STATIONARY DYNAMICS OF NONLINEAR ELECTRON-CYCLOTRON WAVES AT THE PRESENCE OF RESONANT ELECTRONS

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The influence of resonant electrons in phase space on the nonlinear dynamics of electron-cyclotron waves is investigated. It is shown that in the case when the frequency of externally applied electromagnetic waves is close to that of the electron-cyclotron waves, the nonlinear coupling between the electron-cyclotron waves and the resulting longitudinal perturbations is mainly caused by the pondermotive potential of the electrons. It is shown that in the competition of refraction caused by the non-resonant electrons and the relativistic nonlinearity, the influence of resonant electrons will increase strongly. The reaction of such longitudinal waves with electrons that are in resonance with high frequency electromagnetic field changes the character of the soliton propagation in an essential manner.
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INTRODUCTION

Circularly polarized electromagnetic waves are useful for heating of fusion plasma [1], modifications of the lower part of the earth's ionosphere [2]. Accordingly, it is essential to understand the linear as well as the nonlinear propagation characteristics of cyclotron waves. The linear propagation of electron cyclotron waves is fully understood and the results have been summarized in a monograph [3]. The nonlinear propagation involves the study of three wave interaction, wave modulation, self-filamentation, soliton formation, etc.

In the past, there have been substantial efforts [4] in the investigating of electron cyclotron waves. However, most of the studies were concerned with the study of small amplitude case. Nevertheless, the development of ultraintense short pulse lasers allows exploration of fundamentally new parameter regimes for nonlinear laser plasma interaction. In fact, a number of experiments have been carried out in which plasmas are irradiated by laser beams with intensities up to $10^{19} W/cm^2$. At such intensities, the electron quiver velocity approaches the speed of light and a host of phenomena have been predicted such as parametric resonance in an electron plasma [5], the relativistic self focusing [6] and the generation of large amplitude plasma waves (wake field) [7].

Among the nonlinear phenomena, trapping of plasma particles in the trough of low frequency longitudinal waves of plasma exert considerable influence on the propagation characteristics of such waves.

Determination of collisionless distribution function and its moments in nonstationary fields, which describes both free and trapped particles, involves difficult mathematics demanding simplifying assumptions. Gurevich [8], assuming slowly varying longitudinal fields and a collisionless trapping mechanism, calculate the particle distribution function self-consistently. The validity of his solution is demonstrated by simulation and experimental results of Debye shielding and antishielding [9]. When applied field changes slowly, the distribution function is a continuous function of total energy. Then the distribution of untrapped particle is determined by that of particles coming from infinity with a Maxwellian distribution. It then follows from the continuity (continuity of particles current from the trapped to untrapped region) conditions that trapped particle disteribution reduces to a constant. In this way, integrating over the free and trapped particle distributions results in the following total density:

$$\frac{\langle n_\alpha \rangle}{n_0} = e^{-\frac{U_{eff}}{T_\alpha}}\left(1 - erf\sqrt{-\frac{U_{eff}}{T_\alpha}}\right) + \frac{2}{\sqrt{\pi}}\sqrt{-\frac{U_{eff}}{T_\alpha}}, \qquad (1)$$

where U_{eff} describes the potential well and T_α is temperature of particles (α =species index; e,i). We will later explain the meaning of brackets.

In this paper, we consider a class of problems involving the interaction of relativistically intense monochromatic radiation with a magnetized plasma when the effect of trapped electrons is taken into account. To our knowledge, the present study represents the first investigation of the effects of the trapped electrons on the soliton formation.

BASIC EQUATIONS

Consider the propagation of a right-handed circularly polarized electromagnetic wave along the external magnetic field $\vec{B}_0 = B_0\hat{z}$ in the form of,

$$\vec{E} = \frac{1}{2}(\hat{x} + i\hat{y})E(z,t)\exp(-i\omega t) + c.c., \qquad (2)$$

where ω is the frequency of external electromagnetic field and $c.c.$ stands for the complex conjugate.

The complete set of equations is:

$$\vec{\nabla} \times \vec{B} = \frac{1}{c}\partial_t \vec{E} + \frac{4\pi}{c}\vec{J}, \qquad (3)$$

$$\vec{\nabla} \times \vec{E} = -\frac{1}{c}\partial_t \vec{B}, \qquad (4)$$

$$\nabla \cdot E = 4\pi\rho, \qquad (5)$$

$$\rho = e(n_i - n_e), \qquad (6)$$

$$J = e(n_i V_i - n_e V_e), \qquad (7)$$

where $e = |e|$. In order to close the above systems of equations, ρ and J should be properly defined. Since we are going to study the effect of trapped electrons we assume electron temperature, T_e, is much larger that ions one. Therefore, electrons are treated kinetically [eq. (1)] while the fluid treatment is used for the ions dynamics.

In our formalism there are two distinguishable time scales: one associated with the fast motion with the characteristic time of the order of the high frequency oscillation, $\tau = 2\pi/\omega$, and another with the slow motion resulting from the ions motion. Therefore, each quantity can be decomposed into the slowly and rapidly varying parts:

$$A = \langle A \rangle + \tilde{A}, \qquad (8)$$

where the brackets denote averaging over the time interval τ.

We consider the one dimensional case in which $\partial_x = \partial_y = 0$. Therefore, the nonlinear interaction of a pump field with the background plasma that will generate a slowly varying envelope for the amplitude of electromagnetic wave propagating along the external magnetic field involving the weak relativistic quiver velocity is governed by [10]:

$$2i\omega\,\partial_t E + c^2 \partial_{zz} E - \left(\frac{\omega\,\omega_{p_e}^2}{\omega - \Omega_e} - \omega^2\right)E - \frac{\omega\,\omega_{p_e}^2}{\omega - \Omega_e}\left[\frac{\delta n_e}{n_0} - \frac{\omega}{\omega - \Omega_e}\frac{e^2|E|^2}{2m_e^2 c^2(\omega - \Omega_e)^2}\right]E = 0, \qquad (9)$$

where c is light velocity, $\omega_{p_e}^2 = 4\pi e^2 n_0/m_e$ the electron plasma frequency, n_0 the unperturbed electron density, $\Omega_e = eB_0/m_e c$ the electron cyclotron frequency, m_e the electron rest mass and $\delta n_e = \langle n_e \rangle - n_0$.

In order to define the slowly varying deviation of the density, δn_e, taking into account the electron trapping we have to proceed from eq. (1). In eq. (1) we need the definition of U_{eff}. For this purpose one should calculate the average of Vlasov equation over the time interval τ to find [11]:

$$\partial_t \langle f_e \rangle + v_z \partial_z \langle f_e \rangle - \partial_z \left[-e\varphi + \frac{e^2|E|^2}{2m_e\omega(\omega - \Omega_e)}\right]\partial_{p_z}\langle f_e \rangle = 0, \qquad (10)$$

where f_e is the distribution of electrons. From here we can define the effective potential energy as follow:

$$U_{eff} = -e\varphi + \frac{e^2|E|^2}{2m_e\omega(\omega - \Omega_e)}. \qquad (11)$$

We assume the effective potential energy has the form of a single potential well, vanishing at infinity (soliton solution). Then, by substituting eq. (11) in eq. (1) the density of electrons is clearly defined. As it is mentioned, the ions will also be described by the hydrodynamic equations assuming their temperature is negligibly small. Then for the shallow well when $U_{eff} << T_e$ ($\delta n_e << n_0$) and for the case when quasi-neutrality condition is fulfilled ($\delta n_e \approx \delta n_i$), one can find for the δn_e the following expression:

$$\frac{\delta n_e}{n_0} = \frac{c_s^2}{u^2 - c_s^2}\frac{e^2|E|^2}{2m_e T_e \omega(\omega - \Omega_e)} + \frac{4}{3\sqrt{\pi}}\left(\frac{c_s^2}{u^2 - c_s^2}\right)^{5/2}\left[\frac{e^2|E|^2}{2m_e T_e \omega(\omega - \Omega_e)}\right]^{3/2}, \qquad (12)$$

where u is the velocity of stationary envelope, and $c_s^2 = T_e/m_i$ (m_i =mass of ion).

Substituting eq. (12) in eq. (9) and introducing the dimensionless variables as follows

$$\omega t \to t, \quad \frac{\omega}{c} z \to z, \quad \frac{\omega_{p_e}^2}{\omega^2}\frac{E}{\sqrt{8\pi n_0 T_e}} \to E, \qquad (13)$$

will leads to the following equation:

$$2i\partial_t E + \partial_{zz} E - \left[\frac{\omega_{p_e}^2}{\omega(\omega - \Omega_e)} - 1\right]E + R|E|^2 E - P|E|^3 E = 0, \qquad (14)$$

where,

$$R = \frac{\omega^2}{(\omega - \Omega_e)^2}\left[\frac{\omega^2}{(\omega - \Omega_e)^2}\frac{v_{T_e}^2}{c^2} - \frac{c_s^2}{u^2 - c_s^2}\right], \qquad (15)$$

$$P = \frac{4}{3\sqrt{\pi}} \frac{\omega}{\omega_{p_e}} \left(\frac{\omega}{\omega - \Omega_e} \right)^{5/2} \left(\frac{c_s^2}{u^2 - c_s^2} \right)^{5/2}, \qquad (16)$$

and $v_{T_e} = \sqrt{T_e / m_e}$.

The first term in eq. (15) describes the weak relativistic effect and the second one is connected with the density deviation of free electrons. It is essential that these terms have different signs. The last term in eq. (14) describes the influence of trapped electrons. It must be mentioned that, as it is seen from eq. (12), in the case of $\omega > \Omega_e$, the trapping of electrons is possible only in the supersonic regime when $u^2 > c_s^2$.

SOLUTION AND RESULTS

Now, we will consider the case when $\omega > \Omega_e$. To solve the eq. (14) together with expressions (15) and (16) we use the standard method in which the amplitude of the high frequency field is expressed in the following form:

$$E(z,t) = a(z,t)\exp[i\psi\,(z,t)], \qquad (17)$$

where a and ψ are real functions. On substituting eq. (17) in eq. (14), one can find for the real and imaginary parts of eq. (14) the following equations:

$$2\partial_t + 2(\partial_z a)(\partial_z \psi) + a\partial_{zz}\psi = 0, \qquad (18)$$

$$\partial_{zz} a - 2a\partial_t \psi - (\partial_z \psi)^2 - \left[\frac{\omega_{p_e}^2}{\omega(\omega - \Omega_e)} - 1 \right] E + Ra^3 - pa^4 = 0. \qquad (19)$$

In the stationary state when the amplitude and phase depend on the argument,

$$\xi = z - \frac{u}{c} t, \qquad (20)$$

following the boundary conditions:

$$\xi \to \pm\infty, \quad a(\xi) \to 0, \qquad (21)$$

eqs. (18) and (19) are integrable and the equation for the amplitude can be presented in the following form:

$$\frac{1}{2}(\partial_\xi a)^2 + V(a) = 0, \qquad (22)$$

where,

$$V(a) = -\frac{1}{2}\left[\frac{\omega_{p_e}^2}{\omega(\omega - \Omega_e)} - 1 - \frac{u^2}{c^2} \right] a^2 + \frac{1}{4} Ra^4 - \frac{1}{5} Pa^5, \qquad (23)$$

is the effective potential . Equation (22) is identical in form to the energy equation of a single particle with coordinate a and time ξ . The relation between the maximum amplitude of soliton and its velocity is:

$$\frac{u^2}{c^2} = \left[\frac{\omega_{p_e}^2}{\omega(\omega - \Omega_e)} - 1 \right] - \frac{1}{2} Ra_{\max}^2 + \frac{2}{5} Pa_{\max}^3 . \qquad (24)$$

A glance at the effective potential $V(a)$ shows the effect of trapped electron $\left(-1/5\,Pa^5\right)$. As it is seen from eq. (23) to have a bound solution for the eq. (22), the equation ,

$$V(a) = 0, \qquad (25)$$

should at least have two roots. In the competition of refraction caused by the non-resonant electrons and the relativistic nonlinearity, R can becomes enough small that the last term in $V(a)$ (caused by the trapped electrons) dominates R and consequently it can destroy the localized structure of the amplitude.

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