EQUILIBRIUM OF INTENSE ELECTRON BEAM IN COAXIAL UNDULATOR WITH PARTIALLY SHIELDED CATHODE IN GUIDE HOMOGENEOUS MAGNETIC FIELD

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For the initial radii of charged-particle beam in a coaxial magnetic undulator with partially shielded cathode we formulate the equilibrium equations, which enforce the defocusing self-electric field compensation by the longitudinal and azimuthal components of self-magnetic field of the electron beam and account for the undulator focusing through the first (leading) harmonic of the undulator magnetic field. Numerical solutions of the equilibrium equations in a coaxial magnetic undulator with shielded and non-shielded cathode are analyzed and compared.
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INTRODUCTION

During last decades many authors study transportation and limiting currents of charged-particle beam propagation in the conducting grounded drift tube in the longitudinal homogeneous (guide) strong (infinite) or finite magnetic fields [1 - 3]. The limiting current of electron beam in the magnetostatic pump field of the hybrid coaxial free electron laser/maser (FEL/FEM) is discussed in papers [4, 5]. The limiting current of electron beam in the strong (infinite) longitudinal homogeneous magnetic field is estimated for a thin beam in coaxial drift tube in papers [3, 6, 7]. The propagating currents obtained in paper [8] turn out to be smaller than those in the longitudinal homogeneous guide magnetic field; also the currents achieved in the Strathclyde hybrid coaxial FEL [9] are smaller. On the comparison of results received in papers [8, 9] and paper [10] we arrive at the conclusion that the degree of cathode shielding has a substantial influence on the charged-particle beam propagation in the conducting grounded drift tube in the hybrid external guide magnetic field.

The statement of problem and the expression for the self-electric and self-magnetic fields of charged-particle beam in a coaxial drift-tube including the longitudinal component of the self-magnetic field are given in Section 1. In Section 2 we numerically analyze the dependence of equilibrium radii on the beam current, external longitudinal and undulator magnetic fields and compare solutions of equilibrium equations for shielded and non-shielded cathodes.

1. PROBLEM SETUP

In the assumption of laminar flow we obtain the equilibrium equations for the initial radii of electron beam, which enforce the defocusing self-electric field compensation by the undulator focusing force and the longitudinal and azimuthal components of self-magnetic field of electron beam taking into account the first (lead-ing) harmonic of the undulator magnetic field in a coaxial magnetic undulator with partially shielded cathode under the assumptions stated below. We suppose that an angular beam velocity ω_b is constant ("rigid rotor" model $\omega_b \equiv \overline{\upsilon}_\theta(r)/r = const$, where $\overline{\upsilon}_\theta(r)$ is the θ-component of beam velocity, r is the radial coordinate, the overbar "–" denotes the averaging over spatial period l_w of the periodic undulator magnetostatic system). Also we consider charged-particle beams with the constant beam density

$$n_0 = (r_o - r_i)^{-1}\int_{r_i}^{r_o}\overline{n}_0(r)dr \,,$$

where r_i, r_o are the inner and outer beam radii, $\overline{n}_0(r)$ is the beam density averaged over the undulator period. For receiving the analytical expression for the dimensionless θ-component of beam momentum $\overline{\pi}_\theta(\rho) = \overline{p}_\theta(\rho)/(m_q c)$ we have to assume that the relativistic factor $\gamma = \sqrt{1 + (p_r^2 + p_\theta^2 + p_z^2)/(m_q c)^2}$ is constant i.e. $\gamma \equiv \gamma_0 = \sqrt{1 + p_{z0}^2/(m_q c)^2}$, where p_{z0} is z-the component of dimensionless initial moment, p_r, p_θ, p_z are r-, θ-, z - the components of relativistic momentum; m_q is the mass of particles (for electrons $m_q = m_e$); c is the light velocity in vacuum. Those assumptions are natural for thin beams, which can be proved by an expansion in the small parameter $\Delta r = r_o - r_i$ of the expression of self-electric and self-magnetic fields [10, Eqs. (9) - (11)] were obtained for the general case. The equilibrium equations under those assumptions have the form

$$\gamma_0\left(1 - \frac{\overline{\pi_z^2(\rho_{i,eq},\zeta)}}{\gamma_0^2}\right)\overline{e}_r^{self}(\rho_{i,eq}) + \overline{\pi_\theta(\rho_{i,eq},\zeta)b_z^{ext}(\rho_{i,eq},\zeta)} + \overline{\pi_\theta(\rho_{i,eq},\zeta)}(\overline{b}_z^{self}(\rho_{i,eq}) + b_{z0}^{ext}) + \frac{\overline{\pi_\theta^2(\rho_{i,eq},\zeta)}}{\rho_{i,eq}} = 0,$$

$$\gamma_0\left(1 - \frac{\overline{\pi_z^2(\rho_{o,eq},\zeta)}}{\gamma_0^2}\right)\overline{e}_r^{self}(\rho_{o,eq}) + \overline{\pi_\theta(\rho_{o,eq},\zeta)b_z^{ext}(\rho_{o,eq},\zeta)} + \overline{\pi_\theta(\rho_{o,eq},\zeta)}(\overline{b}_z^{self}(\rho_{o,eq}) + b_{z0}^{ext}) + \frac{\overline{\pi_\theta^2(\rho_{o,eq},\zeta)}}{\rho_{o,eq}} = 0. \tag{1}$$

Here $\rho_{i,eq}$, $\rho_{o,eq}$ are the dimensionless equilibrium inner and outer beam radii; $\bar{\pi}_z(\rho) = \bar{p}_z(\rho)/(m_q c)$ are the dimensionless z-components of momentum; $\vec{b}_{z0}^{ext} = \vec{B}_{z0}^{ext}(r,z)qr_2/(m_q c^2)$; $\vec{B}_0^{ext} = (0,0,B_{z0}^{ext})$ is the external homogeneous static magnetic field produced by a solenoid; $\vec{b}_\perp^{ext}(r,z) = \vec{B}_\perp^{ext}(r,z) \times qr_2/(m_q c^2)$; $\vec{B}_\perp^{ext}(r,z) = (B_r^{ext}(r,z), 0, B_z^{ext}(r,z))$ is the periodic undulator magnetostatic field produced by a system of permanent magnets [8, 14, 15];

$$B_r^{ext}(r,z) = -B_\perp^m \sum_{k=0}^{\infty} C_{2k+1} \times \tag{2}$$
$$\times \sin\left((2k+1)(k_w z - \frac{\pi}{4})\right) F_{2k+1}^{(1)}((2k+1)k_w r),$$

$$B_z^{ext}(r,z) = -B_\perp^m \sum_{k=0}^{\infty} C_{2k+1} \times \tag{3}$$
$$\times \cos\left((2k+1)(k_w z - \frac{\pi}{4})\right) F_{2k+1}^{(0)}((2k+1)k_w r),$$

where $k_w = 2\pi/l_w$; $b_\perp^m = B_\perp^m(qr_2/m_q c^2)$; $B_\perp^m$ is the value of longitudinal component of the magnetic induction on cylindrical surfaces of the permanent magnets of the undulator (for simplicity we suppose that these surfaces are located at $r = r_1$ and $r = r_2$); z is the longitudinal coordinate;

$$C_{2k+1} = \frac{4}{(2k+1)\pi} \sin\left[\frac{(2k+1)\pi}{4}\right];$$

$$F_{2k+1}^{(0)}((2k+1)k_w r) = f_{2k+1} I_0((2k+1)k_w r) -$$
$$- g_{2k+1} K_0((2k+1)k_w r),$$

$$F_{2k+1}^{(1)}((2k+1)k_w r) = f_{2k+1} I_1((2k+1)k_w r) +$$
$$+ g_{2k+1} K_1((2k+1)k_w r),$$

$$f_{2k+1} = \frac{K_0((2k+1)k_w r_1) + K_0((2k+1)k_w r_2)}{\Delta_{2k+1}},$$

$$g_{2k+1} = \frac{I_0((2k+1)k_w r_1) + I_0((2k+1)k_w r_2)}{\Delta_{2k+1}},$$

$$\Delta_{2k+1} = I_0((2k+1)k_w r_1) K_0((2k+1)k_w r_2) -$$
$$- I_0((2k+1)k_w r_2) K_0((2k+1)k_w r_1),$$

$I_0(\cdot)$, $I_1(\cdot)$ are Bessel functions of 0- and 1-order; $K_0(\cdot)$, $K_1(\cdot)$ are modified Bessel functions of 0- and 1-order; $\bar{e}_r^{self} \equiv \bar{E}_r^{self}(r)(qr_2/m_q c^2)$, $\bar{b}_\theta^{self} \equiv \bar{B}_\theta^{self}(r)(qr_2/m_q c^2)$, $\bar{b}_z^{self} \equiv \bar{B}_z^{self}(r)(qr_2/m_q c^2)$ are the radial component of the dimensionless self-electric and the θ- and z-components of the dimensionless self-magnetic fields of the beam, respectively;

$$\bar{E}_r^{self}(r) = -\frac{I_0 \gamma_0}{rp_{z0} \ln(r_2/r_1)} \times$$
$$\times \begin{cases} G, & r_1 \le r < r_i; \\ G - 2\ln(r_2/r_1)\dfrac{r^2 - r_i^2}{r_o^2 - r_i^2}, & r_i \le r \le r_o; \\ G - 2\ln(r_2/r_1), & r_o < r \le r_2; \end{cases} \tag{4}$$

$$\bar{B}_\theta^{self}(r) = \frac{\bar{p}_z(r)}{\gamma_0} \bar{E}_r^{self}(r); \tag{5}$$

$$\bar{B}_z^{self}(r) = -\frac{I_0 \bar{p}_\theta(r)}{p_{z0}} \times$$
$$\times \begin{cases} \dfrac{r_o^2 + r_i^2 - 2r_2^2}{r_2^2 - r_1^2}, & r_1 \le r < r_i; \\ 2\dfrac{r^2 - r_i^2}{r_o^2 - r_i^2} + \dfrac{r_o^2 + r_i^2 - 2r_2^2}{r_2^2 - r_1^2}, & r_i \le r \le r_o; \\ \dfrac{r_o^2 + r_i^2 - 2r_1^2}{r_2^2 - r_1^2}, & r_o < r \le r_2; \end{cases} \tag{6}$$

$I_0 = qn_0 \upsilon_{z0} \pi(r_o^2 - r_i^2)$ is the beam current; q is the charge of particles (for electrons $q = -|e|$, e is the electron charge); $\upsilon_{z0} = p_{z0}/(m_q \gamma_0)$ is the initial longitudinal beam velocity; r_1, r_2 are the inner and outer drift-tube radii;

$$G = 1 + 2\ln(r_2/r_o) - \frac{2r_i^2}{r_o^2 - r_i^2} \ln(r_o/r_i)$$

is the geometrical factor. Using the moment conservation law and the assumptions $\gamma \equiv \gamma_0$ we can easy find the following summands in the system (1):

$$\overline{\pi_z^2}(\rho, \zeta) = \pi_{z0}^2 - \overline{\pi_\theta^2}(\rho, \zeta), \tag{7}$$

$$\overline{\pi_\theta(\rho_{i,eq}, \zeta) b_z^{ext}(\rho_{i,eq}, \zeta)} = -\left[1 + \frac{i_0 Q(\rho)}{2\pi_{z0}(1 - \rho_1^2)}\right]^{-1} \times$$
$$\times \frac{(b_\perp^m)^2}{2\kappa_w} C_1^2 F_1^{(0)}(\kappa_w \rho) F_1^{(1)}(\kappa_w \rho), \tag{8}$$

$$\overline{\pi_\theta}(\rho, \zeta) = -\left[1 + \frac{i_0 Q(\rho)}{2\pi_{z0}(1 - \rho_1^2)}\right]^{-1} \frac{\rho b_{z0}^{ext}}{2}\left(1 - \delta_c \frac{\rho_c^2}{\rho^2}\right), \tag{9}$$

$$\overline{\pi_\theta^2}(\rho, \zeta) = \left[1 + \frac{i_0 Q(\rho)}{2\pi_{z0}(1 - \rho_1^2)}\right]^{-2}\left[\frac{\rho^2 (b_{z0}^{ext})^2}{4}\left(1 - \delta_c \frac{\rho_c^2}{\rho^2}\right)^2 + \frac{(b_\perp^m)^2}{2\kappa_w^2} C_1^2 (F_1^{(1)}(\kappa_w \rho))^2\right], \tag{10}$$

where $i_0 = I_0/I_A$ is the dimensionless beam current ($I_A = -17.05$ kA for electrons);

$$Q(\rho) = \frac{(\rho_o^2 - \rho^2)^2 (1 - \rho_1^2)}{(\rho_o^2 - \rho_i^2)} - (1 - \rho^2)\left[\rho_i^2 + \rho_o^2 - 2\rho_1^2\right]$$

is yet another geometrical factor.

2. NUMERICAL SOLUTIONS OF EQUILIBRIUM EQUATION

Previously, it was shown [14, Fig. 2] that, in the absence of a longitudinal magnetic field over a wide range of values of the longitudinal component of the magnetic induction on the cylindrical surfaces of the permanent magnets of the undulator, $B_\perp^m$, and beam current, I_0, there exists the solution to Eq. (15) from [14] corresponding to the equilibrium values of inner and outer radii of charged-particle beam in the coaxial geometry of the drift tube.

In the Fig. 1 the dependence of the equilibrium radii on the beam injection current for different values of the longitudinal component of magnetic induction on cylindrical surface of permanent magnets is shown.
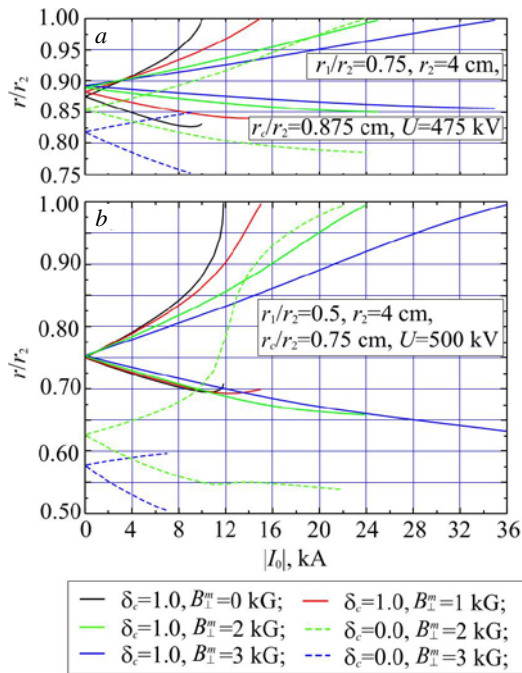
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Fig. 1. Dependence of the equilibrium radii on the beam injection current for different values of the longitudinal component of magnetic induction on cylindrical surface of permanent magnets for $B_{z0}^{ext} = 1\,kG$: Strathclyde FEL/FEM [4, 9] (a) and ubitron considered in [14, 15] (b)

We see that with increasing of the beam injection current the inner beam radius decreases and the outer beam radius increases. Also we note that for the shield cathode the acceptable values of injection current and the inner and outer radii are smaller than for the non-shield cathode.
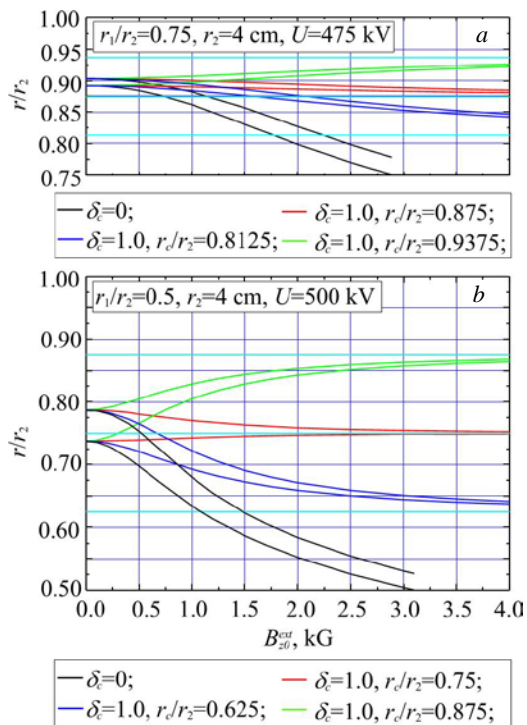

Fig. 2. Dependence of the equilibrium radii on the induction of longitudinal homogenous magnetic field for different values of the cathode radius for $B_{\perp}^{m} = 4\,kG$ and $|\,I_0\,| = 3\,kA$: Strathclyde FEL/FEM [4, 9] (a) and ubitron considered in [14, 15] (b)

In Figs. 2, 3 dependence of the equilibrium radii on the induction of longitudinal homogenous magnetic field for different values of the cathode radius is presented. By choosing the radius r_c of the non-shield cathode and the value of longitudinal guide magnetic field we can control the position of equilibrium radii of electron beam in a drift tube so that one can transport the electron beam in any suitable position. Therefore the electron beam can amplify efficiently arbitrary mode of the coaxial drift tube.
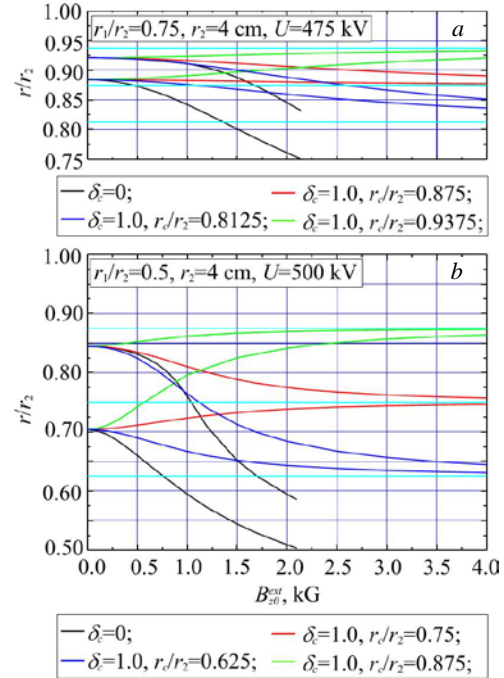

Fig. 3. Dependence of the equilibrium radii on the induction of longitudinal homogenous magnetic field for different values of the cathode radius for $B_{\perp}^{m} = 4\,kG$ and $|\,I_0\,| = 10\,kA$: Strathclyde FEL/FEM [4, 9] (a) and ubitron considered in [14, 15] (b)
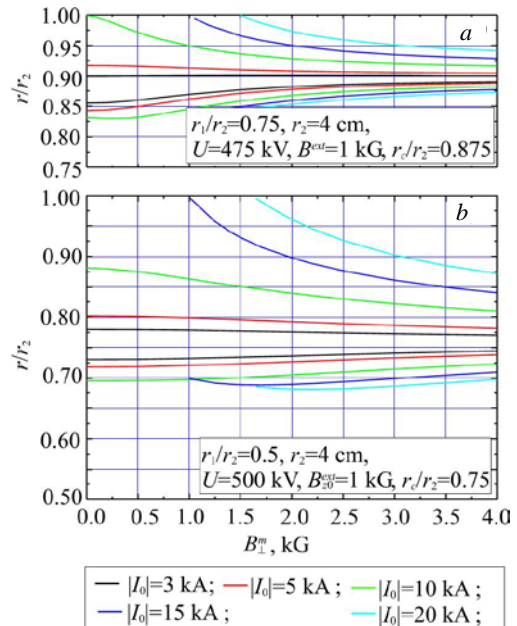

Fig. 4. Dependence of the equilibrium radii on the longitudinal component of magnetic induction on cylindrical surface of permanent magnets for different values of the beam injection current for $B_{z0}^{ext} = 1\,kG$: Strathclyde FEL/FEM [4, 9] (a) and ubitron considered in [14, 15] (b)

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The dependence of the equilibrium radii on the longitudinal component of magnetic induction on cylindrical surface of permanent magnets for different values of the beam injection current is shown in Fig. 4. We see that for all values of the beam current with the increase of the longitudinal component of magnetic induction on cylindrical surface of permanent magnets the inner equilibrium radius increases and the outer equilibrium radius decreases.

CONCLUSIONS

In the approximation of constant beam density and "rigid rotor" type of rotation the equilibrium steady state of high-current electron beam in a coaxial drift tube with a shielded and non-shielded cathodes in the hybrid longitudinal homogeneous (guide) magnetic and magnetostatic undulator fields is studied. The position of equilibrium radii of electron beam in a coaxial drift tube can be controlled by choosing the cathode radii and the value of the longitudinal homogeneous (guide) magnetic field so that the electron beam can be transported in any suitable position and can amplify efficiently arbitrary mode of the coaxial drift tube.

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РАВНОВЕСИЕ СИЛЬНОТОЧНОГО ПУЧКА ЭЛЕКТРОНОВ В КОАКСИАЛЬНОМ ОНДУЛЯТОРЕ С ЧАСТИЧНО ЭКРАНИРОВАННЫМ КАТОДОМ В ВЕДУЩЕМ ОДНОРОДНОМ МАГНИТНОМ ПОЛЕ

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Сформулированы уравнения равновесия для начальных радиусов электронного пучка, на которых дефокусирующее влияние собственного электрического поля компенсируется фокусирующими силами ондулятора и продольной и азимутальной компонент собственного магнитного поля электронного пучка с учетом первой (доминирующей) гармоники ондуляторного магнитного поля в коаксиальном магнитном ондуляторе с частично экранированным катодом. Сравниваются численные решения уравнений равновесия в коаксиальном магнитном ондуляторе с экранированным и неэкранированным катодами.

РІВНОВАГА СИЛЬНОСТРУМОВОГО ПУЧКА ЕЛЕКТРОНІВ У КОАКСІАЛЬНОМУ ОНДУЛЯТОРІ З ЧАСТКОВО ЕКРАНОВАНИМ КАТОДОМ У ВЕДУЧОМУ ОДНОРІДНОМУ МАГНІТНОМУ ПОЛІ

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Сформульовані рівняння рівноваги для початкових радіусів електронного пучка, на яких дефокусуючий вплив власного електричного поля компенсується фокусуючими силами ондулятора та поздовжньої і азимутальної компонент власного магнітного поля електронного пучка з урахуванням першої (домінуючої) гармоніки ондуляторного магнітного поля в коаксіальному магнітному ондуляторі з частково екранованим катодом. Порівнюються чисельні рішення рівнянь рівноваги в коаксіальному магнітному ондуляторі з екранованим та неекранованим катодами.