

MICROSCOPIC DESCRIPTION OF RELAXATION OF IMPURITY MAGNETIC ATOMS IN SPIRAL MAGNETICS

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Kinetics of the impurity magnetic atoms in spiral magnetic is researched. The spectrum of the excitations of the impurity spin is obtained in approximation of weakly coupling. The equation for impurity density matrix up to second order of the perturbation theory is obtained. For impurity spin $\sigma=1/2$ the kinetic coefficients in terms of correlation functions are found.

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1. INTRODUCTION

The kinetics of impurity spin was considered in many papers [1-4]. This paper deals with the kinetics of weakly coupled impurity spin in magnetic dielectric, which is considered as a thermostat. The thermostat has a spiral structure of magnetic ordering. Objects with a spiral magnetic structure are systems with a spontaneously broken symmetry, in which the symmetry of the statistical equilibrium state is lower than the symmetry of the Hamiltonian. Such state is not translationally invariant and does not exhibit invariance to spin rotations around the spiral axes. The method of quasi-averages is a convenient tool for analyzing such structures [5-7]. In this paper, using the reduced description method [8], the equation for impurity spin matrix was obtained. From this equation in main approximation the energy spectrum of impurity spin was found. In last section we describe the evolution of impurity spin $\sigma=1/2$. We define the kinetic coefficients, which are expressed in terms correlation function of the thermostat.

2. KINETIC EQUATION FOR IMPURITY DENSITY MATRIX

The equilibrium state of spiral magnetic is described by statistical operator

$$W_v = \exp(\Omega_v - Y_0(H_m - h_B S^z) - \nu Y_0 \sum_l (S^+_{l-1} \exp(i(px_{l-1} - h_B t)) + h.c.)) \quad (1)$$

where: H_m is the Hamiltonian of matrix; Y_0^{-1} is the temperature; $h_B = h + \mu_0 g B$, h is the magnetic bias field; $\mu_0 = e^{-2} 2mc$ is a Bohr magneton; g' is the gyromagnetic ratio for matrix spin; B is the magnetic field, which is directed parallel to axis Z ; p is the spiral vector; Ω_v is the thermodynamic potential, which is found from the normalizing condition $SpW_v = 1$ (trace is calculated only on thermostat states).

The conditions, which are assumed from the structure of operator (1), are as follows

$$[W, S^z] \neq 0, [W, H_m] \neq 0, [W, (H_m - h_B S^z)] = 0.$$

It means, that statistical operator W depends on time. The statistical operator also has such space symmetry

$$\exp(ipa S^z) U_a W U_a^\dagger \exp(-ipa S^z) = W,$$

where the operator of the translation U_a^\dagger satisfies the following condition

$$U_a^\dagger S_l U_a^\dagger = S_{l+a}^\dagger, U_a^\dagger U_a^\dagger = I,$$

a - pitch of lattice, I - unit operator.

In our case the equilibrium averages of spin components of matrix are as follows

$$\langle S^z \rangle = S_{||}, \langle S^\pm \rangle = S_\perp \exp\{\mp(i(px_l - h_B t))\},$$

where $\langle A \rangle \equiv \lim_{\nu \rightarrow 0} \lim_{V \rightarrow \infty} Sp W_\nu A$ is the quasi averages value of operator A ; S_\perp is the module of the transversal component of spin; $S_{||}$ is the longitudinal component of spin.

We suppose, that the impurity concentration is very low, thus the Hamiltonian of system is as follows

$$H = H_m(B) - \mu_0 g B \sigma^z + V,$$

$$V = - \sum_l (I_{z,l} S_l^z \sigma^z + \frac{1}{2} I_{\perp,l} (S_l^z \sigma^- + S_l^z \sigma^+)),$$

where $H_m(B) = H_m - \mu_0 g' B S^z + V$ Hamiltonian of spiral magnetic in magnetic field, V Hamiltonian of the exchange interaction between the magnetic matrix and the impurity; $-\mu_0 g B \sigma^z$ Hamiltonian of the impurity; g is gyromagnetic ratio of the impurity; $I_{z,l}, I_{\perp,l}$ are the longitudinal and transversal integrals of interaction between impurity spin σ and matrix spin S_l .

From the Liouville equation

$$\frac{\partial \rho(t)}{\partial t} = i[\rho(t), H] \quad (2)$$

we obtain the equation for density matrix $w = Sp \rho$

$$\frac{\partial w}{\partial t} = M, \quad M = iSp[\rho, H] \quad (3)$$

where M is the impurity collision integral.

The impurity matrix w satisfies equation $tr w = 1$, tr is a trace on impurity state. We assume [8], that the statistical operator is the functional of impurity density matrix and phase $\varphi = px - h_B t$, when $t \gg \tau_0$ (τ_0 is the time of relaxation in the thermostat)

$$\rho \rightarrow \rho\{w(t), \varphi(t)\}.$$

It means, that

$$\frac{\partial \rho(w, \varphi)}{\partial t} = \frac{\delta \rho(w, \varphi)}{\delta \varphi} \dot{\varphi} + \frac{\delta \rho(w, \varphi)}{\delta w} \dot{w}. \quad (4)$$

Substituting this equality into Liouville equation (2) and assuming $\dot{\varphi} \approx -h_B$ in main approximation, we obtain jointly with (3) the equations set for impurity density matrix $w(t)$. As boundary condition we use the asymptotic condition for operator $\rho(w, \varphi)$

$$\exp(-iH_0 t) \rho(w_{-\tau}, \varphi) \exp(iH_0 \tau) \xrightarrow{\tau \gg \tau_0} W(\varphi) w, \quad (5)$$

where $w_{-\tau} = \exp(-i\mu_0 g B \sigma^z) w \exp(i\mu_0 g B \sigma^z)$,

$$H_0 = H_m - \mu_0 g B \sigma^z - h_B S^z.$$

Thus the Liouville equation jointly with (4) is as follows

$$\frac{\delta \rho(w, \varphi)}{\delta w} (M_0 + M_V) = i[\rho, H_0 + V],$$

where $M_0 = -i\Omega [w, \sigma^z]$, $M_V \equiv iSp[\rho, V]$, $\Omega = \mu_0 g B$.

We take into account, that $\frac{\delta \rho(w, \varphi)}{\delta \varphi} \dot{\varphi} = ih_B [\rho, S^z]$. As a result we obtain the integral equation for $\rho(w, \varphi)$ in common with (5)

$$\rho(w, \varphi) = W(\varphi) w + \int_{-\infty}^0 d\tau e^{iH_0 \tau} \left\{ i[\rho, V] - \frac{\delta \rho}{\delta w} M_V \right\}_{w \rightarrow w_{-\tau}} e^{-iH_0 \tau}$$

This equation and the equation (2) form the closed set of the equations for impurity density matrix $w(t)$.

The zero and the first approximation for $\rho(w, \varphi)$ equal correspondently to

$$\rho^{(0)}(w, \varphi) = W(\varphi) w, \quad (6a)$$

$$\rho^{(1)}(w, \varphi) = \int_{-\infty}^0 d\tau e^{iH_0 \tau} [W(\varphi) w, V - \langle V \rangle]_{w \rightarrow w_{-\tau}} e^{-iH_0 \tau}, \quad (6b)$$

Here $\langle V \rangle \equiv Sp W(\varphi) V$.

The collision integral subject to (6a), (6b) is as follows correspondently

$$M^{(1)} = iSp[\rho^{(0)}(W, \varphi), V] = i[w, \langle V \rangle],$$

$$M^{(2)} = iSp[\rho^{(1)}, V] = - \int_{-\infty}^0 d\tau Sp[[W(\varphi) w, V_{\tau} - \langle V \rangle]_{\tau}, V] \quad (7)$$

$$V_{\tau} = e^{iH_0 \tau} V e^{-iH_0 \tau}.$$

As a result the kinetic equation for w up to the second order on interaction take on form

$$\dot{w} = M^{(0)} + M^{(1)} + M^{(2)},$$

where $M^{(0)}$, $M^{(1)}$, $M^{(2)}$ are defined by formulae (7).

In order to calculate $\langle V \rangle$ we use the unitary transformation

$$\tilde{W} = U_p^- W U_p^+, \quad U_p^- \equiv \exp(-ip \sum_l \tilde{x}_l S_l^z).$$

Then the calculation of the averages in spiral magnetic can be obtained with the operator

$$\tilde{W} = \exp(\Omega_y - Y_0 (H_m^- - h_B S^z) - Y_0 \sum_l (S_l^z \exp(-ih_B t) + h.c.))$$

where $H_m^p \equiv U_p^- H_m U_p^+$.

This statistical operator will be the space homogeneous operator. In order to get rid of dependence on time in the statistical operator we must turn into rotating sys-

tem of coordinates. It means, that we must consider the operator

$$W' = e^{ih_B S^z t} W e^{-ih_B S^z t}, \quad \langle a \rangle = Sp W' a.$$

Taking these notes into account and using the reduced description method [8], the equation for impurity density matrix in the described condensed matter was obtained:

$$\dot{w} = -i\omega_0 [w, n_\alpha \sigma_\alpha] - \left(\int_{-\infty}^0 d\tau K_{\alpha\beta}(\tau) [w \sigma_\alpha(\tau), \sigma_\beta] + h.c. \right). \quad (8)$$

Frequency ω_0 and ort n are defined by formulae:

$$\omega_0 = \sqrt{(\tilde{I}_z(0) S_{\parallel} + h + \mu_0 g' B + \mu_0 g B)^2 + (\tilde{I}_{\perp}(p) S_{\perp})^2},$$

$$n = \left(\frac{\tilde{I}_{\perp}(p) S_{\perp}}{\omega_0}, 0, \frac{(\tilde{I}_z(0) S_{\parallel} + h + \mu_0 g' B + \mu_0 g B)}{\omega_0} \right),$$

$\tilde{I}_z(p)$, $\tilde{I}_{\perp}(p)$ are the Fourier components of the longitudinal $I_z(x)$ and transversal $I_{\perp}(x)$ exchange integrals; $K_{\alpha\beta}$ is the spin correlation function

$$K_{ij}(\tau) = \int_{-\infty}^0 d\tau' \int_{i,l} I_i(x_i) I_j(x_e) \langle S_i(x_i, \tau') S_j(x_e) \rangle \langle S_i(x_i) \rangle \langle S_j(x_e) \rangle,$$

$$I_{\alpha} = \delta_{\alpha z} I_z + \delta_{\alpha \pm} I_{\perp}, \quad \sigma_{\alpha}(\tau) = e^{iH_0 \tau} \sigma_{\alpha} e^{-iH_0 \tau}.$$

Here the averaging is performed with homogeneous and time independent statistical operator of magnetic matrix.

3. ENERGY SPECTRUM OF ANY IMPURITY SPIN IN MAGNETIC

The equations (8) allow us to find the spectrum of any impurity spin in magnetic in main approximation. We perform the turn in spin space so as

$$U_{\alpha} n \sigma U_{\alpha}^+ = \sigma^z,$$

where $U_{\alpha} = e^{i\alpha \sigma_y}$ is the unit operator of turn, $\tan \alpha = n_x / n_z$.

In a result we obtain the equation for impurity density matrix \tilde{w}

$$\dot{\tilde{w}}(t) = -i\omega_0 [\tilde{w}(t), \sigma^z], \quad (9)$$

$$\tilde{w}(t) = U_{\alpha} w(t) U_{\alpha}^+.$$

The Hermit operator \tilde{w} is characterized by the set of the matrix elements

$$\tilde{w}_{mn} \equiv \langle m | \tilde{w} | n \rangle, \quad m, n = -\sigma, \dots, +\sigma,$$

where $|m\rangle$ is the eigenvector of the operator σ^z ,

$$\sigma^z |m\rangle = m |m\rangle, \quad -\sigma \leq m \leq \sigma.$$

The equation (9) in terms of the matrix elements is as follows

$$\dot{\tilde{w}}_{mn}(t) = -i\omega_0 (m - n) \tilde{w}_{mn}(t).$$

From this equation we define the energy spectrum of the impurity spin.

$$\omega = \omega_0 (m - n), \quad m, n = -\sigma, \dots, +\sigma.$$

The spectrum we obtain has equidistant character. This spectrum is nondegenerated spectrum, when

$m = \sigma, n = -\sigma$ and $m = -\sigma, n = \sigma$. When $m = n$ the repetition factor of the degenerate equals $2\sigma + 1$.

4. KINETIC COEFFICIENTS FOR IMPURITY SPIN $\sigma = 1/2$

It is very difficult to solve equation (8) in general case. Now we pay main attention to the case, when the impurity spin is equal to $1/2$. In this case: $w = (1/2)(1 + P\tau_i)$, where τ_i is the Pauli matrix and P is the polarization vector. From (8) we get the equation for a motion of polarization vector

$$\frac{\partial P_\alpha}{\partial t} = -\varepsilon_{\alpha\beta\gamma}(n_\beta\omega_0 - \frac{1}{2}D_\beta)P_\gamma - D_{\alpha\beta}^S P_\beta \quad (10)$$

$$D_\alpha = \varepsilon_{\alpha\beta\gamma}D_{\alpha\beta}, D_{\alpha\beta}^S = \frac{1}{2}(D_{\alpha\beta} + D_{\beta\alpha}),$$

$$D_{\alpha\beta} = \text{Re}(\delta_{\alpha\beta}K_{\lambda\lambda} - K_{\alpha\beta}).$$

The first item in round brackets describes the precession of the polarization vector. The coefficients D_α define the shift and the thin structure of levels, and the coefficients $D_{\alpha\beta}^S$ – their width.

Now we will find the solutions of the equations (10) in form

$$P \sim \exp(i\omega t).$$

In main approximation in the interaction between the matrix and the impurity spin we have obtained dispersion equation

$$\omega(\omega^2 - \omega_0^2) = 0.$$

From this equation we have two solutions: $\omega = 0$ and $\omega = \omega_0$. Next approximation gives frequency shift $\delta\omega$ and damping decrement γ .

The solution for $\omega = 0$ is as follows:

$$\delta\omega_1 = 0, \gamma_1 = n_x^2 D_{11}^S + 2n_x n_z D_{13}^S + n_z^2 D_{33}^S.$$

And the solution for $\omega = \omega_0$ is

$$\delta\omega_2 = -n_x D_1 - n_z D_3, \gamma_2 = (1/2)(D_{ii}^S - \gamma_1).$$

These results correspond the results obtained in [9].

Now we consider the particular case, corresponding to the case of a magnetic with “light axes” ordering type. For this case we have $\langle S^\pm \rangle = 0, \langle S^z \rangle \neq 0$. For fundamental frequency $\omega = 0$ we have

$$\delta\omega_1 = 0, \gamma_1 = D_{33}^S.$$

For frequency $\omega = \omega_0$ correspondently

$$\delta\omega_2 = -D_3, \gamma_2 = (1/2)(D_{11}^S + D_{22}^S).$$

The results correspond the results obtained in [10].

Using the Goldstein – Primakov form for matrix spin we can find the temperature dependence of the obtained decrements. In quadratic approximation by f (the magnon distribution function) we have only decrement γ_2 , which is not equal to zero:

$$\gamma_2 = \text{Re}K_{zz}(0) = \pi \sum_{pq} \tilde{T}_{p-q}^2 \delta(\varepsilon_p - \varepsilon_q) f_p (1 + f_q).$$

Performing the integration we get in spin-wave approximation

$$\gamma_2 = \left(\frac{\tilde{T}^2}{(2\pi)^4 \theta_c} \right) \left(\frac{kT}{\theta_c} \right)^2 \ln \left(\frac{kT}{\varepsilon(0)} \right),$$

where $\varepsilon_p \equiv \varepsilon(p) = \theta_c (ap)^2 + \mu g' B$ is the energy of magnon, θ_c is the Curie energy, $\tilde{T} = \tilde{T}(0)$ is the zero Fourier component of the longitudinal exchange integral.

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