ON THE EXPERIMENTAL VALUE OF THE Δ^+(1232) MASS FROM $\gamma p \rightarrow n\pi^+(p\pi^0)$ REACTIONS

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The reliability of the phenomenological estimates for the Δ^+(1232) mass is questioned coming from the of π^+ and π^0 mesons photoproduction data off proton target till the end of 2001. The origin of an old discrepancy for the Δ^+(1232) mass presented in tables of the Particle Data Group is discussed.

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1. INTRODUCTION

Up to present time the single pion photoproduction reactions are the only source of information about the parameters of the Δ^+(1232) resonance [1]. This information was obtained from the resonance fitting of the resonant multipoles leading to the final πN state with isotopic spin T=3/2 and total momentum J=3/2 that are obtained in several energy independent analyses at condition that Watson's theorem is not used in treating these multipoles. During the last decades the most striking point here was the significant disagreement between the Δ^+(1232) mass 1234.9±1.4 MeV obtained by authors of work [2] using the results of their preceding energy independent analysis with "free" imaginary part of the magnetic dipole resonant amplitude [3], and other papers cited in [1]. In particular, this value is considered as inconsistent with Δ^0 and Δ^{++} masses, 1233.6±0.5 MeV and 1230.9±0.3 MeV respectively found in more recent pion nucleon analyses [4].

It should be noted that Δ^0 and Δ^{++} masses are defined as energy at which the corresponding πN phase shifts are crossing the value π/2, or, equivalently, the real parts of the resonant partial waves equal zero (the "experimental observed" values). They can be found, for example, by fitting the P_{33} amplitudes with using the Breit-Wigner resonance formulas. Of course, one can try to get more underlying resonance parameters by separating the amplitude into the "proper resonance" and a background. Such problem is rather complicated and model dependent. The masses obtained in this way have different physical meaning. Usually they are very different from the values mentioned above (for example, 1241 MeV [5], 1251.1 MeV [6], 1235.1 MeV and 1256.0 MeV [7]). In general there is the whole hierarchy of mass and width parameters describing the Δ(1232) isobar: a) the "experimental" values we have just discussed refer to the "dressed" resonance with unitary addition of the background; b) parameters of the resonance, dressed by independently introduced background; c) parameters of the "pure" resonance propagating in the absence of any background; d) mass of the "bare" resonance possessing zero width (relevant calculations in the framework of a nonrelativistic model [8,9]can be found in [10]). It is clear that one can discuss some effect of the isotopic splitting only comparing the parameters corresponding to the same level of the dynamical description. In particular the mentioned before Δ^+ mass has to be searched as the energy at which the real parts of the resonant multipoles pass through zero.

Here we estimate the role of different factors at determination of the "experimental" Δ^+(1232) parameters by direct fitting the modern photoproduction data. In this calculations the resonance parameters were determined as ones of the Walker's formula [11] invoked to describe the resonant multipole amplitudes, with a little generalization introduced to test the energy excitation function.

2. THE RESONANCE MODEL OF THE Δ^+(1232) PHOTOEXCITATION

In the first resonance region the real parts of the background single pion photoproduction multipoles were taken as the Born approximation completed in *s*-, *p*- and *d*-multipoles by the following cubic polynomials (indexes + and – correspond to the total angular momenta j=l±1/2):

$$\mathrm{Re}\, M^I_{l\pm}(E_\gamma) =$$

$$\sum_{i=1}^{i=4} \mathrm{Re}\, M^I_{l\pm}(E_\lambda^{(i)}) \prod_{\substack{j=1 \\ (j\neq i)}}^{j=4} (E_\gamma - E_\lambda^{(j)}) \Big/ (E_\gamma^{(i)} - E_\gamma^{(j)}). \tag{1}$$

Here $M^I_{l\pm} \equiv A^I_{l\pm}, B^I_{l\pm}$ are defined according to [11] spiral multipoles with following isospin structure

$A^{1/2} = 1/3\, A(\pi^0) + \sqrt{2/3}\, A(\pi^+)$,
$A^{3/2} = A(\pi^0) - 1/\sqrt{2}\, A(\pi^+)$.

Index *l* is the orbital angular momentum, $E_\gamma^{(i)}$ is a knot value of the current photon energy E_γ. Taking the P_{33} πN scattering partial amplitudes to be purely elastic imaginary parts of the spiral background multipoles up to *l*=3 were calculated according the Watson's theorem with using the phase shifts $\delta_{2I,\,2(l\pm)}$. from the πN elastic scattering analyses:

$$\mathrm{Im}\, M^{I}_{l\pm} = \mathrm{Re}\, M^{I}_{l\pm}\, tg(\delta_{2I,\,2(l\pm)}). \qquad (2)$$

The resonant multipole amplitudes $A^{3/2}_{1+}$ and $B^{3/2}_{1+}$ were written using the Walker's formula for contribution of the Δ(1232) resonance:

$$M^{3/2}_{1+}(W) =$$

$$C_M\, \mathrm{Im}\, M^{3/2}_{1+}(W_0)\sqrt{\frac{k_0 q_0}{kq}}\;\frac{W_0\sqrt{\Gamma_\pi \Gamma_\gamma}}{W_0^2 - W^2 - i W_0 \Gamma_\pi}. \qquad (3)$$

Here W and W_0 are the total c. m. energy and its value at resonance, respectively, $C_M \equiv \mathrm{Im}\, M^{3/2}_{1+}(W_0)$ is the resonance constant. The energy dependent widths were parameterized with introduction two phenomenological parameters X_π, X_γ:

$$\Gamma_\gamma(W) = \Gamma_0 \left(\frac{k}{k_0}\right)^{2g} \left(\frac{k_0^2 - X_\gamma^2}{k^2 - X_\gamma^2}\right), \qquad (4)$$

$$\Gamma_\pi(W) = \Gamma_0 \left(\frac{q}{q_0}\right)^{3} \left(\frac{q_0^2 - X_\pi^2}{q^2 - X_\pi^2}\right). \qquad (5)$$

In Eqs. (4)-(5) k, q are the c. m. momenta of the photon and pion, respectively, k_0, q_0 being the corresponding quantities at W_0. The parameter g is controlling the energy dependence of the resonant multipoles via ratio k/k_0: at g=0.5 such a contribution in energy dependence is absent at all, at g=1 and $X_\pi = X_\gamma \equiv X$ the Walker's formula for Δ(1232) is reproduced, and g=1 corresponds to the kq factor appearing in the resonant multipoles in the Born approximation.

In our approach this resonance mechanism represents the whole resonant multipoles without any additional background contributions. Accordingly, parameters M_0 and Γ_0 are the "experimentally observed" mass and width discussed in Introduction.

3. NUMERICAL CALCULATIONS

The calculations were fulfilled for several variants involving one at a time a change in different factors concerning the model used or modification in the load of the experimental data to compare results with some looking to be realistic standard variant. The initial experimental information on pion photoproduction on proton was taken from compilation [12] up December 2001 in photon energy interval from 280 MeV to 400 MeV (differential cross section and polarization data). The energy knots 280, 320, 360, and 400 MeV in Eq. (1) were introduced for the real parts of the background multipoles. At calculating the imaginary parts of the multipoles via Eq. (2) the πN phase shifts from the Arndt's analysis [13] were used.

In Table 1 the χ^2 value per number of degree of freedom N_{df}, values of the resonance mass and width obtained are presented at level with the corresponding ratio EMR of the electrical quadrupole and the magnetic dipole amplitudes at resonance. We used this ratio as some additional criterion for quality estimation of our solutions.

For the standard variant (the first line in Table 1) the resonance quantities at the level with the knot values of the real parts of background multipoles A_{0+}, A_{1+}, A_{1-}, A_{2-}, B_{1+}, B_{2-} with isotopic I=1/2, and A_{0+}, A_{1-} for T=3/2 were determined by minimization of standard χ^2 without introducing rating factors for any type of observable. All other multipoles were taken into account in electrical Born approximation being unitarized up to the partial waves with orbital momentum l=3. Parameter X was fixed at the Walker's value 185 MeV, and g=1. We had to ignore the data provided by some experiments with χ^2/N_{df} significantly exceeding 9.0. These are mentioned in Table 2 in terms of the labels from compilation [12] with addition of two letters from the name of the laboratory. Accordingly, we used 3124 experimental points in this run.

Each subsequent line in Table 1 presents a run involving some change in the data or modification in the background or in the resonance description. The concise clarification is presented in the relevant comment. Some additional remarks are given below.

*Line 1.*The experimental $\boldsymbol{\Delta}^{+}$ mass for the variant practically coincides with the $\boldsymbol{\Delta}^{0}$ mass.

*Line 2.*Restoration of the "bad" data increased $\chi^2/\mathrm{N}_{\mathrm{df}}$, with a small change of the $\boldsymbol{\Delta}^{+}$ width only.

*Line 3.*In the *s*-, *p*-wave variant the $\boldsymbol{\Delta}^{+}$ mass is situated in the middle of the interval ($\boldsymbol{\Delta}^{++}$, $\boldsymbol{\Delta}^{o}$). Besides, the large value of theEMR ratio is coming close to the upper values for E2/M1 cited in [1].

*Line 4.*Involving in fitting all background *d*-waves makes $\boldsymbol{\Delta}^{+}$ mass smaller. Large number of iterations.

*Line 5.*Introducing of the full Born approximation makes our fit worse with significant enlargement of the mass.

*Line 6.*The data before 1967 allow determination neither the $\boldsymbol{\Delta}^{+}$ mass nor the width.

*Line 7.*The database available at time of work [3].

Line 8. At this time the Kharkov experiments in the first resonance region have already been done. The best-looking result for the mass splitting. The ratio EMR (≅ – 1.5%) is close to the E2/M1 for this years [1,14].

*Line 9.*X= 300.30 ± 15.11, significant correlations, many iterations. Weak dependence on X.

Line 10.X_{γ}=185 MeV, free parameter X_π goes to ∞.

Line 11.X_π=185 MeV, free parameter X_γ goes to zero and is strongly correlated.

*Line 12.*Introducing the *kq* dependence of the resonance multipoles gives unreasonably small mass.

*Line 13.*Relaxation of the resonance energy dependence via ratio k/k_{o} gives too large mass.

*Line 14.*Variant with taking into account the true π^0 mass in the *q* factor for cross section and in the resonance contribution at π^0 production. The mass shift is three times greater then statistical error.

*Line 15.*Cross section data of some laboratories have been multiplied by the normalization factors N_{lab} being defined from the fit simultaneously with other parameters of the standard run (results for N_{lab} are given in Table 3). The correlation factors for N_{lab} are close to 1 and do not exceed 1.2. A significant decrease of the $\chi^2/\mathrm{N}_{\mathrm{df}}$ is observed; the answer for the $\boldsymbol{\Delta}^{+}$ mass is practically the same as in the standard run (to compare with effect of the normalization procedure in the recent work [15]).

Table 1. *Δ^+ "experimental" parameters for different variants of the resonance model*

№	Comment	χ^2/N_{df}	M_0, MeV	Γ_0, MeV	EMR, %
1	Standard	3.1176	1233.46 ± .15	116.26 ± .64	-2.07 ± .05
2	All data	3.8424	1233.39 ± .15	114.59 ± .59	-2.14 ± .05
3	*s*-, *p*-waves	3.6635	1232.31 ± .13	130.50 ± .53	-3.36 ± .05
4	*s*-, *p*-, *d*-waves	2.3768	1231.09 ± .28	121.66 ± 1.04	-3.75 ± .06
5	Full Born	3.0392	1234.57 ± .14	122.01 ± 0.65	-2.30 ± .06
6	Data before 1967	1.2836	1255.2 ± 48.4	201.51 ± 255.1	4.29 ± 7.37
7	Data before 1977	2.5019	1233.78 ± .29	111.02 ± 1.07	-.60 ± .10
8	Data before 1984	2.8545	1232.71 ± .22	119.28 ± .90	-1.43 ± .08
9	Free $X = X_\pi = X_\gamma$	3.0837	1233.59 ± .16	123.86 ± .96	-2.06 ± .06
10	Free $X = X_\pi$	2.8429	1242.63 ± .28	155.80 ± 1.37	-2.80 ± .08
11	Free $X= X_\gamma$	3.0180	1237.33 ± .35	116.92 ± .71	-2.31 ± .09
12	g=3/2	3.5209	1227.18 ± .11	109.38 ± .51	-1.95 ± .05
13	g=1/2	2.9686	1239.36 ± .20	120.05 ± .76	-2.42 ± .07
14	Corrected π^0 mass	3.1371	1234.06 ± .15	118.09 ± .66	-2.13 ± .06
15	Normalization	2.7476	1233.43 ± .15	116.84 ± .64	-2.08 ± .06

Table 2. *The data removed from the database*

Reaction	Observable	Data	N_{pnt}	Energy interval, MeV		Angle interval, deg.		χ^2/N_{pnt}
$\gamma p \to n\pi^+$	dσ/dΩ	BL01LE	32	286	322	20.0	170.0	29.2
	dσ/dΩ	KN63UC	11	290	290	.0	160.0	16.0
	Sigma	LU64ST	3	330	330	45.0	135.0	27.4
	Sigma	ZD72ST	1	390	390	135.0	135.0	22.5
$\gamma p \to p\pi^0$	dσ/dΩ	HA97MA	43	283	379	10.0	170.0	19.0
	dσ/dΩ	BL01LE	35	286	334	70.0	130.0	12.8
	dσ/dΩ	HE73TO	2	350	400	6.0	6.1	9.1

Table 3. *The cross section data normalized on factors N_{lab}*

Reaction	Data	N_{lab}
$\gamma p \to n\pi^+$	FU77TO	0.9172 ± .0040
	AV66ST	0.9424 ± .0189
	AL63BO	0.9296 ± .0089
	BU94BO	0.9948 ± .0054
	AK71LE	1.0566 ± .0146
	DU80BO	0.9298 ± .0058
	DA01BO	0.9557 ± .0060
	FR63BO	1.0721 ± .0173
	BT68OR	1.0433 ± .0045
$\gamma p \to p\pi^0$	BW71BO	1.0395 ± .0049
	MO69OR	1.1051 ± .0103
	DO76LU	0.9253 ± .0086
	DO77LU	0.9448 ± .0156
	AK78LE	1.0673 ± .0073
	HR74BO	0.9493 ± .0060
	DO75LU	0.9499 ± .0257
	BC67FR	0.9572 ± .0708
	BC73BO	0.8835 ± .0646

4. DISCUSSION

Of coarse, the use of the resonance model for determination of the $\Delta^+(1232)$ mass is a constrained method in comparison with the energy independent analysis without Watson's theorem for resonant multipoles. Nevertheless in the latter case one has to use some resonance model to get the resonance parameters, too. So, direct application of the resonance model is the simplest way to take into account correlation of the resonance parameters with the initial experimental data.

The smooth parameterization of multipoles achieved instead of histogram description with some arbitrary energy bins can be treated as an additional advantage.

Concerning the result of results submitted in [2] we would like to mention that the resonance parameters in this work do not apply to the experimentally "measured" ones, as Δ^0 and Δ^{++} parameters in πN scattering, but actually refers to the problem of explicit separation of the resonance and the background. To describe the resonant magnetic dipole amplitude $M^{(3/2)}_{1+}$ the authors of [2] invoked the Olsson's Eq. (8) [16] (but with usual addition of the resonance and background phase shifts for the elastic reaction. In [16] the corresponding algorithm has another form, which is roughly subtraction instead of summation.). The Olsson's receipt does not agree with the relevant formula from the Noille's unitary approach [17]. From physical point of view the latter one seems to be more convincing. However, the important point is that in both approaches the Watson's theorem is treated as underlying property. So, it is principally inadequate to use them to analyze the resonant multipole obtained with refusal of the Watson's theorem. Certainly, one can treat such a formula as some empirical parameterization. But in this case one has to determine the experimental Δ^+ mass as the energy point at which the real part of resonant multipole passes through zero. In analysis [3], which allowed releasing from the Watson's theorem for the T=3/2 magnetic dipole amplitude only, this occurs for the photon energy near 350 MeV. So, the experimental mass would even exceed 1234.9 MeV. It looks quite natural keeping in mind the experimental database before 1977 (at that time, for example, the main polarization experiments of Kharkov were yet not fulfilled). Nevertheless, our relevant retrospective run (line 7 in Table 1) has given the Δ^+ mass 1233.78 ± .29 MeV looking more acceptable.

5. CONCLUSIONS

In the region of excitation of the first nucleon resonance the photoproduction on proton allows good determination of the main *s*-, *p*-wave partial amplitudes of pion photoproduction on proton target with taking into account *d*-wave correction, too. Stable determination of the Δ^+(1232) mass and width can be treated as an encouraging moment. In spite of the simplest form used to describe the resonant multipoles the value EMR occurred to be close to the E2/M1 ratio defined with using the Watson's theorem (usually in the Noelle's approach). But there are many factors that have significant influence on the experimental mass and width values of the Δ^+ resonance. This is the problem of selection and possible normalization of the data used, and different possibilities concerning the background and description of the resonance contribution in the amplitude. The mass in several cases occurred to be beyond the interval restricted by Δ^0 and Δ^{++} masses. Nevertheless that is not the case for variants looking the most reliable from the point of view of reasonable balance between the existing data and the amount of the parameters to define from the fits. It is interesting to remark that between tested modifications of the Breit-Wigner formula one used by Walker occurred to be the most adequate.

Comparison fits with the data restricted by different years shows that the new data are entailing the evident shift of the Δ^+ mass value. As modern data about some single polarization observables are yet absent a new precise experiments in the reaction of pion photoproduction on proton are extremely desirable.

On the other side, the results of the calculation show that the phenomenological treatment of the present-day photoproduction database for the Δ(1232) excitation region needs the formalism taking into account difference of π^+ and π^0 masses.

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