ON SPIRAL SUPERFLUIDITY IN THE FERMI-LIQUID MODEL

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In this work we propose a model of description of a superfluid Fermi liquid with a spiral ordering by spins. The method of description is similar to that of description of spiral magnetics. Self-consistency equations for the order parameter are obtained. The transition temperature and order parameter in case of interaction similar to the Skyrme interaction are numerically calculated. The transition temperature and the order parameter, are proved to reach their maximal value under spirality distinct from zero.

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The term "spiral ordering" means that the state of a system remains invariant after an arbitrary spatial shift on a vector $\boldsymbol{a}$ and a simultaneous rotation of spins on an angle $\boldsymbol{aq}$, with $\boldsymbol{q}$ the spirality vector, i. e. the state of the system is invariant relatively the unitary transformation

$$\hat{V} = \exp i\boldsymbol{a}(\hat{\boldsymbol{p}} - \boldsymbol{q}\hat{\sigma}_3) : \tag{1}$$

$$\hat{V}\hat{f}\hat{V}^{+} = \hat{f}, \tag{2}$$

where

$$\hat{f} = \begin{pmatrix} f & g \\ g^{+} & 1-\tilde{f} \end{pmatrix}, \tag{3}$$

$\hat{\sigma}_3$ – Pauli matrix and $\hat{\boldsymbol{p}}$ – operator of momentum, generalized on a superfluid Fermi liquid:

$$\hat{\sigma}_3 = \begin{pmatrix} \sigma_3 & 0 \\ 0 & -\sigma_3 \end{pmatrix} \quad \hat{\boldsymbol{p}} = \begin{pmatrix} \boldsymbol{p} & 0 \\ 0 & -\boldsymbol{p} \end{pmatrix}. \tag{4}$$

Here f and g stand for normal and anomal distribution functions of quasi-particles in the superfluid Fermi liquid.

Such states of the superfluid Fermi liquid are analogous to the spiral ordering of magnetics [1] and can be presumably realized in liquid ^{3}He or in neutron stars.

Generally, such a state must be spatially unhomogeneous. It is a superposition of singlet and triplet spin states of the superfluid Fermi liquid.

In construction of the theory of spiral ordering for the superfluid Fermi liquid we followed the works [2,3], where the distribution function of the normal and superfluid components is presented in the form of defined above supermatrix $\hat{f}$; the quasi-particle energy $\hat{\varepsilon}$ and the operators of physical values $\underline{a}$ read as

$$\hat{\varepsilon} = \begin{pmatrix} \varepsilon & \Delta \\ \Delta^{+} & -\tilde{\varepsilon} \end{pmatrix}, \quad \hat{a} = \begin{pmatrix} \underline{a} & 0 \\ 0 & -\underline{\tilde{a}} \end{pmatrix}. \tag{5}$$

The unitary transformation

$$\hat{f}_{\boldsymbol{q}} = \hat{U}\hat{f}\hat{U}^{+}, \tag{6}$$

where $\hat{U} = \exp(-i\boldsymbol{q}\hat{\boldsymbol{x}}\sigma_3)$, preserves the structure of the supermatrix $\hat{f}_{\boldsymbol{q}}$ analogical to the matrix $\hat{f}$:

$$\hat{f}_{\boldsymbol{q}} = \begin{pmatrix} f_{\boldsymbol{q}} & g_{\boldsymbol{q}} \\ g^{+}_{\boldsymbol{q}} & 1-\tilde{f}_{\boldsymbol{q}} \end{pmatrix} \tag{7}$$

and transfers the system into a homogeneous state:

$$[\hat{f}_{\boldsymbol{q}}, \hat{\boldsymbol{p}}] = 0. \tag{8}$$

Normal and anomalous distribution functions f_q and g_q that define the supermatrix $\hat{f}_{\boldsymbol{q}}$ in (7) are expressed through f and g in the following way:

$$f_{\boldsymbol{q}} = e^{-i\boldsymbol{q}\boldsymbol{x}\sigma_3} f e^{i\boldsymbol{q}\boldsymbol{x}\sigma_3}, \tag{9}$$

$$g_{\boldsymbol{q}} = e^{-i\boldsymbol{q}\boldsymbol{x}\sigma_3} g e^{-i\boldsymbol{q}\tilde{\boldsymbol{x}}\sigma_3}. \tag{10}$$

The general structure of the "homogeneous" anomalous distribution function g_q and the order parameters Δ_q for case of one kind of fermions is:

$$g_{\boldsymbol{q}}(\boldsymbol{p}) = (g_0(\boldsymbol{p}) + g(\boldsymbol{p})\sigma)\sigma_2, \tag{11}$$

$$\Delta_{\boldsymbol{q}}(\boldsymbol{p}) = (\Delta_0(\boldsymbol{p}) + \Delta(\boldsymbol{p})\sigma)\sigma_2. \tag{12}$$

This describes a superposition of singlet and triplet states of the system. On the other hand, the quantity $g_q(\boldsymbol{p})$ can be expanded into components $g_{\parallel}$ and $g_{\perp}$ that is convenient for further calculations:

$$g_{\boldsymbol{q}} = g_{\parallel} + g_{\perp}, \tag{13}$$

where

$$g_{\parallel} = (g_0 + g_3\sigma_3)\sigma_2, \tag{14}$$

and

$$g_{\perp} = (g_1\sigma_1 + g_2\sigma_2)\sigma_2. \tag{15}$$

The order parameter $\Delta_q(\boldsymbol{p})$ can be expanded in the same way:

$$\Delta_{\boldsymbol{q}} = \Delta_{\parallel} + \Delta_{\perp}, \tag{16}$$

where

$$\Delta_{\|} = \left(\Delta_0 + \Delta_3 \sigma_3\right)\sigma_2 , \tag{17}$$

and

$$\Delta_{\perp} = \left(\Delta_1 \sigma_1 + \Delta_2 \sigma_2\right)\sigma_2 . \tag{18}$$

By means of these matrices, it will be easy to reconstruct the unhomogeneous distribution functions f and g, the order parameter Δ and to calculate the spatially unhomogeneous characteristics of the system.

In fact, the matrix elements of the distribution functions f, g, f_q and g_q between the impulse states $\langle \boldsymbol{p}|$ and $|\boldsymbol{p}'\rangle$ have form:

$$f_{\boldsymbol{pp}'} = \sum_{\substack{\eta = \pm 1 \\ \kappa = \pm 1}} \frac{1+\eta\sigma_3}{2} f_{\boldsymbol{q}}(\boldsymbol{p} - \eta\,\boldsymbol{q}) \frac{1+\kappa\sigma_3}{2} \times \delta\left(\boldsymbol{p} - \boldsymbol{p}' - (\eta - \kappa)\boldsymbol{q}\right) , \tag{19}$$

$$g_{\boldsymbol{pp}'} = \sum_{\substack{\eta = \pm 1 \\ \kappa = \pm 1}} \frac{1+\eta\sigma_3}{2} g_{\boldsymbol{q}}(\boldsymbol{p} - \eta\,\boldsymbol{q}) \frac{1+\kappa\sigma_3}{2} \times \delta\left(\boldsymbol{p} + \boldsymbol{p}' - (\eta + \kappa)\boldsymbol{q}\right) \tag{20}$$

The matrix elements of the order parameter Δ_{12} and the energy ε_{12} are of analogous form. They are defined through "homogeneous" order parameters $\Delta_{\boldsymbol{q}}(\boldsymbol{p})_{12}$ and $\varepsilon_{\boldsymbol{q}}(\boldsymbol{p})_{12}$, which are diagonal matrices in the impulse space.

The energy of the system can be represented in the following form:

$$E\left(\hat{f}\right) = E_0(f) + E_{\text{int}}(g) . \tag{21}$$

In this work we do not take into account influence of the normal amplitudes of interaction on the superfluid properties.

The energy $E_0(f)$ of the free Fermi liquid quasi-particles possesses the following form:

$$E_0(f) = \sum_{\boldsymbol{p}} tr\varepsilon\left(\boldsymbol{p}\right) f_{\boldsymbol{pp}} , \tag{22}$$

where $\varepsilon\left(\boldsymbol{p}\right) = \boldsymbol{p}^2/2m$. Note, that the energy $\varepsilon_{\boldsymbol{pp}'}(\boldsymbol{p})$ and $\varepsilon_{\boldsymbol{q}}(\boldsymbol{p})$ have diagonal form in the impulse space:

$$\varepsilon_{\boldsymbol{pp}'}\left(\boldsymbol{p}\right) = \varepsilon\left(\boldsymbol{p}\right)\delta\left(\boldsymbol{p} - \boldsymbol{p}'\right). \tag{23}$$

Then

$$\varepsilon_{\boldsymbol{q}}\left(\boldsymbol{p}\right) = \frac{\left(\boldsymbol{p} + \boldsymbol{q}\sigma_3\right)^2}{2m} = \varepsilon_0\left(\boldsymbol{p}\right) + \sigma_3 \varepsilon_3\left(\boldsymbol{p}\right) . \tag{24}$$

The term $\sigma_3 \varepsilon_3\left(\boldsymbol{p}\right)$ in the energy $\varepsilon_{\boldsymbol{q}}(\boldsymbol{p})$ is not equal to zero and substantially influences the process of diagonalization of the distribution function $\hat{f}_{\boldsymbol{q}}\left(\boldsymbol{p}\right)$.

We choose the term of (21) that is responsible for the superfluidity in the form which is quadratic in the distribution function:

$$E_{\text{int}}(g) = \frac{1}{2}\sum_{1234} v(1234) g^{+}_{21} g_{34} . \tag{25}$$

We choose the interaction between the Fermi liquid quasi-particles being translationally invariant and invariant under rotations in the spin space:

$$\begin{aligned} v\left(\boldsymbol{p}_1\boldsymbol{p}_2\boldsymbol{p}_3\boldsymbol{p}_4\right)_{1234} &= v_0\left(\boldsymbol{k},\boldsymbol{k}'\right)\delta_{13}\delta_{24} + \\ &+ v_1\left(\boldsymbol{k},\boldsymbol{k}'\right)\sigma_{13}\sigma_{24} = \\ &= \mathrm{u}_0(\boldsymbol{k},\boldsymbol{k}')\delta_{13}\delta_{24} + \mathrm{u}_1(\boldsymbol{k},\boldsymbol{k}')\mathrm{P}_{\sigma\ 1234} , \end{aligned} \tag{26}$$

where

$$P_{\sigma\,1234} = \left(\delta_{13}\delta_{24} + \sigma_{13}\sigma_{24}\right)/2 \tag{27}$$

and

$$\boldsymbol{k} = \frac{\boldsymbol{p}_1 - \boldsymbol{p}_2}{2}, \qquad \boldsymbol{k}' = \frac{\boldsymbol{p}_3 - \boldsymbol{p}_4}{2}, \qquad \boldsymbol{p}_1 + \boldsymbol{p}_2 = \boldsymbol{p}_3 + \boldsymbol{p}_4 . \tag{28}$$

We suppose that the interaction between quasi-particles does not depend on the total momentum of the interacting particles. As a model of interaction we consider the effective Skyrme force that depends on angles between momenta of the interacting quasi-particles and quadratically – on absolute values of their momenta. Under such a choice of interaction one can say formally that one examines the proton-neutron pairing in a symmetric nuclear matter, unless the interaction is considered as an example of enough simple force between quasi-particles.

As is known from the works [2,3], the order parameter Δ_{12} and anomalous distribution function are connected by the following equation:

$$\Delta_{12} = \sum_{3,4} v(1234) g_{34} . \tag{29}$$

By expressing g through Δ, one can obtain a self-consistency equation for the order parameter.

In the case of spiral ordering the order parameter $\Delta_{\boldsymbol{pp}'}$ and the anomalous distribution function $g_{\boldsymbol{pp}'}$ express through $\Delta_{\boldsymbol{q}}(\boldsymbol{p})$ and $g_{\boldsymbol{q}}(\boldsymbol{p})$ accordingly, and the "homogeneous" normal distribution function $g_{\boldsymbol{q}}(\boldsymbol{p})$ was obtained in the work [4] as a function of the singlet and triplet order parameters Δ_0 and Δ .

For the order parameter $(\Delta_0 \pm \Delta_3)$ we have the following equation:

$$\Delta_0(\boldsymbol{p}_1) \pm \Delta_3(\boldsymbol{p}_1) = \frac{1}{V}\sum_{\boldsymbol{p}_2} \Phi\left(\boldsymbol{p}_1 \pm \boldsymbol{q}, \boldsymbol{p}_2 \pm \boldsymbol{q}\right) \times \left(g_0(\boldsymbol{p}_2) \pm g_3(\boldsymbol{p}_2)\right) , \tag{30}$$

where

$$\Phi\left(\boldsymbol{p},\boldsymbol{p}'\right) = v_0\left(\boldsymbol{p},\boldsymbol{p}'\right) - v_1\left(\boldsymbol{p},\boldsymbol{p}'\right) - 2v_1\left(\boldsymbol{p},-\boldsymbol{p}'\right) , \tag{31}$$

$$g_0\left(\boldsymbol{p}\right) \pm g_3\left(\boldsymbol{p}\right) = -\frac{\Delta_0\left(\boldsymbol{p}\right) \pm \Delta_3\left(\boldsymbol{p}\right)}{2E^{\pm}_{\|}}\, th\frac{E^{\pm}_{\|}}{2T} , \tag{32}$$

$$E_{\parallel}^{\pm} = \sqrt{\left(\frac{(\boldsymbol{p}+\boldsymbol{q})^2}{2m}-\mu\right)^2 + \left|\Delta_0(\boldsymbol{p}) \pm \Delta_3(\boldsymbol{p})\right|^2} \,. \qquad (33)$$

For the order parameter $\Delta_2 \pm \Delta_1$ we have the equation:

$$\Delta_2(\boldsymbol{p}_1) \pm i\Delta_1(\boldsymbol{p}_1) = \frac{1}{V}\sum_{\boldsymbol{p}_2} \Psi(\boldsymbol{p}_1, \boldsymbol{p}_2) \times \left(g_2(\boldsymbol{p}_2) \pm i g_1(\boldsymbol{p}_2)\right), \qquad (34)$$

where

$$\Psi(\boldsymbol{p}, \boldsymbol{p}') = v_0(\boldsymbol{p}, \boldsymbol{p}') + v_1(\boldsymbol{p}, \boldsymbol{p}'), \qquad (35)$$

$$g_2(\boldsymbol{p}) \pm i g_1(\boldsymbol{p}) = -\frac{\Delta_2(\boldsymbol{p}) \pm i\Delta_1(\boldsymbol{p})}{2E_{\perp}^{\pm}}\, th\frac{E_{\perp}^{\pm}}{2T}, \qquad (36)$$

$$E_{\perp}^{\pm} = \sqrt{\left(\frac{\boldsymbol{p}^2+\boldsymbol{q}^2}{2m}-\mu\right)^2 + \left|\Delta_2(\boldsymbol{p}) \pm i\Delta_1(\boldsymbol{p})\right|^2} \,. \qquad (37)$$

Equations (30, 34) describe two particular cases. For description of general spiral ordering of superfluid Fermi liquid we have to obtain a system of 4 nonlinear integral self-consistency equations for the singlet (Δ_0) and triplet (Δ) order parameters. Nevertheless, the true spirality is possible only in the case when all 4 components Δ_j, where j=0,….3 do not vanish. Then the systems of equations are closely coupled between themselves. However, in the present work we do not investigate this case.

It is easy to see that the spirality parameter $\boldsymbol{q}$ defines directly the equations for Δ_1 and Δ_2 only, so we will examine this unhomogeneous case. Note that equation (34) does not depend on the sign "±", hence Δ_1=0.

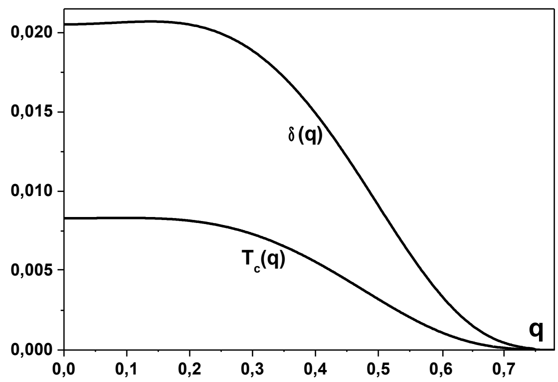

Fig. 1

The equation (30) for Δ_2 has been numerically solved for the Skyrme SKP force. For this force we have

$$\begin{aligned} u_0(\boldsymbol{p}, \boldsymbol{p}') &= a_0 + a_1\boldsymbol{p}\boldsymbol{p}' + a_2\left(\boldsymbol{p}^2 + \boldsymbol{p}'^2\right), \\ u_1(\boldsymbol{p}, \boldsymbol{p}') &= x_0 a_0 + x_1 a_1 \boldsymbol{p}\boldsymbol{p}' + x_2 a_2\left(\boldsymbol{p}^2 + \boldsymbol{p}'^2\right). \end{aligned} \qquad (38)$$

The constants a_i and x_i (i=0,….,3) are defined by parametrisation of the potential.

Under such a choice of potential we should seek for the order parameter in the form $\Delta_2(p)=\delta_2\, p\, z$, where z is the cosine of angle between the vectors $\boldsymbol{p}$ and $\boldsymbol{q}$.

The order parameter Δ_2 has been calculated at $T=0$ as a function of the spirality vector $\boldsymbol{q}$ and is presented on the Fig.1. At the same figure we present the transition temperature versus $\boldsymbol{q}$. At $\boldsymbol{q} \sim 0{,}75$ the superfluidity disappears (δ_2=0, T_c=0).

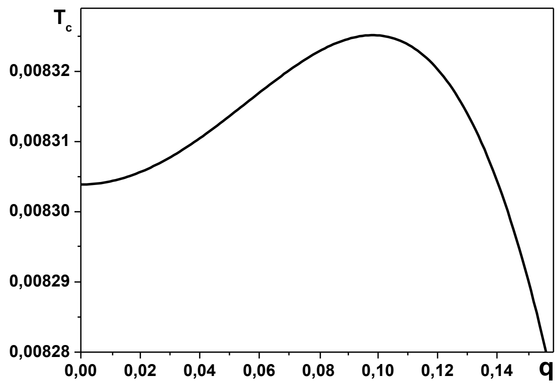

Fig. 2

The transition temperature (and δ_2 as well) reaches its maximal value at $\boldsymbol{q}$~0,1. It means that the most stable state should be under $\boldsymbol{q}$ distinct from zero, unless the maximum of T_c is very tiny. Similarly behaves the order parameter δ_2 at $T=0$. The area of extremum of the transition temperature is shown on the Fig.2. Whether the appearance of the extermum is model-dependent, remains an open question.

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