SUPERFLUIDITY OF A CONDENSATE WITH *np* PAIRING CORRELATIONS IN ASYMMETRIC NUCLEAR MATTER

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Influence of asymmetry on superfluidity of nuclear matter with triplet-singlet pairing of nucleons (in spin and isospin spaces) is considered within the framework of a Fermi-liquid theory. Solutions of self-consistent equations for the energy gap at $T=0$ are obtained. It is shown, that if the chemical potentials of protons and neutrons are determined in the zero gap width approximation, then the energy gap for some values of density and asymmetry parameter of nuclear matter demonstrates double-valued behavior. However, with account for the feedback of pairing correlations through the normal distribution functions of nucleons two-valued behavior of the energy gap turns into universal one-valued behavior. At $T=0$ the energy gap has a discontinuities as a function of density in a narrow layer model. These discontinuities depend on the asymmetry parameter.

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1. GENERAL EQUATIONS OF THE THEORY

It is well established, that neutron-proton (*np*) pairing plays an essential role in description of superfluidity of finite nuclei with *N*=*Z* (see Ref. [1] and references therein) and symmetric nuclear matter [2-4]. In this report we shall investigate influence of asymmetry on *np* superfluidity of nuclear matter. Earlier this problem was treated with the use of various approaches and potentials of *NN* interaction. In particular, the cases of $^3S_1$-$^3D_1$ and $^3D_2$ pairing were considered in Refs. [2,5] on the base of the Thouless criterion for the thermodynamic *T* matrix. As a potential of *NN* interaction, the Graz II and Paris potential were chosen respectively. Superfluidity in $^3S_1$-$^3D_1$ pairing channel was studied also in Ref. [6] within BCS theory of superconductivity with the use of the Paris potential in the separable form. Investigations, based on the Thouless criterion, deduce the suppression of *np* pairing correlations with increase of isospin asymmetry. However, the Thouless criterion can be exploited for finding the critical temperature only, but does not permit to draw any conclusions about superfluidity with a finite gap. The studies in Ref. [6], based on the BCS theory, were carried out with the use of the bare interaction and the single particle spectrum of a free Fermi gas and give, thus, overestimated values of the energy gap. The effect of ladder-renormalized single particle spectrum on the magnitude of the energy gap in $^3S_1$-$^3D_1$ pairing channel was investigated in Ref. [7]. The Argonne V_{14} potential was explored as input for determination of the single particle energy and the bare interaction in the form of the Paris potential was used to evaluate the energy gap. The use of the bare interaction in the gap equation seems to be a very strong simplification, because medium polarization strongly reduces the magnitude of the gap. In principle, the effective pairing interaction should be obtained by means of Brueckner renormalization, which gives the correct interaction after modifying the bare interaction for the effect of nuclear medium. However, the issue of microscopic many-body calculation of the effective pairing potential is a complex one and still is not solved. For this reason, it is quite natural step to develop some kind of a phenomenological theory, where instead of microscopical calculation of the pairing interaction one exploits the phenomenological effective interaction. We shall investigate influence of asymmetry on superfluid properties of nuclear matter, using Landau's semiphenomenological theory of a Fermi-liquid (FL). In the Fermi-liquid model the normal and anomalous FL interaction amplitudes are taken into account on an equal footing. This will allow us to consider consistently influence of the FL amplitudes on superfluid properties of nuclear matter. Besides, as a potential of NN interaction we choose the Skyrme effective forces, describing interaction of two nucleons in the presence of nucleon medium. The Skyrme forces are widely used in description of nuclear system properties and, in particular, they were exploited for description of superfluid properties of finite nuclei [8,9] as well as infinite symmetric nuclear matter [10-12].

The basic formalism is laid out in more detail in Ref. [11], where superfluidity of symmetric nuclear matter was studied. As shown there, superfluidity with triplet-singlet (TS) pairing of nucleons (total spin *S* and isospin *T* of a pair are equal $S=1$, $T=0$) is realized near the saturation density in symmetric nuclear matter with the Skyrme interaction. Therefore, further we shall study influence of asymmetry on superfluid properties of TS phase of nuclear matter. For the states with the projections of total spin and isospin $S_z=T_z=0$, the normal distribution function *f* and the anomalous distribution function *g* have the form

$$f(p)=f_{00}(p)\sigma_0\tau_0+f_{03}(p)\sigma_0\tau_3, \qquad (1)$$
$$g(p)=g_{30}(p)\sigma_3\sigma_2\tau_2,$$

where σ_i and τ_k are the Pauli matrices in spin and isospin spaces. The operator of quasiparticle energy ε and the matrix order parameter Δ of the system for the energy functional, being invariant with respect to rotations in spin and isospin spaces, have the analogous structure:

$$\varepsilon(p) = \varepsilon_{00}(p)\sigma_0\tau_0 + \varepsilon_{03}(p)\sigma_0\tau_3, \tag{2}$$
$$\Delta(p) = \Delta_{30}(p)\sigma_3\sigma_2\tau_2.$$

Using the minimum principle of the thermodynamic potential and procedure of block diagonalization [13], one can express evidently the distribution functions f_{00}, f_{03}, g_{30} in terms of the quantities ε and Δ:

$$f_{00} = \frac{1}{2} - \frac{\xi_{00}}{4E}\left(\tanh\frac{E+\xi_{03}}{2T} + \tanh\frac{E-\xi_{03}}{2T}\right), \tag{3}$$

$$f_{03} = -\frac{1}{4}\left(\tanh\frac{E+\xi_{03}}{2T} - \tanh\frac{E-\xi_{03}}{2T}\right), \tag{4}$$

$$g_{30} = -\frac{\Delta_{30}}{4E}\left(\tanh\frac{E+\xi_{03}}{2T} + \tanh\frac{E-\xi_{03}}{2T}\right). \tag{5}$$

Here

$$E = \sqrt{\xi_{00}^2 + \Delta_{30}^2}, \quad \xi_{00} = \varepsilon_{00} - \mu_{00}^0, \quad \xi_{03} = \varepsilon_{03} - \mu_{03}^0,$$
$$\mu_{00}^0 = \frac{\mu_p^0 + \mu_n^0}{2}, \quad \mu_{03}^0 = \frac{\mu_p^0 - \mu_n^0}{2},$$

T is temperature, μ_p^0 and μ_n^0 are chemical potentials of protons and neutrons. To obtain the closed system of equations for the quasiparticle energy ε and Δ, it is necessary to express the quantities ε and Δ through the distribution functions f and g. For this purpose one has to set the energy functional $E(f,g)$ of the system. In the case of asymmetric nuclear matter with TS pairing of nucleons the energy functional is characterized by two normal U_0, U_2 and one anomalous V_1 FL amplitudes [11]. Differentiating the functional $E(f,g)$ with respect to g [13] and using Eq. (5), one can obtain the gap equation in the form

$$\Delta_{30}(\vec{p}) = -\frac{1}{4V}\sum_q V_1(\vec{p},\vec{q})\frac{\Delta_{30}(q)}{E(q)}$$
$$\times\left\{\tanh\frac{E+\xi_{03}}{2T} + \tanh\frac{E-\xi_{03}}{2T}\right\} \tag{6}$$

The anomalous interaction amplitude V_1 in the Skyrme model reads [11]

$$V_1(\vec{p},\vec{q}) = t_0(1+x_0) + \frac{1}{6}t_3\rho^{\beta}(1+x_3) + \frac{1}{2\hbar^2}t_1(1+x_1)(\vec{p}^{\,2}+\vec{q}^{\,2}), \tag{7}$$

where ρ is density of nuclear matter, t_i, x_i, β are some phenomenological parameters, which differ for various versions of the Skyrme forces (later we shall use the SkP potential [8,14]). Eq. (6) should be solved jointly with equations

$$\frac{1}{V}\sum_p\left\{2 - \frac{\xi_{00}(p)}{E(p)}\left(\tanh\frac{E(p)+\xi_{03}(p)}{2T} + \tanh\frac{E(p)-\xi_{03}(p)}{2T}\right)\right\} = \rho, \tag{8}$$

$$\frac{1}{V}\sum_p\left\{\tanh\frac{E+\xi_{03}}{2T} - \tanh\frac{E-\xi_{03}}{2T}\right\} = \alpha\rho \tag{9}$$

being the normalization conditions for the normal distribution functions f_{00}, f_{03}. In Eq. (9) the quantity $\alpha = (\rho_n - \rho_p)/\rho$ is the asymmetry parameter of nuclear matter, ρ_p, ρ_n are the partial number densities of protons and neutrons. Note that account of the normal FL amplitudes in the case of the effective Skyrme interaction, being quadratic in momenta, is reduced to renormalization of free nucleon masses and chemical potentials. Expressions for the quantities ξ_{00}, ξ_{03}, which enter in Eqs. (6,8,9), with regard for the explicit form of the amplitudes U_0, U_2 [11] read

$$\xi_{00} = \frac{p^2}{2m_{00}} - \mu_{00}, \quad \xi_{03} = \frac{p^2}{2m_{03}} - \mu_{03},$$

where the effective nucleon mass m_{00} and effective isovector mass m_{03} are defined by the formulas

$$\frac{\hbar^2}{2m_{00}} = \frac{\hbar^2}{2m_{00}^0} + \frac{\rho}{16}\left[3t_1 + t_2(5+4x_2)\right], \tag{10}$$
$$\frac{\hbar^2}{2m_{03}} = \frac{\alpha\rho}{16}\left[t_1(1+2x_1) - t_2(1+2x_2)\right],$$

m_{00}^0 being the bare mass of a nucleon. The renormalized chemical potentials μ_{00}, μ_{03} should be determined from Eqs. (8,9) and in the leading approximation on the ratios T/ε_F, Δ/ε_F have the form

$$\mu_{00} = \frac{1}{2}(\mu_p + \mu_n), \quad \mu_{03} = \frac{1}{2}(\mu_p - \mu_n), \tag{11}$$
$$\mu_{p,n} = \frac{\hbar^2 k_{F_{p,n}}^2}{2m_{p,n}}$$

where $k_{F_{p,n}} = (3\pi^2\rho_{p,n})^{1/3}$, m_p and m_n are the proton and neutron effective masses, defined as

$$\frac{2}{m_{00}} = \frac{1}{m_p} + \frac{1}{m_n}, \quad \frac{2}{m_{03}} = \frac{1}{m_p} - \frac{1}{m_n}.$$

The proton and neutron effective masses, according to Eqs. (10), can be written in the form

$$m_p = \frac{m_{00}}{1+\beta}, \quad m_n = \frac{m_{00}}{1-\beta}, \tag{12}$$
$$\beta = \alpha\rho r, \quad r = \frac{m_{00}^0\left[t_1(1+2x_1) - t_2(1+2x_2)\right]}{8\hbar^2 + \rho m_{00}^0\left[3t_1 + t_2(5+4x_2)\right]}.$$

In Eq. (12) the dependence on the asymmetry parameter α is contained through the quantity β. With account of Eqs. (11,12) for the renormalized chemical potentials μ_{00}, μ_{03} we obtain

$$\mu_{00} = \frac{\mu_0}{2}\left\{(1-\alpha)^{2/3}(1+\beta) + (1+\alpha)^{2/3}(1-\beta)\right\}, \quad (13)$$
$$\mu_{03} = \frac{\mu_0}{2}\left\{(1-\alpha)^{2/3}(1+\beta) - (1+\alpha)^{2/3}(1-\beta)\right\},$$
$$\mu_0 \equiv \frac{\hbar^2}{2m_{00}}\left(\frac{3\pi^2\rho}{2}\right)^{2/3}.$$

2. ENERGY GAP AT ZERO TEMPERATURE

We write now the self-consistent equations for determining the energy gap and effective chemical potentials at $T = 0$. Considering, that the interaction amplitude V_1 is not equal to zero only in a narrow layer near the Fermi-surface, $|\xi_{00}| \le \theta_0$ (we shall set $\theta_0 = 0.1\mu_{00}$) and entering new dimensionless variables $\xi = \xi_{00}/\mu_{00}$, $\Delta = \Delta_{30}/\mu_{00}$, $\xi_3 = \xi_{03}/\mu_{00}$, we present these equations in the form

$$1 = \frac{g}{4}\int_{-\theta}^{\theta}\frac{d\xi}{E}\left[\mathrm{sgn}(E+\xi_3) + \mathrm{sgn}(E-\xi_3)\right] \quad (14)$$

$$\nu_F\mu_{00}\int_{-1}^{\infty} d\xi\,\sqrt{1+\xi} \quad (15)$$
$$\times\left\{2 - \frac{\xi}{E}\left[\mathrm{sgn}(E+\xi_3) + \mathrm{sgn}(E-\xi_3)\right]\right\} = \rho,$$

$$\nu_F\mu_{00}\int_{-1}^{\infty} d\xi\,\sqrt{1+\xi}\left[\mathrm{sgn}(E+\xi_3) - \mathrm{sgn}(E-\xi_3)\right] = \alpha\rho. \quad (16)$$

Here

$$E = \sqrt{\xi^2 + \Delta^2}, \quad g = -\nu_F V_1(p = p_F, q = p_F),$$
$$\nu_F = \frac{m_{00}p_f}{2\pi^2\hbar^3}, \quad p_F = \sqrt{2m_{00}\mu_{00}},$$

and $\theta = \theta_0/\mu_{00}$ is dimensionless cut-off parameter. For $\alpha > 0$ it holds $m_{03} > 0$, $\mu_{03} < 0$, and, hence

$$\mathrm{sgn}(E+\xi_3) + \mathrm{sgn}(E-\xi_3) = 1 + \mathrm{sgn}(E^2 - \xi_3^2),$$
$$\mathrm{sgn}(E+\xi_3) - \mathrm{sgn}(E-\xi_3) = 1 - \mathrm{sgn}(E^2 - \xi_3^2).$$

The contribution to the gap equation (14) gives the domain on ξ, for which the expression $E^2 - \xi_3^2$ is positive. Its roots are equal

$$\gamma^2\xi_\pm = \beta\varphi \pm \sqrt{\varphi^2 - \gamma^2\Delta^2}.$$

Here

$$\varphi = \frac{m_{00}}{m_{03}} - \frac{\mu_{03}}{\mu_{00}}, \quad \gamma^2 = 1 - \beta^2.$$

If $\varphi^2 < \gamma^2\Delta^2$, then $\mathrm{sgn}(E^2 - \xi_3^2) = 1$ and, hence, the whole domain from $-\theta$ to θ gives the contribution to the integral in Eq. (14). In this case we arrive at the equation of the BCS type at $T = 0$ with the solution $\Delta = \theta/\sinh(1/g)$. Eqs. (14)-(16) can be solved in two approximations: when one disregards by the dependence from Δ in chemical potentials μ_{00}, μ_{03} and when this dependence is taken into account exactly. First, we consider the former case. Using Eqs. (15,16), it is easy to see that the quantity φ in the main approximation on the ratio Δ/ε_F and $\alpha \ll 1$ reads

$$\varphi = \frac{2}{3}\alpha + O(\alpha^3)$$

Thus, at $\Delta \ge \Delta_c \equiv \frac{2}{3}\alpha$ we arrive at the BCS solution at $T = 0$. If $\Delta < \Delta_c$, the gap equation reads

$$1 = \frac{g}{2}\ln\frac{\sqrt{\xi_-^2 + \Delta^2} + \xi_-}{\sqrt{\theta^2 + \Delta^2} - \theta}\cdot\frac{\sqrt{\theta^2 + \Delta^2} + \theta}{\sqrt{\xi_+^2 + \Delta^2} + \xi_+}, \quad (17)$$
$$\xi_\pm \approx \pm\sqrt{\Delta_c^2 - \Delta^2},$$

assuming that $\alpha \ll 1$ and quantities $\xi_\pm$ lye in the interval $(-\theta, \theta)$. Solution of Eq. (17) can be presented in the form (for $\Delta < \Delta_c$)

$$\Delta^2 = \frac{4\lambda}{(\lambda^2+1)^2}(\lambda\Delta_c - \theta)(\lambda\theta + \Delta_c), \quad \lambda = \exp(1/g). \quad (18)$$

For the densities where $\lambda\Delta_c < \theta$, we have $\Delta \equiv 0$. Thus, in the zero gap size approximation in chemical potentials we have two solutions for the energy gap: one solution corresponds to BCS solution at $\Delta \ge \Delta_c$ and the second one is described by Eq. (18).

Now we present the results of numerical integration of the gap equation for the given case (Fig. 1).

In the case of symmetric nuclear matter ($\alpha = 0$) we obtain the phase curve with one-valued behavior of the gap. For small values of asymmetry α there exist such regions of large and low densities of nuclear matter (excluding some vicinity of the point $\rho = 0$), for which we have two values of the energy gap, where one of these values is the solution of the BCS type and practically coincides with the corresponding value of the gap for the case $\alpha = 0$. When α increases, these regions begin to approach and at some value $\alpha = \alpha_c$ it takes place contiguity of the regions with two solutions. For $\alpha > \alpha_c$ two branches of the phase curves are separated

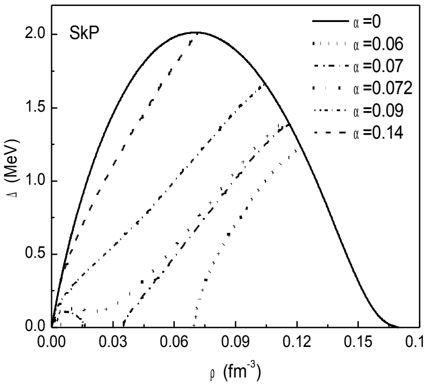

Fig. 1. *Energy gap as a function of density in uncoupled calculations*

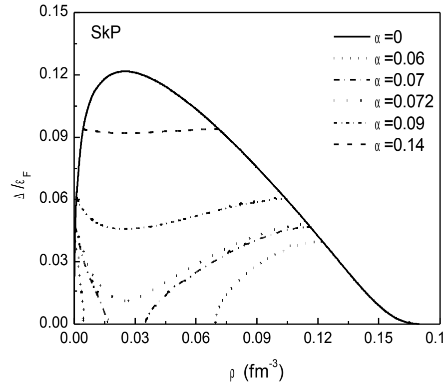

Fig. 2. *The ratio $\Delta_{30}/\varepsilon_F$ as a function of density in uncoupled calculations*

from the density axis and combine to one curve, beginning and ending in some points of the phase curve with $\alpha = 0$. When α increases further, these points move towards and at some $\alpha = \alpha_m$ the branches of solutions contract to a point. The value α_m determines the maximum value of the asymmetry parameter, when TS superfluidity exists at $T = 0$ ($\alpha_m \approx 0.179$).

To find this value, let us denote as λ_{min} the minimum value of the parameter λ, corresponding to the maximum value g_{max} of the coupling constant. The value α_m is found from the relationship $\Delta_{BCS}^{max} = \Delta_c$, whence we get

$$\alpha_m = \frac{3\lambda_{min}\theta}{\lambda_{min}^2 - 1}.$$

The position of the boundary points ρ_{min}, ρ_{max}, limiting the interval, where solutions exist, is determined from the requirement $\Delta_{BCS} = \Delta_c$, from here we obtain the equation

$$\lambda(\rho) = \frac{\theta + \sqrt{\theta^2 + \Delta_c^2}}{\Delta_c}.$$

The points ρ'_{min}, ρ'_{max}, at which the gap vanishes, are found from the relationship $\lambda(\rho) = \frac{\theta}{\Delta_c}$. For $\alpha > \frac{3\theta}{2\lambda_{min}} \equiv \alpha_c$ the points ρ'_{min}, ρ'_{max} are absent, and, hence, for the lower branch $0 < \Delta \le \Delta_c$. At $\alpha = \alpha_c$ we have $\rho'_{min} = \rho'_{max}$. All these peculiarities are qualitatively seen from Fig. 2.

Let us find now the solutions of Eqs. (14-16) with account for the influence of the finite size of the gap on the chemical potentials μ_{00}, μ_{03}. Calculations show, that in self-consistent treatment of Eqs. (14-16) it will be realized only the case $\varphi^2 - \Delta^2\gamma^2 < 0$, and, hence, we have only solution of the BCS type. Since $\varphi^2 - \Delta^2\gamma^2 < 0$, Eq. (16) will have nontrivial solution for the chemical potential μ_{03}, if the roots $\xi_\pm^0 \equiv \xi_\pm(\Delta = 0)$ of the subintegrand function at $\Delta = 0$ will be located outside the interval $(-\theta, \theta)$.

The results of self-consistent integration of the gap equation are presented in Fig. 3.

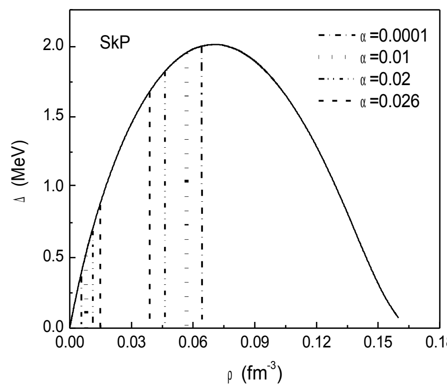

Fig. 3. *Energy gap as a function of density in self-consistent scheme*

Here the dashed lines are drawn through the boundary points of the curves, being the solutions of Eqs. (14-16) for different α. One can see, that taking into account the feedback of the finite size of the gap through the normal distribution functions f_{00}, f_{03} in Eqs. (3,4), leads to the qualitative change: instead of two-valued behavior of the gap we have universal one-valued behavior. The first solution of the BCS type, obtained in uncoupled calculation, remains practically unchanged in self-consistent treatment of the gap equation (14) and with sufficiently high accuracy equals to its value in the self-consistent determination. The second solution in uncoupled scheme, to which corresponds the smaller gap width, tends to the first solution of the BCS type under simultaneous iterations of Eqs. (14,16). Taking into account the finiteness of the gap results in reduction of the threshold asymmetry, at which superfluidity disappears, to the value $\alpha_m \approx 0.03$. Thus, in spite of the smallness of the ratio Δ/ε_F for all densities ρ, the backward influence of pairing correlations is significant. This is explained by the fact, that if the quantity Δ in Eqs. (15,16) differs from zero, then the absolute value of the chemical potential μ_{03} increases a few times as against its value at $\Delta=0$. The increase of $|\mu_{03}|$ is equivalent to the increase of the effective shift between neutron and proton Fermi surfaces that leads to significant reduction of the threshold asymmetry. Note, that in a narrow layer model the gap is everywhere finite as a function of density.

In summary, we studied TS superfluidity of asymmetric nuclear matter in FL model with density-dependent Skyrme effective interaction (SkP force). In FL approach the normal and anomalous FL amplitudes are taken into account on an equal footing and this allows to consider consistently within the framework of a phenomenological theory the influence of medium effects on superfluid properties of nuclear matter. It is shown, that in the case, when the chemical potentials μ_{00}, μ_{03} (half of a sum and half of a difference of the proton and neutron chemical potentials, respectively) are determined in the approximation of ideal Fermi-gas, the energy gap demonstrates for some values of density and asymmetry parameter the double-valued behavior. If to consider the feedback of pairing correlations through the dependence of the normal distribution functions of nucleons from the energy gap, then the energy gap drastically changes its behavior from two-valued to universal one-valued character. In spite of relative smallness of the ratio Δ/ε_F, taking into account of the finite size of the gap in chemical potentials leads to the significant increase of absolute value of μ_{03} and, hence, to considerable reduction of the threshold asymmetry, at which superfluidity at T=0 disappears. In self-consistent determination the energy gap at T=0 as a function of density has a finite width. Among the other problems we note here the study of multi-gap superfluidity [12] in asymmetric nuclear matter.

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