ON EQUILIBRIUM STATES
OF SUPERFLUID WITH d-WAVE PAIRING

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The equilibrium states of quantum fluid with d-pairing classification is carried out on the basis of the quasiaverages concept. It is shown, that the set of such an equilibrium states can be classified in the terms of the orbital momentum quantum number, relevant to the projection of the orbital momentum of the superconducting electron pair on an anisotropy direction. The explicit expression of three admissible unbroken symmetry generators and relevant order parameter equilibrium values are found.
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INTRODUCTION

Classification of condensed media equilibrium states which, using the phenomenological Ginzburg-Landau approach, requires knowledge of a free energy as functions of the order parameter and essentially depends on an considered model aspect. Other group-theoretic approach is based on the unbroken symmetry representation of an equilibrium degenerated state as on the subgroup of a normal phase symmetry. Corresponding transformational properties of an order parameter at the transformations of Hamiltonian symmetry are essential in this approach. This review is free from any model suppositions about a free energy aspect. The classification of homogeneous states within the framework of both indicated approaches was carried out for superfluid ^{3}He [1-3], which is featured by a tensor order parameter. The problem of classification of possible nonuniform superfluid equilibrium states of this quantum fluid is researched in [4]. Other important example of the degenerated condensed matter is the superfluid in a state of d-pairing, which also is featured by the tensor order parameter [5]. The possible states classification for this pairing is carried out on the basis of the phenomenological approach in [5-6]. The microscopic approach for homogeneous states classification of an equilibrium which is based on the quasiaverages concept [7-8,4] was proposed in this work. The admissible symmetry properties of a quantum fluid equilibrium state and relevant order parameter structures are at nonzero values of an order parameter from the unbroken symmetry requirements.

THE BASIC EQUATIONS

The statistical physics of condensed matter with the spontaneously broken symmetry theoretical base is the concept of quasiaverages by N.N. Bogoliubov [7], which extends Gibbs statistical operator on degenerated condensed states. According to [7], the quasiaverages of a physical value in a state of a statistical equilibrium with the broken symmetry is determined by the formula

$$\left\langle \hat{a}(x)\right\rangle \equiv \lim_{\nu\to 0}\lim_{V\to\infty} Sp\hat{w}_V\, \hat{a}(x), \tag{1}$$

$$\hat{w}_V \equiv \exp(\Omega_V - Y_a\hat{\gamma}_a - \nu\hat{F}). \tag{2}$$

Here $\hat{\gamma}_a$ - additive integrals of motion ($\hat{H}$ – Hamiltonian, $\hat{P}_k$ – impulse operator, $\hat{N}$ – particle number operator, $\hat{S}_\alpha$ – spin operator), $(Y_a \equiv Y_0, Y_k, Y_4, Y_\alpha)$ - thermodynamic forces, relevant to them. For convenience of the further account we suppose, that in a laboratory co-ordinates in an equilibrium state there are no macroscopic fluxes, i.e. $Y_k = 0$ and the external magnetic field is equal to zero $Y_\alpha = 0$. The thermodynamic potential Ω_V is defined from a normalization conditions $Sp\hat{w} = 1$. The operator $\hat{F}$ has the considered phase symmetry of a condensed media and represents as linear functional of the order parameter operator

$$\hat{F} = \int d^3x\left(f_{ik}(x)\hat{\Delta}_{ik}(x) + h.c.\right). \tag{3}$$

Here $f_{ik}(x)$ - some function of coordinates, conjugate order parameter operator, which sets its equilibrium values in sense quasiaverages $\Delta_{ik}(x) = \left\langle \hat{\Delta}_{ik}(x)\right\rangle$. The functions structure $f_{ik}(x)$ is determined by properties of the quantum fluid researched states symmetry. It allows within the microscopic theory framework to define the additional thermodynamic parameters into Gibbs statistical operator. We define the order

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parameter operator of d-pairing $\hat{\Delta}_{ik}(x)$ in the terms of the operators of creation and annihilation of Fermi - particle in a point x :

$$\hat{\Delta}_{ik}(x) \equiv \nabla_i \hat{\psi}(x)\sigma_2 \nabla_k \hat{\psi}(x) + \nabla_k \hat{\psi}(x)\sigma_2 \nabla_i \hat{\psi}(x) - \frac{2}{3}\delta_{ik} \nabla_j \hat{\psi}(x)\sigma_2 \nabla_j \hat{\psi}(x) \quad (4),$$

where σ_2 - Pauli matrix. We define the operators of number of particles $\hat{N}$, impulse $\hat{P}_k$, spin $\hat{S}_\alpha$ and orbital moment $\hat{L}_k$

$$\hat{N} = \int d^3x \hat{n}(x), \hat{P}_i = \int d^3x \hat{\pi}_i(x), \hat{S}_i = \int d^3x \hat{s}_i(x), \hat{L}_i = \int d^3x \hat{l}_i(x), \quad (5)$$

where the relevant densities of motion integrals $\hat{n}(x)$, $\hat{\pi}_i(x)$, $\hat{s}_i(x)$, $\hat{l}_i(x)$, in the terms of the operators of creation and annihilation $\hat{\psi}^+(x), \hat{\psi}(x)$,are

$$\hat{n}(x) = \hat{\psi}^+_\sigma(x)\hat{\psi}_\sigma(x),$$
$$\hat{\pi}_i(x) = -\frac{i}{2}\left\{\hat{\psi}^+_\sigma(x)\nabla_i\hat{\psi}_\sigma(x) - \nabla_i\hat{\psi}^+_\sigma(x)\hat{\psi}_\sigma(x)\right\}, \quad (6)$$
$$\hat{s}_\alpha(x) = \hat{\psi}^+_\sigma(x)(s_\alpha)_{\sigma\sigma'}\hat{\psi}_{\sigma'}(x), \hat{l}_i(x) = \varepsilon_{ikl}x_k\hat{\pi}_l(x).$$

Using definitions (4) - (6) we obtain operator algebra:

$$\left[\hat{N}, \hat{\Delta}_{ik}(x)\right] = -2\hat{\Delta}_{ik}(x),$$
$$\left[\hat{S}_\alpha, \hat{\Delta}_{ik}(x)\right] = 0,$$
$$i\left[\hat{P}_l, \hat{\Delta}_{ik}(x)\right] = -\nabla_l \hat{\Delta}_{ik}(x),$$
$$i\left[\hat{L}_l, \hat{\Delta}_{ik}(x)\right] = -\varepsilon_{lij}\hat{\Delta}_{jk}(x) - \varepsilon_{lkj}\hat{\Delta}_{ji}(x) - \varepsilon_{lkj}x_k\nabla_j\hat{\Delta}_{ik}(x) \quad . \quad (7)$$

The mean values of the order parameter $\Delta_{ik}(x,\hat{\rho}) = Sp\hat{\rho}\hat{\Delta}_{ik}(x)$, ($\hat{\rho}$ - the arbitrary statistical operator) have properties $\Delta_{ik}(x,\hat{\rho}) = \Delta_{ki}(x,\hat{\rho})$, $\Delta_{ii}(x,\hat{\rho}) = 0$. We have ten independent values in this case. We choose a parameterization of $\Delta_{ik}(x,\hat{\rho})$ in the form

$$\Delta_{ik}(x,\hat{\rho}) \equiv Q_{ik}(x,\hat{\rho}) + i\underline{Q}_{ik}(x,\hat{\rho}),$$
$$Q_{ik} \equiv A[n_i n_k - \frac{1}{3}\delta_{ik}] + B[m_i m_k - \frac{1}{3}\delta_{ik}], \quad (8)$$
$$\underline{Q}_{ik} \equiv C\left(n_i n_k - \frac{1}{3}\delta_{ik}\right) + D\left(m_i m_k - \frac{1}{3}\delta_{ik}\right) + E\left(m_i n_k + m_k n_i\right) + F\left(n_i l_k + n_k l_i\right) + G\left(l_i m_k + l_k m_i\right).$$

Here $A, B, .., G$ - the modules of the order parameter, n, m, l are the axes of an anisotropy, also represent unitary and mutually perpendicular vectors $\vec{n}^2 = \vec{m}^2 = \vec{l}^2 = 1$, $nm = 0$, $lm = 0$, $nl = 0$.

The condensed matter in a normal equilibrium state is characterized by symmetry properties

$$\left[\hat{w}, \hat{P}_k\right] = 0, \left[\hat{w}, \hat{L}_k\right] = 0, \left[\hat{w}, \hat{S}_\alpha\right] = 0, \left[\hat{w}, \hat{N}\right] = 0. \quad (9)$$

These properties reflect a translational invariance, spatial and spin isotropy, phase invariance of a normal phase equilibrium state of condensed matter. A requirement of a spatial isotropy in (9) and quantum brackets algebra (7) give the equality

$$Sp\hat{w}\hat{\Delta}_{ik}(x) = 0,$$

in an aspect of the chosen directions lack in the equilibrium state.

We consider possible equilibrium order parameter structures in the translational-invariant states of a superfluid state equilibrium. The analysis of translational-invariant subgroups of the equilibrium states unbroken symmetry, according to [4] is feasible from relations

$$\left[\hat{w}, \hat{T}\right] = 0, \left[\hat{w}, \hat{P}_k\right] = 0, \quad (10)$$

where $\hat{T}$ - the generator of the unbroken symmetry, which represents the linear motion integrals combination

$$\hat{T} \equiv a_i \hat{L}_i + b_\alpha S_\alpha + c\hat{N}, \quad (11)$$

here a_i, b_α, c - the real parameters. The unitary transformations $U = \exp i\hat{T}(a)$ form continuous subgroups of the equilibrium state unbroken symmetries $U(a)U(a') = U(a''(a,a'))$. Agrees (10), (11) we get $Sp\left[\hat{w}, \hat{T}\right]\hat{\Delta}_{ik}(x) = 0$. Therefore, taking into account algebra (7), we get the system of linear and homogeneous equations

$$F^{ik}_{jl}\Delta_{jl} = 0, \quad (12)$$

where

$$a_l\left(\varepsilon_{lij}\delta_{kl} + \varepsilon_{lkj}\delta_{il}\right) + 2ic\delta_{kl}\delta_{ji} \equiv F^{ik}_{jl}.$$

Passing in the formula (12) from double summation to unary, at which the indexes $ik; jl$ of summation possess the values α, β : $11 \equiv 1$, $12 \equiv 2$, $...33 \equiv 9$ we obtain the equation

$$F^\beta_\alpha \Delta_\alpha = 0. \quad (13)$$

The requirements $\det\left|F^\beta_\alpha\right| = 0$ (it is the condition of existing of a nontrivial system linear and homogeneous equations solution (13) $\Delta_{ik}(x) \neq 0$) are fulfilled at c=0, $\pm a$, $\pm a/2$. The three solutions of the equation $\det\left|F^\beta_\alpha\right| = 0$ are as follows

$$1)\ c = 0,\ 2)\ c = \pm a,\ 3)\ c = \pm \frac{1}{2} a. \qquad (14)$$

Hence, the unbroken symmetry generators of the superfluid fluid equilibrium state with d-pairing are

$$\left[\hat{w}, \hat{S}_{\alpha}\right] = 0, \left[\hat{w}, \frac{a_i}{a}\hat{L}_i - \frac{m_l}{2}\hat{N}\right] = 0. \qquad (15)$$

Here m_l - quantum number accepting the values 0, 1, 2. Using the second relation (15) and taking into account (8) we obtain the equations, defining the explicit form of the order parameter in equilibrium

$$\frac{a_i}{a}\left(\varepsilon_{iuj}\underline{Q}_{jv} + \varepsilon_{ivj}\underline{Q}_{ju}\right) - m_l Q_{uv} = 0, \qquad (16)$$

$$\frac{a_i}{a}\left(\varepsilon_{iuj}Q_{jv} + \varepsilon_{ivj}Q_{ju}\right) + m_l \underline{Q}_{uv} = 0.$$

SOLUTION OF THE EQUATIONS AND DISCUSSION OF THEM

Solving obtained set of equations and using representation (8) we find the order parameter structure in the equilibrium state in this case

$$\Delta_{uv} = (A + iC)\left\{n_u n_v - \frac{1}{3}\delta_{uv}\right\}, \qquad (17)$$

here for simplicity the vectors a and n are chosen collinear. The equality in (17) is similar to an order parameter structure of a liquid crystal body with complex amplitude.

At $m = \pm 2$ we obtain the solution

$$\Delta_{uv} = G\left\{\mp n_u n_v \mp 2 m_u m_v \pm \delta_{uv} + i\left(l_v m_u + l_u m_v\right)\right\}. \qquad (18)$$

For a determination of the solution at the quantum number value $m = \pm 1$, the vector a on the indicated coordinate is decomposable $a/a = \alpha n + \beta m + \gamma l$. Here α, β, γ are bound by equality $\alpha^2 + \beta^2 + \gamma^2 = 1$. From (13) we get the solution

$$\Delta_{uv} = A\left\{n_u n_v + 2 m_u m_v - \delta_{uv} \pm \right.,$$

$$\left. \pm \frac{i}{\sqrt{2}}\left[m_v n_u + m_u n_v + n_v l_u + n_u l_v\right]\right\}, \qquad (19)$$

thus $\beta = \gamma = \pm 2^{-1/2}, \alpha = 0$.

To compare the obtained outcomes with results of works [5,6] we normalize the order parameter by a relation

$$\Delta_{ik}\Delta^{*}_{ki} = 1. \qquad (20)$$

Agrees (17) - (19), (20) we find $A^2 + C^2 = 3/2$, $G = \pm 1/2$, $A = \pm 1/2$. According to the paper [5] we define mean value

$$\Delta \equiv k_i \Delta_{ij} k_j, \qquad (21)$$

where the unit vector k is defined by equality

$$k \equiv \{\cos\theta\, l, \sin\theta \sin\varphi\, m, \sin\theta \cos\varphi\, n\} \equiv \{k_z, k_y, k_x\}. \qquad (22)$$

For a solution (17) according to (21), (22) we write

$$\Delta^{(1)} = (A + iC)\left(k_z^2 - 1/3\right).$$

This solution corresponds the "real" state. The solution (18) in according to (21) (22) gives the equality

$$\Delta^{(2)} = \left(k_x + ik_y\right)^2/2,$$

which coincides with "axial" state of works [5,6]. At last, in a case (19) we obtain

$$\Delta^{(3)} = (\beta + \gamma)\left(i\sqrt{2}k_z + k_y - k_x\right). \qquad (23)$$

Comparing this result with "cyclic" state from papers [5,6], we can't notice the equivalence of them. They coincide only with α=0 and ζ==0.

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