

From analysis of growth rate we can see the term causes by the trapped particles has a negative sign so we need a harder restriction for the amplitude of the homogeneous wave.

CONCLUSION

In this paper we considered the propagation of electron cyclotron waves, propagating along an external constant magnetic field in plasma. We included the adiabatic trapping of the electrons in the longitudinal field. As it is mentioned, according to the existence of two natural time scales in the plasma, an envelope equation has been found for the slowly varying part of the wave amplitude. It is contained both the trapping of electrons and their relativistic mass variations.

It was shown that this system is modulationally unstable, but the trapped electrons have a stabilizing influence. It means that at the presence of trapped electrons for the creation of nonlinear structures, e.g. solitons, we will encounter with harder conditions.

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