THE NONLINEAR THEORY OF THE ELECTRON AND ION BEAMES INTERACTION IN A SUPERCRITICAL MODE

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The problem of REB and ion beam interaction at currents above space charge limiting values has been mathematically formulated. The method of Lagrangian variables has been used for the description of selfconsistent nonlinear dynamics of electrons and ions in collective fields of the beams. The simulation has been performed for the parameters of the experimental installation.
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1. INTRODUCTION

For implementation of collective methods of acceleration of ions by quasistatic electrical fields of high-current relativistic electron beam (REB), in particular in a mode of formation of a virtual cathode (VC) [1-4], the research low frequency (LF) processes of interaction of ions flows with REB has a great important. The similar situation arises at injection of plasma in area of VC for destruction of VC potential well and time modulation of REB [1]. The self-consistent influence of ions on VC can result in to synchronic motion of a potential well of VC and essential increase of final energy of accelerated ions [2,3]. The transversal oscillations of ions in two-dimensional quasistatic electrical field of VC under certain conditions cause the low frequency (LF) relaxation oscillations of VC field and by that provide deep LF modulation of REB density. This phenomenon can be laid in the fundamentals of the operation of the first stage of a two-stage collective ion accelerator working on a principle of simultaneous spatial and temporal LF modulation of REB density [1,4].

In the present paper some results of non-linear dynamics investigation of the interaction of ions flow with VC field produced by REB is represented. The system of the non-linear integral-differential equations describing indicated process is formulated and is obtained and its solution and analysis is carried out by numerical methods.

2. PROBLEM FORMULATION

Problem will be decided in the following statement. In the semi-infinite metallic drift chamber, closed at z=0 by the conductive diaphragm, it is injected along axis in direction z > 0 thin-wall tubular electron and ion beams. These beams have different radii, energy of particles and currents. The drift chamber is under zero electric potential. Electron and ion beams are propagating in a homogeneous leading magnetic field. The beam is considered as magnetized ones (motion strictly along of magnetic field lines), and influence of the magnetic field on motion of ions is neglected.

For the theoretical description of non-linear dynamics of electron and ion beams interaction in the drift chamber the following approach was used. At first we find the electric potential created in the drift chamber by infinitely thin driving charged ring with particle density

$$dn = dN_0 \frac{\delta(r - r_L)}{2\pi r_L} \delta(z - z_L) \tag{1}$$

where dN_0 – number of particles in a ring, $r_L(t,t_0)$ – ring radius, that varies during motion, t – current time, t_0 – time moment of an entry of a ring in the drift chamber, $r_L(t,t_0) = r_0$ – initial value of radius of a ring, $z_L(t,t_0)$ – Lagrangian longitudinal coordinate of a ring, $z_L(t_0,t_0) = 0$, r,z – radial and longitudinal coordinate in the drift chamber. The electric potential ϕ_G of a ring (1) is governed by a Poisson equation

$$\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial \phi_G}{\partial r} + \frac{\partial^2 \phi_G}{\partial z^2} = 2dN_0 e \frac{\delta(r - r_L)}{r_L} \delta(z - z_L) \tag{2}$$

For determinacy we consider a potential (Green function) for negatively charged ring (electron ring). The electric potential becomes zero on a lateral area of the drift chamber and also at the conductive butt-end of the drift chamber.

For Green function we receive

$$\phi_G(r,z) = \frac{2edN_0}{a} \cdot \sum_{n=1}^{\infty} \frac{J_0(\lambda_n \frac{r_L}{a}) J_0(\lambda_n \frac{r}{a})}{\lambda_n J_1^2(\lambda_n)} \left[e^{-\frac{\lambda_n}{a}(z+z_L)} - e^{-\frac{\lambda_n}{a}|z-z_L|} \right] \tag{3}$$

The number of particles in each ring dN_0 is connected with a current of the electron beam $I_e(t_0)$ by a simple relationship

$$dN_0 = I_e(t_0) dt_0 / e \tag{4}$$

The electric potential of the electron beam is being found by integrating of a right side of eq. (3) over time of an entry of "macroparticles" formed by elementary rings with number of particles (4). The contribution to a potential of the ion beam is being similarly determined. In result for the electric potential of a system consisting of infinitely thin electron and ion beams moving in the conductive drift chamber, we shall receive the following expression

$$\phi(r,z)=\frac{2}{a}\sum_{n=1}^{\infty}\frac{J_0(\lambda_n \frac{r}{a})}{\lambda_n J_1^2(\lambda_n)}\cdot$$

$$\cdot\left\{\begin{aligned}&J_0(\lambda_n\frac{r_0}{a})\int_0^t I_e(t_0)dt_0\left[e^{-\frac{\lambda_n}{a}(z+z_{Le})}-e^{-\frac{\lambda_n}{a}|z-z_{Le}|}\right]-\\&-\int_0^t I_i(t_0)J_0(\lambda_n\frac{r_{Li}}{a})dt_0\left[e^{-\frac{\lambda_n}{a}(z+z_{Li})}-e^{-\frac{\lambda_n}{a}|z-z_{Li}|}\right]\end{aligned}\right\} \quad (5)$$

Let's comment briefly the expression (5). The integrating over time of an entry in right side is carried out from time of entry in the drift chamber of the first particles $t_0=0$ up to current time $t_0=t$. The radial coordinates of electrons (in (5) indexes "e") remain unchanged, i.e. -independent of an entry time. In an integral over an entry time we has taken into consideration this circumstance. As to ions (in (5) indexes "i"), under an integral over an entry time of ions both their radial $r_{Li}(t,t_0)$, and longitudinal $z_{Li}(t,t_0)$ coordinates are contained.

Knowing electric potential, it is not difficult to determine longitudinal and radial component of electrical field. Inserting expressions for the components of quasistatic electrical field in motion equations for electrons and ions we obtain self-consistent nonlinear equations set, which describes complex dynamics of interaction of overlimiting REB with ion flow.

$$\frac{dP_{Le}}{dt}=e\frac{d\phi}{dz}, \quad \frac{dz_{Le}}{dt}=v_{Le}, \quad v_{Le}=\frac{cP_{Le}}{\sqrt{P_{Le}^2+m^2c^2}} \quad (6)$$

$$\frac{d^2z_{Li}}{dt^2}=-\frac{e}{M}\frac{d\phi}{dz}, \quad \frac{d^2r_{Li}}{dt^2}=-\frac{e}{M}\frac{d\phi}{dr} \quad (7)$$

The dependencies of electron $I_e(t_0)$ and ion $I_i(t_0)$ currents upon time at the entry into the drift chamber are taken in the form $I_{e,i}(t_0)=I_{e,i}\psi_{e,i}(T_0)$, where the functions $\psi_{e,i}(T_0)$ describe the shape of pulses of the corresponding currents. If in the system the direct currents are continuously injected, then. $\psi_{e,i}(T_0)=1$.

3. NUMERICAL RESULTS

The set of motion equations (5-7) was solved by Runge-Kutta method of second order. Integrals in right sides were approximated by Simpson method. Depending on parameters the step in time was varied.

In Fig. 1 (curve 1) phase portrait of electron beam in absence of ions is pictured. Parameters of electron beam are the followings: energy $E_e=280keV$, current $I_e=5.6kA$, radius $R_{0e}=1.6cm$. For indicated values of energies, radii of the beam and drift chamber the limiting current is equal $I_{\lim}=3.8kA$ that is less than the current of injected REB.

From Fig. 1 it is seen, that in the drift chamber the VC is formed. It is possible to see passed through and reflected particles. The value of voltage drop (see Fig. 1, curve 2) coincides with initial energy of the beam.

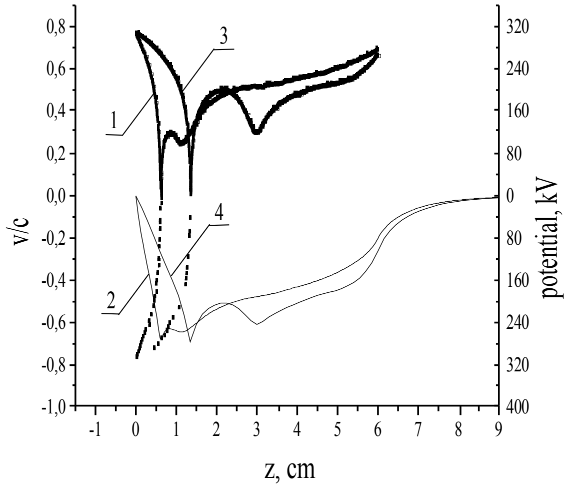

Fig.1 Phase plane of electron beam without ion beam (1) and with ion beam (3). Electrical potentials for these cases, accordingly (2 and 4). $I_e=I_i=5.6kA$, $E_e=280keV$, $E_i=25keV$, $m/M=40$, $R_e=R_{0i}=1.6cm$, $N_i/N_e=1.5$

Let's consider now the case, interesting for us, of simultaneous injection REB and ion flux. We begin from the analysis of the simplest situation, when the initial radii of beams coincide $R_{0e}=R_{0i}=1.6cm$. The initial energy of ions was selected equal $E_i=25keV$, and linear density (number of particles per unit length of beam) $N_i/N_e=1.5$, where $N_{e,i}=I_{e,i}/ev_{e,i}$, where $v_{e,i}$ - initial velocities of particles. The model mass ratio was selected equal $m/M=40$. These conditions correspond to ion current $I_i=540A$. In Fig. 1 (curve 3) the phase portrait of electrons for this case is pictured. In the same Fig. 1 the distribution of the electric potential in the region of VC is shown (curve 4). It is seen, that the influence of ions has resulted in displacement of VC inside drift chamber. The distance from the injection plane up to VC was increased more than twice comparatively to the case of ion flow absence. By that the depth of a voltage drop has not changed. On the curve of the potential additional minimum has appeared. Accordingly, this minimum has found its reflection on a phase portrait of electrons in a view of slowing and gathering of electrons in this area.

In Fig. 2 the phase portrait of ions is shown. It is well seen that starting from injection plane the ions are experienced accelerating by VC field. Then, having reached the bottom of the potential well and having received gain of energy, approximately equal to REB energy, the ions start slowing. Their density increases, that results in distorting of the potential and formation with the help of electrons of the next minimum of the potential. Energy of ions again slightly grows up. The level of these oscillations of energy is rather low.

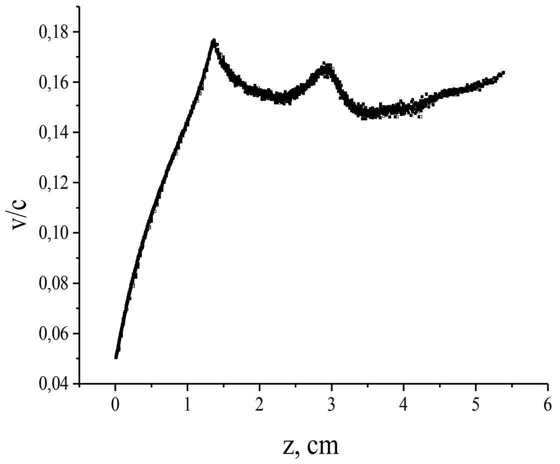

Fig.2 Phase plane of ion beam with electron beam. $I_e=I_i=5.6kA$, $E_e=280keV$, $E_i=25keV$ $m/M=40$, $R_e=R_{0i}=1.6cm$, $N_i/N_e=1.5$

Let's consider now the dynamics of VC when ion current is increasing. In Fig. 3 the phase portrait of electrons and potential distribution along the system is represented for $N_i / N_e = 4.5$.

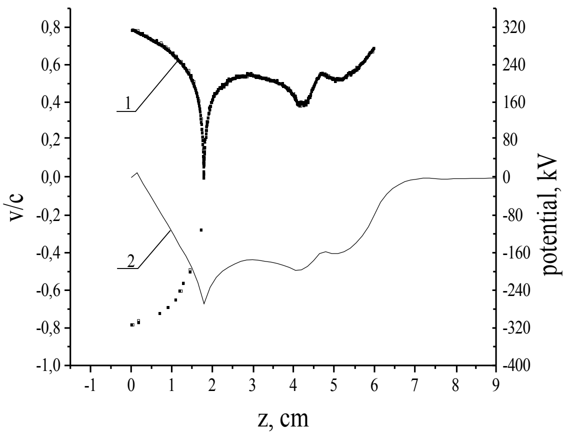

Fig.3 Phase plane of electron beam with ion beam (1) and electrical potential (2). $I_e=I_i=5.6kA$, $E_e=280keV$, $E_i=25keV$ $m/M=40$, $R_e=R_{0i}=1.6cm$, $N_i/N_e=4.5$

It is seen, that the displacement of VC into chambers increases. The distribution of potential, as well as in the previous case, is not monotonic.

In Fig. 4 (curve 1) the phase portrait of ions is shown. It is well seen, that near the plane of injection the ion virtual anode (VA) is formed. The part of ions was reflected, and the others are involved in process of acceleration. On the curve of the electric potential (curve 2) in proximity of the injection plane it is visible the positive maximum formed by ion VA.

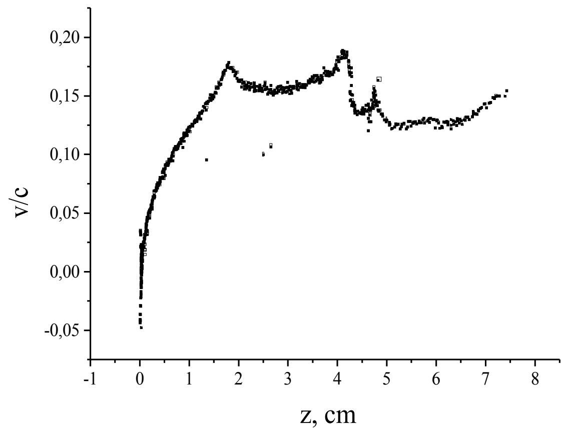

Fig.4 Phase plane of ion beam with electron beam. $I_e=I_i=5.6kA$, $E_e=280keV$, $E_i=25keV$ $m/M=40$, $R_e=R_{0i}=1.6cm$, $N_i/N_e=4.5$

The ion VA can be considered as an emitter of ions with space charge restriction. Therefore passed ion current will be determined by a voltage drop in the region of electron VC and spacing interval from an ion VA (plane of injection) up to an electron VC (Child-Langmuir law).

4. CONCLUSIONS

In the paper the system of non-linear integral-differential equations describing the non-linear interaction of an ion flux with a VC, produced by REB, is formulated, in two-dimensional approximation for ions and one-dimensional for electrons (the electrons are supposed to be magnetized). Using a numerical code differing from both the method of "macroparticles" and the full numerical simulation, but combining some of their advantages, the results of dynamics of electron and ion beams are obtained.

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