ADVANCED FLUID MODELLING OF TRANSPORT IN TOKAMAKS

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The need for and conditions for fluid modelling of tokamak transport is discussed. In particular emphasis is put on fluid closure, off diagonal transport fluxes and non- Markovian effects. Some recent results of a fluid model using a reactive closure are also discussed including theory and simulations of zonal flows and temperature and particle pinches.

Introduction

Although research on transport in magnetic confinement devices has been a high priority research area since the beginning of fusion research, the agreement between theory and experiment has been poor until the development of advanced fluid models started in the end of the 1980s [1]. The reason for this has mainly been the fact that toroidal effects play a very important role in toroidal devices and that including these fully, in principle, requires a kinetic description [2–4]. Although it is relatively easy to apply a linear gyrokinetic description, this problem is quite difficult since a nonlinear kinetic description is needed for transport calculations. Nonlinear kinetic codes can not yet be run on the confinement timescale with realistic computation times. Thus some kind of approximation of kinetic theory is needed. The most successful methods so far has been the development of *advanced fluid models*.

Advanced fluid models

As indicated above, we need to deal with the kinetic resonance in some approximate way. Simple fluid models in general are applicable either in collision dominated cases or in the adiabatic (fast process) or isothermal (slow process) limits in collisionless plasmas. Thus a fluid model that can be applied for intermediately fast processes in collisionless plasmas can be defined as an advanced fluid model. Due to the dramatic improvement in agreement with experiments found when the flat density regime was included, we can also tie the definition of advanced fluid models to the ability to include this regime as was done in Ref [4].

Energy equation and closure

The highest moment we will consider here is the heat flow. We then write the energy equation for the collisionless case (no thermal force)

$$\frac{2}{3}n\left(\frac{\partial}{\partial t}+\mathbf{v}\cdot\boldsymbol{\nabla}\right)T+P\boldsymbol{\nabla}\cdot\mathbf{v}=-\boldsymbol{\nabla}\cdot\mathbf{q}. \qquad (1)$$

Here the choice of $\mathbf{q}$ defines the closure. In our reactive model we use the Braghinskii diamagnetic heat flux:

$$\mathbf{q}=\mathbf{q}^{*}=\frac{5}{2}\frac{P}{m\Omega}\left(\mathbf{e}_{\parallel}\times\boldsymbol{\nabla}T\right). \qquad (2)$$

Gyrofluid models here add an imaginary part representing the contribution to Landaudamping and Magnetic drift resonances from all higher moments. This imaginary part is fitted to linear kinetic theory. We will shortly give a motivation for our choice of $\mathbf{q}$ in a collisionless state. After replacing $\boldsymbol{\nabla}\cdot\mathbf{v}$ by the continuity equation we find that convective diamagnetic effects cancel. We then arrive at the temperature perturbation:

$$\frac{\delta T}{T}=\frac{\omega}{\omega-5/3\omega_D}\left[\frac{2}{3}\frac{\delta n}{n}-\frac{\omega_{*e}}{\omega}\left(\frac{2}{3}-\eta\right)\frac{e\phi}{T_e}\right]. \qquad (3)$$

Here fluid resonance due to nonadiabatic part is kept! $\delta n/n-(\omega_{*e}/\omega)(e\phi/T_e)$ is a difference due to compression and $(\omega_{*e}/\omega)\eta(e\phi/T_e)$ is a convective part.

The same model is used for ions, trapped electrons and impurity ions. We will now outline the derivation of the closure (2) in the collisionless case.

Collisionless closure

Continuing the fluid hierarchy up to the heat flow equation we have the highest moment:

$$\langle v_i v_j v_k v_l \rangle = \langle v_i v_k \rangle \langle v_j v_l \rangle + \ldots + G,$$

where $G = \langle v_i v_j v_k v_l \rangle_{\mathrm{irr}}$ is the irreducible, genuine fourth moment.

If the particle velocities are stochastic we can make the **Random phase** approximation

$$\langle v_i v_k \rangle = |v_i|^2 \eth(i-k), \qquad G = 0.$$

This is exactly fulfilled for a Maxwell distribution.

In a recent paper [5] it was shown by a **Maximum entropy principle** that $G = 0$ defines a closure in an **isolated** unmagnetized solid state plasma.This focuses interest on the *external sources.*

If we *include only moments with external sources* (n and T) we also arrive at the condition $G = 0$ under the condition that no internal sources are created for G. An example of when an external source of a lower moment can create a source for a higher moment is Ohmic heating. However, this is a completely random process which fulfills the Random phase approximation. In order to create a source for G we need a source that generates correlations. An example of this is instabilities driven by the external heating such as beam driven modes. Here the beam fills in velocity space continuously so that flattening or particle trapping are balanced by the source in velocity space. For modes driven by ideal particle and heat sources we do not expect sources in velocity space to play a role. Furthermore, since we include the lowest order velocity resonances, nonlinear relaxation of the resonance due to higher order moments will not mean complete flattening of velocity space. If we further assume isotropic temperature perturbations we again arrive at the closure (2). The argument for doing this is that there are nonlinear equilibration terms for the parallel and perpendicular temperatures perturbations in the energy equations. Effects of a nonlinear closure were recently studied by generalizing the method of Mattor and Parker [6] to include diffusion [7].

Linear properties of ITG modes

Combining Eq (2) with the usual perpendicular fluid drifts and parallel ion motion leads to a three pole ion density response. Then using Boltzmann electrons we arrive at an eigenvalue problem that can be solved analytically in the strong ballooning limit. The stability limits are shown in Fig 1.

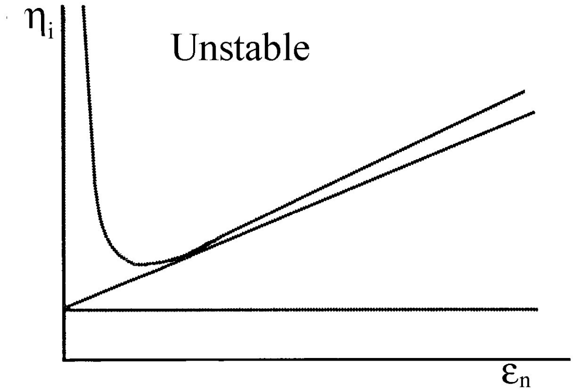

Fig 1

The upper curve in Fig 1 shows the local stability boundary, the tangent line is the nonlocal stability boundary and the horizontal line is the stability boundary for the adiabatic model. The ion conductivity can be written:

$$\chi_i = \frac{1}{\eta_i}\left(\eta_i - \frac{2}{3} - \frac{10}{9\tau}\varepsilon_n\right)\frac{\gamma^3/k_x^2}{(\omega_r - 5\omega_{Di}/3)^2 + \eta^2}. \quad (4)$$

The three parts in paretesis represent in order diagonal, off diagonal and convective parts. We note that the convective part gives a strong inward flux component for hot ions The real frequency in the denominator (non-Markovian effect) increases stiffness and introduces a *selectivity* regarding transport from modes with different real eigenfrequency [8,9]. It comes from the denominator of (3). At nonlocal marginal stability the first factor in (4) and the denominator of (3) both vanish.

Fig 2 shows typical radial profiles of χ for experiments and simple fluid models. An example of this is shown in Ref 10. The behaviour of advanced fluid models was obtained from Eq (4) for the parameters in Ref 10.

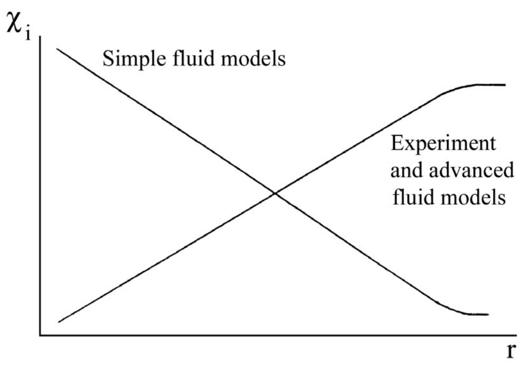

Fig 2

Pinch fluxes

In connection with radial $E \times B$ convection we can write the temperature perturbation as:

$$\delta T = -\xi \bullet \nabla T + \alpha \xi \bullet \nabla n + \beta \xi \bullet \nabla B, \qquad (5)$$

where ξ is the $E \times B$ displacement and the coefficients α and β are compressibility coefficients. The fist term on the right hand side is the convective temperature perturbation, the second is due to expansion after convection in the density gradient and the last term is due to expansion after convection in the magnetic field gradient. The last two terms will reduce the temperature perturbation. Linearly they will create a threshold in Ln/Lt or Lb/Lt. for instability. The nonlinear fluxes are obtained by multiplying (5) with the radial $E \times B$ velocity. The last two parts will then give off diagonal fluxes that tend to counteract the flux due to the first, diagonal term. If the off diagonal fluxes dominate we would get a pinch. However then there is no thermal instability. If we, however, have two different species that can give thermal instability where one is stable and the other is unstable, the unstable feedback loop will drive the stable one due to $E \times B$ convection and we get a pinch from the stable loop [11]. An example of an electron heat pinch simulation with our transport model was presented in Ref 12. This was from off axis ECH heating in RTP where the ECH source was modulated in time. The electron heat pinch was driven by the ITG mode and both average temperature and the first three harmonics of the modulation frequency were in good agreement with the experiment.

Zonal flows

Nonlinearly generated zonal flows were recently found to give a nonlinear upshift in the critical temperature gradient in nonlinear gyrokinetic simulations of ion heat transport in a D- III-D H-mode [13]. We have recently simulated this case with our fluid model [14]. We verified the nonlinear (Dimits) upshift within 10% of the fluxtube nonlinear gyrokinetic simulations in Ref 13. This is particularly interesting since the zonal flows are excited resonantly just above the linear threshold. The resonance is due to the fluid resonance in the energy equation and is thus sensitive to the fluid closure. We note that the IFS-PPPL-98 model had a deviation of 50% in the nonlinear upshift in the simulations in Ref 13. It is also important that the general transport level in our model, as compared to the nonlinear gyrokinetic results in Ref 13, was verified in Ref 14.

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