EFFECT OF AC ELECTRIC FIELD WITH THE CHANGEABLE IN TIME FREQUENCY ON TRAPPED PARTICLES

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The effect of alternating electric field on the particle detrapping in two kinds of magnetic configurations with different | **B** | modulation is considered here.
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1. INTRODUCTION

The processes of detrapping / retrapping of the charged particles under the high-frequency parallel electric field can be considered as the resonance between the bounce frequency of the trapped particle and the frequency of the externally applied parallel electric field [1]. Principally new effect studied here is the following. Linearly increasing frequency is used for detrapping of charged particle. The initial value of the frequency of the electric field is close to the bounce frequency of the trapped particle. Detrapping / retrapping processes of the charged particle under the alternating parallel electric field is studied by numerical integration of guiding center equations in two types of magnetic configurations. Outward shifted configuration and inward shifted configuration [2] are considered and compared.

2.1 Magnetic field model

The components of the magnetic field are as follows:

$$B_r = B_0 \left(- R/m \right) \varepsilon_{lm} (l/a)(r/a)^{l-1} \sin(l\vartheta - m\varphi) \qquad (1)$$

$$B_\vartheta = B_0 \left(- R/m \right) \varepsilon_{lm} (l/a)(r/a)^{l-1} \cos(l\vartheta - m\varphi) \qquad (2)$$

$$B_\varphi = B_0 \left\{ 1 + (r/R)\cos\vartheta + \varepsilon_{lm} (r/a)^l \cos(l\vartheta - m\varphi) \right\} \qquad (3)$$

where B_0 is the magnetic field at a circular axis of the torus with major radius R and minor radius a. The coordinates r, ϑ, φ are connected with circular axis of the torus: radial variable r, toroidal and poloidal angles ϑ, φ ; m is the number of magnetic field periods along the torus, l is the number of the helical winding poles; ε_{lm} are the coefficients of harmonics of the magnetic field. The magnetic field absolute value is given by $B \approx B_\varphi$. The vertical magnetic field is added to the main magnetic field.

$$B_{r(vert)} = B_\perp \sin\vartheta ;$$

$$B_{\vartheta (vert)} = B_\perp \cos\vartheta . \qquad (4)$$

Inward shifted configuration and outward shifted configuration are obtained by varying of the vertical magnetic field $B_\perp$.

2.2 Electric field model

Electric field which acts on the charged particle is taken in such form:

$$E_\varphi = E_{lm} \cos(l\vartheta - m\varphi) \cos(\Omega t + \delta) \qquad (5)$$

where E_{lm} is electric field amplitude, Ω is the frequency and δ is the phase of the electric field oscillation. During the process of detrapping the frequency of the electric field Ω is linearly increasing from $\omega_{bounce}^{toroidal}$ to $\omega_{bounce}^{helical}$, where $\omega_{bounce}^{toroidal}$ is the bounce frequency of toroidally trapped particle and $\omega_{bounce}^{helical}$ is the bounce frequency of helically trapped particle.

2.3 Guiding Center equations

Guiding center equations in coordinates r, ϑ, φ can be reduced to the following form:

$$\dot{r} = v_\parallel \frac{B_r}{B} - \frac{c}{B^2} E_\varphi B_\vartheta - \frac{Mc(2v_\parallel^2 + v_\perp^2)}{2eB^3} B_\varphi (\nabla B)_\vartheta ; \qquad (6)$$

$$r\dot{\vartheta} = v_\parallel \frac{B_\vartheta}{B} + \frac{Mc(2v_\parallel^2 + v_\perp^2)}{2eB^3} B_\varphi (\nabla B)_r ; \qquad (7)$$

$$(R - r\cos\vartheta)\dot{\varphi} = v_\parallel \frac{B_\varphi}{B} - \frac{Mc(2v_\parallel^2 + v_\perp^2)}{2eB^3} B_\vartheta (\nabla B)_r ; \qquad (8)$$

$$\dot{v}_\parallel = \frac{1}{v_\parallel} \left[\frac{eE_\varphi}{M} (R - r\cos\vartheta)\dot{\varphi} - \frac{\mu}{2} \left(\dot{r}\frac{\partial B}{\partial r} + \dot{\vartheta}\frac{\partial B}{\partial \vartheta} \right) \right] ; \qquad (9)$$

$$\dot{v}_\perp = \frac{\mu}{2v_\perp} \left(\dot{r}\frac{\partial B}{\partial r} + \dot{\vartheta}\frac{\partial B}{\partial \vartheta} \right) . \qquad (10)$$

where $v_\parallel$ is the parallel velocity of the charged particle, $v_\perp$ is its perpendicular velocity and $\mu = v_\perp^2 / B$ is the adiabatic invariant.
Runge-Kutta method is used for numerical integration of guiding center equations.

3. ANALYSIS OF THE TRANSITION PROCESS

Using guiding center equations and expressions for magnetic field components we obtain magnetic surface

function Ψ. Here is the expression for the function Ψ^0 without vertical magnetic field:

$$\Psi^0 = B_0 \left(\frac{mr^2}{2R} + \frac{R}{m} \varepsilon_{lm} \left(\frac{r}{a} \right)^l \cos(l\vartheta - m\varphi) \right). \qquad (11)$$

For further studies we consider $l=2$ case in order to simplify the expressions.

Using the equation $\mathbf{B}\nabla\Psi = 0$, considering only linear perturbing terms, we can write $\left(\mathbf{B}^0 + \mathbf{B}^1\right)\nabla\left(\Psi^0 + \Psi^1\right) = 0$, where $\mathbf{B}^0$ is the main magnetic field, $\mathbf{B}^1$ is the small adding to the main magnetic field due to the vertical magnetic field, and Ψ^1 is the small adding to the function Ψ due to the vertical magnetic field. Full function Ψ takes the following form:

$$\Psi = B_0 \left[\frac{mr^2}{2R} + \frac{R}{m} \varepsilon_{lm} \left(\frac{r}{a} \right)^l \cos(2\vartheta - m\varphi) \right] +$$
$$+ B_\perp \left[\left(A^2 - 1\right) r\cos\vartheta + Ar\cos(\vartheta - m\varphi) \right] \qquad (12)$$

where $A = m^2 a^2 / 2\varepsilon_{2m} R^2$. Using the equation for the contour of the magnetic surface and considering $A \ll 1$, we found the expression for the shift of the magnetic axis Δ:

$$\Delta = B_\perp \Big/ B_0 \left(\frac{m}{R} + \frac{R}{m} \varepsilon_{2m} \frac{2}{a^2} \right) \qquad (13)$$

The modulation of the magnetic field can take the following form:

$$B = B_0 \left\{ 1 + \frac{r_0}{R} \cos\vartheta + \right.$$
$$\left. + \left(\frac{r_0}{a} \right)^2 \varepsilon_{2m} \cos(2\vartheta - m\varphi) \left[1 + \sigma \cos\vartheta \right] \right\}, \qquad (14)$$

where $\sigma = \left(2\Delta - R/m^2 \right)/r_0$, r_0 is averaged radius of the shifted magnetic surface.

As trapped or blocked particle alters the direction of its motion, the trajectory has reflection points. While altering the direction of the particle motion the longitudinal velocity changes its sign and equals zero at the reflection point. Thus the analysis of the equation (9) for $\dot{v}_\parallel$ is important for understanding the condition of particle transition from the trapped state to the passing one. Now we pass to the analysis of the equation (9). Using the expressions (6), (7) for $\dot{r}$ and $r\dot{\vartheta}$ we can write the expression for $\dot{v}_\parallel$ in such form:

$$\dot{v}_\parallel = E_\varphi \left(\frac{e}{M} + \frac{cF^*}{B^3} v_\parallel \right) +$$
$$+ \frac{v_\perp^2 Mc \left(2v_\parallel^2 + v_\perp^2 \right)}{4eB^4 v_\parallel} F^* (\nabla B)_\varphi - \frac{v_\perp^2}{2B^2} G^* \qquad (15)$$

where

$$F^* \equiv B_o^2 \left[\frac{1}{m} \varepsilon_{2m} \frac{2r}{a^2} \cos(\vartheta - m\varphi) + \frac{R}{m} \varepsilon_{2m}^2 \frac{4r^2}{a^4} \right] -$$
$$- B_0 B_\perp \left[\frac{1}{R} + \varepsilon_{2m} \frac{2r}{a^2} \cos(\vartheta - m\varphi) \right]; \qquad (16)$$

$$G^* \equiv - B_0 \varepsilon_{2m} \frac{2r}{a^2} \sin(\vartheta - m\varphi) \left(\frac{B_0}{m} + B_\perp \right). \qquad (17)$$

As one can see, the expressions for F^* and G^* contain harmonics $\sin(\vartheta - m\varphi)$ and $\cos(\vartheta - m\varphi)$. Therefore it is possible to choose the form of the electric field model to contain harmonics $\sin(\vartheta - m\varphi)$ or $\cos(\vartheta - m\varphi)$.

4. Particle orbits in Inward Shifted and Outward Shifted configurations

For the further numerical study we take the magnetic configuration with the following parameters: $R = 1000$ cm, $a = 200$ cm, $B_0 = 3$T, $l = 2$, $m = 10$. The configuration with such parameters is chosen to understand the physics of considered phenomena in the magnetic systems with large aspect ratio.

Inward shifted configuration.
Fig.1 presents the magnetic field modulation in this configuration with $B_\perp / B_0 = -0,016$. When the proton
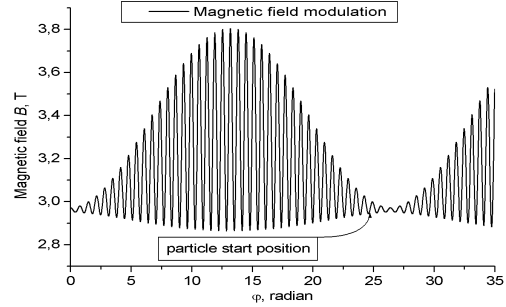

Fig. 1 Magnetic field modulation in INWARD SHIFTED configuration

with the energy 1 keV, $v_\parallel / v = 0,2$ moves in such magnetic configuration it is trapped on the toroidal inhomogenity of the magnetic field (blocked particle).
The high-frequency externally applied electric field with linearly increasing frequency is used to detrap the particle. The motion of the particle changes considerably.
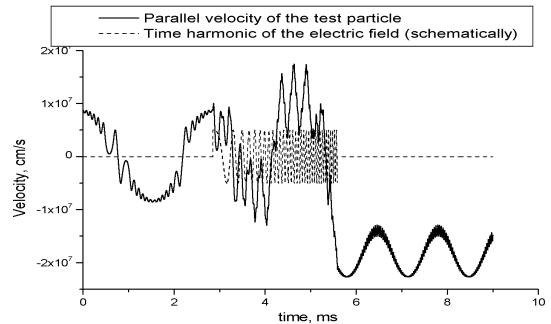

Fig. 2 Illustration of the transition of the test particle from the blocked state into the passing one in INWARD SHIFTED configuration.

On the Fig.2 one can see the parallel velocity of the test particle while transiting from the blocked state into the passing one. The vertical cross-section of the orbit of the passing particle is shown on Fig.3.
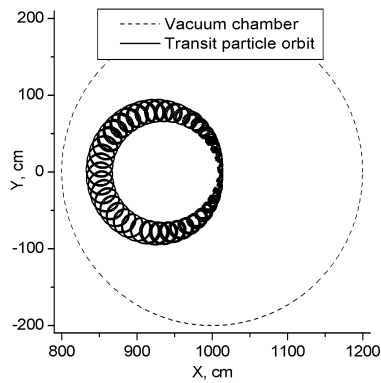
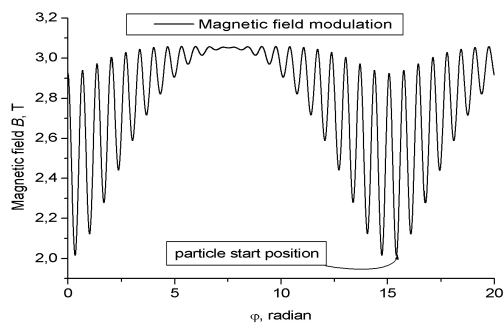Fig. 3 Transit particle orbit in INWARD SHIFTED configuration.

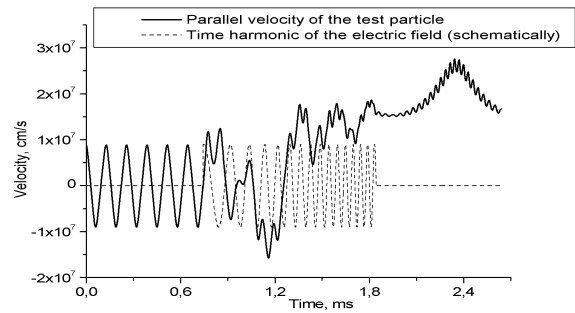Fig. 4 Magnetic field modulation in OUTWARD SHIFTED configuration.

Fig. 5 Illustration of the transition of the test particle from the trapped state into the passing one in OUTWARD SHIFTED configuration.
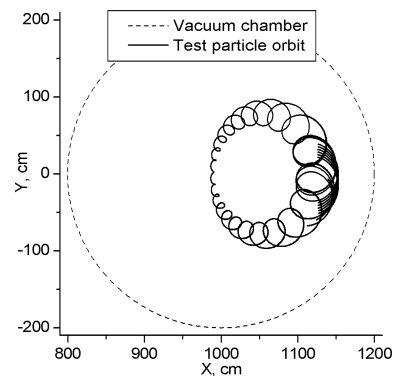

Fig. 6 Test particle orbit while transiting from the trapped state into the passing one in OUTWARD SHIFTED configuration.

Outward shifted configuration.

On the Fig.4 it is shown the magnetic field modulation, corresponding to the outward shifted configuration with $B_\perp/B_0 = 0,019$. When the test particle (1keV proton with $v_\parallel/v = 0,2$) moves in the magnetic field of a such configuration, it is helically trapped. But, as it is seen from the Fig.5, when the electric field with linearly increasing frequency (dashed line on the Fig.5) acts on the test particle it becomes the passing one. The Fig.6 shows the vertical cross-section of the test particle orbit.

In real stellarator type devices including the helical devices the magnetic axis is shifted from the circular axis of the torus. This shift is increased under the effect of the finite β, where β is the ratio of the plasma gas kinetic pressure to the magnetic pressure ($\beta = 8\pi nT/B^2$). The displacement of magnetic axis Δ is the measure of β.

CONCLUSION

1. The principally new thing considered here is the application of the alternating electric field with variable in time frequency with the goal to force the transition of the charged particle from the trapped state into the passing one.

2. The process of transition is studied in two configurations with different magnetic field modulation. It is shown that particle transition from the trapped state into one takes place in both configurations.

3. Magnetic field model with $l=2$ is considered in numerical studies and it is shown that the AC electric field has to contain $l=1$ harmonics.

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