

ON SOUND WAVE SPECTRA IN BIAXIAL NEMATIC CRYSTALS

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On the basis of the Hamiltonian approach dynamics of biaxial nematics surveyed and the deduction of the nonlinear equations of ideal hydrodynamics, taking into account the shape, (disk-shaped and rod-shaped of a molecule) and conformation degree of freedom are given. The densities of additive integrals of motion and relevant fluxes are represented in the terms of a thermodynamic potential. The spectra of linear oscillations in these liquid crystals surveyed. Two branches of ultrasonic oscillations are found and the character of an anisotropy of both velocities of sounds is found out.

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I. INTRODUCTION

The liquid crystal (LC) dynamic equations in the traditional phenomenological approach are obtained proceeding from the symmetry considerations as accounted for by the conservation laws. In this case, the accurate accounting for non-linear items presents a problem. The Hamiltonian approach is exactly more consistent [1]. This approach provides for an effective method for construction of the non-linear dynamic equations describing the transfer phenomena in various condensed media. In the Hamiltonian approach, the Poisson bracket (PB) structure for reduced description parameters that settles completely the medium state at the macroscopic level is of primary importance.

Selection of the reduced description parameters in the case of LC depends on a number of factors. Some hydrodynamic parameters are associated with the Hamiltonian symmetry properties, manifested as the presence of dynamic equations conditioned by the differential conservation laws. The molecular shape is another factor influencing the set of hydrodynamic parameters. In LC, there is a correlation between the molecular shape and the structure of hydrodynamic equations. The Poisson bracket structure of hydrodynamic parameters has been shown to be of different form for disc- and rod-shaped molecules [2-6]. This molecular shape effect is manifested physically as different signs of the reactive coefficient in the hydrodynamic equations, different possibilities of ferroelectric state realization and spectral peculiarities of the polarized light absorption. In [5,6], a specific feature of the mentioned correlation between the molecular shape and hydrodynamics has been clarified, manifested, firstly, as distinctions in the Poisson brackets structure for the reduced description parameters, and, secondly, as an increased number of the mentioned parameters at the hydrodynamic evolution stage. From the mathematical standpoint, the appearance of an additional quantity is due to the requirement of the closed Poisson bracket algebra for

the entire set of hydrodynamic variables. Physically, this is associated with the fact that several characteristic lengths and relaxation time values are present in such a medium.

Near the phase transition temperature in sufficiently strong electric or magnetic external fields, additional components of the order parameter are to be taken into account in low-dimensional cases ($n < 3$), the parameter being a symmetric and hole-free tensor [7-10]. In this connection, analogy with the super-fluid Bose-liquid should be noted; for the latter, near the phase transition region, or in the super-fluid aerogel state, the set of the reduced description parameters should also be extended in order to take into account not only the order parameter phase but also its module [11,12]. At last, the parameter set is associated with the character of spontaneous symmetry disturbance in the system. The elasticity theory is formulated as part of the continuous medium mechanics, based on the concept of spontaneously disturbed translation symmetry. It is just the deformation tensor that represents the dynamic quantity in the set of reduced description parameters associated with such symmetry disturbance. That tensor can be presented in a certain manner in terms of the distortion one [13]. The latter quantity reflects completely the strain character of the continuous medium, but its introduction as an additional dynamic quantity is redundant as a rule. The LC hydrodynamic theory can also be viewed as continuous medium mechanics with spontaneously disturbed symmetry. In this case, there is the symmetry disturbance with respect to rotations in the configuration space. It has been shown [4-6] that additional hydrodynamic parameters associated with this symmetry disturbance can also be presented in terms of the distortion tensor for many uniaxial LC.

The hydrodynamics of biaxial LC has been considered in [14-17]. The liquid crystalline ordering is described by a field of three unit vectors. For this LC class, the typical case is the full spontaneous symmetry

disturbance with respect to rotations in the configuration space $O(3)$. The mentioned works, however, do not present in explicit form the expressions for all reactive flux densities of the motion integrals in terms of the energy functional. The molecular shape effect on the dynamic equations for this LC class is also not elucidated.

In this work, some additional hydrodynamic quantities associated with spontaneous symmetry disturbance with respect to rotations in the configuration space are introduced in terms of the distortion tensor. The densities and fluxes of the additive motion integrals are presented in terms of the thermodynamic potential. The ideal hydrodynamic equations are derived. The spectra of collective excitations for biaxial LC are studied taking into account their molecular shape.

Firstly, let the Hamiltonian formalism be considered briefly for the case of LC. Let the system Lagrangian be presented as

$$L = L_k(\varphi, \dot{\varphi}) - H \equiv \int d^3x F_\alpha(x, \varphi(x')) \dot{\varphi}_\alpha(x) - H$$

,where $L_k(\varphi, \dot{\varphi})$ is the kinematic part of the Lagrangian; $H(\varphi) = \int d^3x \varepsilon(\varphi)$ - the Hamiltonian; $F_\alpha(x, \varphi(x'))$, a certain functional of dynamic variables $\varphi_\alpha(x)$. From the stationary action principle, the motion equations for the $\varphi_\alpha(x)$ field components are presented as follows:

$$\dot{\varphi}_\alpha(x) = \int d^3x' J_{\alpha\beta}^{-1}(x, x', \varphi) \frac{\delta H}{\delta \varphi_\beta(x')} \\ J_{\alpha\beta}(x, x', \varphi) \equiv \frac{\delta F_\beta(x')}{\delta \varphi_\alpha(x)} - \frac{\delta F_\alpha(x)}{\delta \varphi_\beta(x')} \quad (1.1)$$

Let the Poisson brackets for the arbitrary functional values $A(\varphi)$, $B(\varphi)$ be defined by the equality

$$\{A, B\} \equiv \int d^3x \int d^3x' \frac{\delta A}{\delta \varphi_\alpha(x)} J_{\alpha\beta}^{-1}(x, x') \frac{\delta B}{\delta \varphi_\beta(x')}.$$

Then, the equations (1.1) take on the Hamiltonian form:

$$\dot{\varphi}_\alpha(x) = \{\varphi_\alpha(x), H\} \quad (1.2)$$

The Hamiltonian approach allows one to construct differential conservation laws without specifying explicitly the Hamiltonian form, but accounting only for its symmetry properties. To this purpose, note that the motion equation for the density $a(x)$ of the arbitrary physical quantity $A = \int d^3x a(x)$ can be presented [4] as:

$$\dot{a}(x) = \{a(x), H\} \equiv \{A, \varepsilon(x)\} - \nabla_k a_k(x), \\ a_k(x) = \int d^3x' x'_k \int_0^1 d\lambda \{a(x + \lambda x'), \varepsilon(x - (1 - \lambda)x')\} \quad (1.3)$$

These relationships permit to express the flux densities in terms of the Poisson brackets for the corresponding densities of the additive motion integrals.

Assuming in (1.3) $a(x) = \rho(x)$, where $\rho(x)$ is the substance density, and regarding

$$M = \int d^3x \rho(x), \quad \{M, \varepsilon(x)\} = 0,$$

we obtain the differential law of mass conservation:

$$\dot{\rho}(x) = -\nabla_k j_k(x), \\ j_k(x) = \int d^3x' x'_k \int_0^1 d\lambda \{\rho(x + \lambda x'), \varepsilon(x - (1 - \lambda)x')\} \quad (1.4)$$

Here, $j_k(x)$ is the mass flux density. At $a(x) = \pi_k(x)$, (where $\pi_k(x)$ is the momentum density) and under condition of the translation invariance of energy density,

$$P_i = \int d^3x \pi_i(x), \quad \{P_i, \varepsilon(x)\} = \nabla_i \varepsilon(x)$$

we obtain the differential law of momentum conservation

$$\dot{\pi}_i(x) = -\nabla_k t_{ik}(x), \\ t_{ik}(x) = -\varepsilon(x) \delta_{ik} + \\ + \int d^3x' x'_k \int_0^1 d\lambda \{\pi_i(x + \lambda x'), \varepsilon(x - (1 - \lambda)x')\} \quad (1.5)$$

Here, $t_{ik}(x)$ is the momentum flux density. At $a(x) = \varepsilon(x)$, we obtain the differential law of energy conservation from (1.3):

$$\dot{\varepsilon}(x) = -\nabla_k q_k(x), \\ q_k(x) = \frac{1}{2} \int d^3x' x'_k \int_0^1 d\lambda \{\varepsilon(x + \lambda x'), \varepsilon(x - (1 - \lambda)x')\} \quad (1.6)$$

where $q_k(x)$ is the energy flux density.

2. THE POISSON BRACKET STRUCTURE FOR CLASSIC CONTINUOUS MEDIA

In terms of the Lagrangian variables ζ_i , the position of a particle in a medium is characterized by the functions $x_k(\zeta, t)$. The Lagrangian coordinates have the physical sense of particle coordinates in the initial position complying with the strain-free medium state. Let the system Lagrangian be written as:

$$L = L_k - H, \quad H = \int d^3\zeta \varepsilon(\pi, \frac{\partial x}{\partial \zeta}),$$

$$L_k = \int d^3\zeta \pi_i(\zeta) \dot{x}_i(\zeta),$$

where $\varepsilon(\zeta) = \varepsilon(\pi(\zeta), \frac{\partial x}{\partial \zeta})$ is the energy density and

$\pi_k(\zeta)$ is the momentum density in the Lagrangian variables; L_k - the kinematic Lagrangian part. Let the latter expression be transformed using the Eulerian

variables $x_i(\zeta, t)$ and introducing the displacement vector of continuous medium particles $u_i(x, t)$:

$$x_i(t) = \zeta_i + u_i(x, t). \quad (2.1)$$

Bearing in mind that

$$\dot{x}_j = b_{ji}^{-1}(x) \dot{u}_i(x), \quad b_{ij}(x) \equiv \delta_{ji} - \nabla_j u_i(x) \quad (2.2)$$

let the kinematic Lagrangian part be presented in the form of $L_k = p_k(x) \dot{u}_k(x) - \sigma(x) \dot{\psi}(x)$, where $p_k(x) = (\pi_i(x) - \sigma(x) \nabla_i \psi(x)) b_{ik}^{-1}(x)$ is the generalized momentum density in the Eulerian coordinates. The variable $\psi(x)$ that is canonically conjugate with the entropy density $\sigma(x)$ is believed to be cyclic. Using this kinematic part, one can obtain the PB for the variables $\sigma(x), \psi(x), \pi_i(x), u_i(x)$, proceeding from such transformations that retain the invariant kinematic part of the Lagrangian

$$\begin{aligned} \delta u_i(x) &= a_i(x), \quad \delta \psi(x) = b(x), \\ \delta p_i(x) &= \delta \sigma(x) = 0 \end{aligned}$$

where the functions $a_i(x), b(x)$ are independent of $p_i(x), \sigma(x), u_i(x), \psi(x)$. As a result, we have the PB set:

$$\begin{aligned} \{u_k(x), \pi_i(x')\} &= b_{ki}(x') \delta(x - x'), \\ \{\psi(x), \pi_i(x')\} &= -\delta(x - x') \nabla_i \psi(x), \\ \{\pi_i(x), \sigma(x')\} &= -\sigma(x) \nabla_i \delta(x - x'), \\ \{\sigma(x), \psi(x')\} &= \delta(x - x'), \\ \{\pi_i(x), \pi_j(x')\} &= \pi_j(x) \nabla'_i \delta(x - x') - \\ &- \pi_i(x') \nabla'_j \delta(x - x') \end{aligned} \quad (2.3)$$

Now, let us take into account the fact that if $\tilde{\rho}$ is the substance density referred to the strain-free unit volume, then the true density ρ is defined as:

$$\rho(x) = \tilde{\rho} \det b_{ij}(x) \quad (2.4)$$

Let's find PB for the variables $\pi_k(x), \rho(x')$ taking into account (2.2), (2.3), (2.4):

$$\{\pi_i(x), \rho(x')\} = \rho(x) \nabla'_i \delta(x - x') \quad (2.5)$$

In the follow-up, the energy density $\varepsilon(x)$ is assumed to be translationally invariant, thus, not being dependent on the displacement vector $u_k(x)$, but rather on its derivatives $\nabla_i u_k \equiv w_{ik}$ (w_{ik} is the distortion vector) or, the same, on the strain matrix $b_{st}(x)$. Therefore, it is convenient to select directly the $b_{st}(x)$ as a dynamic variable along with others. For $b_{st}(x)$, the only non-zero PB has the form

$$\{\pi_i(x), b_{kj}(x')\} = -b_{ki}(x) \nabla'_j \delta(x - x'). \quad (2.6)$$

The PB set (2.3), (2.5), (2.6) is the construction base for hydrodynamic type equations for normal liquids, crystals and LC.

3. DYNAMICS OF BIAxIAL NEMATIC CRYSTALS

In the LC biaxial nematic phase, the complete rotational symmetry spontaneous disturbance takes place. Along with the dynamic variables describing the isotropic liquid state, namely, the mass density $\rho(x)$, the momentum density $\pi_i(x)$, the entropy density $\sigma(x)$, additional parameters associated with the rotational symmetry disturbance in the configuration space should be determined. Two anisotropy axes have been included in the reduced description parameters in [14–17]; these works have a phenomenological character that is not connected with the Hamiltonian formalism. In the case of disc-shaped LC, let the unit and orthogonal symmetry axes, characterizing the rotational invariance disturbance $m(x), n(x)$ and the shape factor $p(x)$, be defined as

$$\begin{aligned} n_i(x) &\equiv \frac{a(x) b_{i1}(x) + b(x) a_i(x)}{|a(x) b(x) + b(x) a(x)|}, \\ m_i(x) &\equiv \frac{a(x) b_{i2}(x) - b(x) a_i(x)}{|a(x) b(x) - b(x) a(x)|}, \\ p(x) &\equiv \frac{1}{2} \left(1 - \frac{a(x) b(x)}{a(x) b(x)} \right). \end{aligned} \quad (3.1)$$

The vectors $a(x), b(x)$ in terms of the strain matrix $b_{ij}(x)$ have the form $a_j(x) \equiv e_{1k} b_{kj}(x)$, $b_j(x) \equiv e_{2k} b_{kj}(x)$. Here, e_{1k}, e_{2k} are the orthogonal constant vectors defining the anisotropy axes in the strain-free state $a(x) = |a(x)|$, $b(x) = |b(x)|$. The orientation degrees of freedom $m(x), n(x), p(x)$ are defined not as local functions of the strain field u , but as those of their derivatives $\nabla_i u_k(x)$. Therefore, the local relations of four quantities $m(x), n(x), p(x)$ with nine $\nabla_i u_k(x)$ ones will not result in overflow of the dynamic equation system, and the quantities $m(x), n(x), p(x)$ can be considered as independent variables. According to (3.1) and taking into account (2.6), we obtain the PB for the variables $\pi_k(x), n_k(x), m_k(x)$:

$$\begin{aligned} \{\pi_i(x), n_j(x')\} &= \delta(x - x') \nabla'_i n_j(x) + \\ &+ f_{ilj}(x') \nabla'_l \delta(x - x') \end{aligned} \quad (3.2)$$

$$\begin{aligned} \{\pi_i(x), m_j(x')\} &= \delta(x-x') \nabla_i m_j(x) + \\ &+ g_{i\lambda j}(x') \nabla'_\lambda \delta(x-x'), \end{aligned}$$

where the quantities $f_{i\lambda j}(x)$, $g_{i\lambda j}(x)$ and $\delta_{kj}^\perp(f(x))$ are defined by the equalities:

$$\begin{aligned} f_{i\lambda j}(x) &\equiv n_i \delta_{j\lambda}^\perp(n(x)) - \\ &- p(x) m_j(x) (n_i(x) m_\lambda(x) + n_\lambda(x) m_i(x)), \\ g_{i\lambda j}(x) &\equiv m_i \delta_{j\lambda}^\perp(\vec{m}(x)) - \\ &- (1-p(x)) n_j(x) (n_i(x) m_\lambda(x) + n_\lambda(x) m_i(x)), \\ \delta_{kj}^\perp(f(x)) &\equiv \delta_{kj} - f_k(x) f_j(x). \end{aligned}$$

The scalar parameter $p(x)$ is defined, according to (3.1), by the angle between the strained LC axes. It follows from (3.1) accounting for (2.6),(2.3) that the nonzero PB for the variable $p(x)$ with other biaxial nematic crystal hydrodynamic parameters has the form

$$\begin{aligned} \{\pi_i(x), p(x')\} &= \delta(x-x') \nabla_i p(x) - 2p(x') \times \\ &\times (1-p(x')) (n_i(x') n_l(x') - m_i(x') m_l(x')) \nabla'_l \delta(x-x') \end{aligned} \quad (3.3)$$

The Poisson brackets (3.2),(3.3) along with the (2.3),(2.5),(2.6) ones form closed algebra of hydrodynamic quantities for the biaxial nematic crystals with disc-shaped molecules. It is just this instance that makes it necessary to include the $p(x)$ quantity in this set of variables.

Now let us consider the motion equation in the Hamiltonian form (1.2), taking into account the (3.2),(3.3),(2.3) PB form and explicit expressions for the flux density of additive integrals (1.4),(1.5). Assuming the Hamiltonian has the Galilean invariant form

$$H = \int d^3x \left(\frac{\pi_i^2(x)}{2\rho(x)} + \Phi(\rho, \sigma, \vec{n}, \vec{m}, \nabla \vec{n}, \nabla \vec{m}, p) \right). \quad (3.4)$$

we obtain the following dynamic equations for this biaxial phase:

$$\begin{aligned} \dot{\sigma}(x) &= -\nabla_i (\sigma(x) v_i(x)), \quad \dot{\rho}(x) = -\nabla_i \pi_i(x), \\ \dot{\pi}_i(x) &= -\nabla_k t_{ik}(x), \\ \dot{n}_j(x) &= -v_s(x) \nabla_s n_j(x) - f_{i\lambda j}(x) \nabla_\lambda v_i(x), \\ \dot{m}_j(x) &= -v_s(x) \nabla_s m_j(x) - g_{i\lambda j}(x) \nabla_\lambda v_i(x), \\ \dot{p}(x) &= -v_s(x) \nabla_s p(x) + 2p(x) (1-p(x)) \times \\ &\times (n_k(x) n_l(x) - m_k(x) m_l(x)) \nabla_k v_l(x). \end{aligned} \quad (3.5)$$

Here, $v_i(x) \equiv \frac{\partial \varepsilon(x)}{\partial \pi_i(x)} = \frac{\pi_i(x)}{\rho(x)}$ is the medium unit

mass velocity. The momentum flux density $t_{ik}(x)$ can be found using the PB set (2.3),(2.5),(3.2),(3.3) and the formula (1.5). As a result, we obtain the following expression for this flux in terms of energy density:

$$\begin{aligned} t_{ik}(x) &= t_{ik}^0(x) + t'_{ik}(x), \\ t_{ik}^0 &= \frac{\pi_i \pi_k}{\rho} + \left(\sigma \frac{\partial \Phi}{\partial \sigma} + \rho \frac{\partial \Phi}{\partial \rho} - \Phi \right) \delta_{ik}, \\ t'_{ik} &= \frac{\partial \varepsilon}{\partial \nabla_k n_j} \nabla_i n_j + \frac{\partial \varepsilon}{\partial \nabla_k m_j} \nabla_i m_j + \\ &+ f_{ikl} \left(\frac{\partial \varepsilon}{\partial n_l} - \nabla_j \frac{\partial \varepsilon}{\partial \nabla_j n_l} \right) + g_{ikl} \left(\frac{\partial \varepsilon}{\partial m_l} - \nabla_j \frac{\partial \varepsilon}{\partial \nabla_j m_l} \right) - \\ &- 2p(1-p) \frac{\partial \varepsilon}{\partial p} (n_i n_k - m_i m_k). \end{aligned} \quad (3.6)$$

According to (3.4), the second law of thermodynamics has the form

$$\begin{aligned} d\varepsilon &= \mu d\rho + \rho T d\sigma + v_i d\pi_i + \frac{\partial \varepsilon}{\partial p} dp + \\ &+ \left(\frac{\partial \varepsilon}{\partial n_i} \right) \delta_{ik}^\perp(\vec{n}) dn_k + \nabla_j \left(\frac{\partial \varepsilon}{\partial \nabla_j n_i} \delta_{ik}^\perp(\vec{n}) dn_k \right) + \\ &+ \left(\frac{\partial \varepsilon}{\partial m_i} \right) \delta_{ik}^\perp(\vec{m}) dm_k + \nabla_j \left(\frac{\partial \varepsilon}{\partial \nabla_j m_i} \delta_{ik}^\perp(\vec{m}) dm_k \right) \end{aligned} \quad (3.7)$$

where T - is the temperature; μ - the chemical potential.

The formulae (3.5,6) represent a complete set of equations for the ideal hydrodynamics of biaxial nematic crystals consisting of disc-shaped molecules.

In the case of rod-shaped molecules, let the unit and orthogonal anisotropy axes characterizing the rotational invariance disturbance, $m(x), n(x)$ and the shape factor $p(x)$ be defined as

$$n_i(x) = \frac{A(x) B_{\underline{i}}(x) + B(x) A_{\underline{i}}(x)}{|A(x) B(x) + B(x) A(x)|}, \quad (3.8)$$

$$m_i(x) = \frac{A(x) B_{\underline{i}}(x) - B(x) A_{\underline{i}}(x)}{|A(x) B(x) - B(x) A(x)|},$$

$$p(x) = \frac{1}{2} \left(1 - \frac{A(x) B(x)}{A(x) B(x)} \right). \quad (3.9)$$

Here, the vectors $A_j(x) \equiv e_{1k} b_{kj}^{-1}(x)$ and $B_j(x) \equiv e_{2k} b_{kj}^{-1}(x)$ are set in terms of the reverse matrix $b_{kj}^{-1}(x)$. Taking into account this definition (3.9) and the relation (2.6), we obtain the Poisson brackets

$$\begin{aligned}
\{\pi_i(x), n_j(x')\} &= \delta(x-x') \nabla_i n_j(x) + \bar{f}_{ij}(x') \nabla'_\lambda \delta(x-x'), \\
\{\pi_i(x), m_j(x')\} &= \delta(x-x') \nabla_i m_j(x) + \bar{g}_{ij}(x') \nabla'_\lambda \delta(x-x'), \\
\{\pi_i(x), p(x')\} &= \delta(x-x') \nabla_i p(x) + 2p(x') \times \\
&\times (1-p(x')) (n_i(x') n_l(x') - m_i(x') m_l(x')) \nabla'_\lambda \delta(x-x')
\end{aligned} \tag{3.10}$$

where the following notations are used:

$$\begin{aligned}
\bar{f}_{ij}(x) &\equiv -n_\lambda \delta_{ij} \frac{1}{ij} (n(x)) + p(x) m_j(x) \times \\
&\times (n_i(x) m_\lambda(x) + n_\lambda(x) m_i(x)), \\
\bar{g}_{ij}(x) &\equiv -m_\lambda \delta_{ij} \frac{1}{ij} (m(x)) + (1-p(x)) n_j(x) \times \\
&\times (n_i(x) m_\lambda(x) + n_\lambda(x) m_i(x)).
\end{aligned}$$

Making further treatment in a manner similar to the above case of disc-shaped molecules, it is not difficult to derive equations for ideal hydrodynamics of rod-shaped molecules:

$$\begin{aligned}
\dot{\sigma}(x) &= -\nabla_i (\sigma(x) v_i(x)) \quad , \quad \dot{\rho}(x) = -\nabla_i \pi_i(x), \\
\dot{\pi}_i(x) &= -\nabla_k t_{ik}(x), \\
\dot{n}_j(x) &= -v_s(x) \nabla_s n_j(x) - \bar{f}_{ij}(x) \nabla_\lambda v_i(x), \\
\dot{m}_j(x) &= -v_s(x) \nabla_s m_j(x) - \bar{g}_{ij}(x) \nabla_\lambda v_i(x), \\
\dot{p}(x) &= -v_s(x) \nabla_s p(x) - 2p(x) (1-p(x)) \times \\
&\times (n_k(x) n_l(x) - m_k(x) m_l(x)) \nabla_k v_l(x)
\end{aligned} \tag{3.11}$$

In this case, the momentum flux density has the form

$$\begin{aligned}
t_{ik}(x) &= t_{ik}^0(x) + t'_{ik}(x), \\
t_{ik}^0 &= \frac{\pi_i \pi_k}{\rho} + \left(\sigma \frac{\partial \Phi}{\partial \sigma} + \rho \frac{\partial \Phi}{\partial \rho} - \Phi \right) \delta_{ik}, \\
t'_{ik} &= \frac{\partial \varepsilon}{\partial \nabla_k n_j} \nabla_i n_j + \frac{\partial \varepsilon}{\partial \nabla_k m_j} \nabla_i m_j + \bar{f}_{ikl} \left(\frac{\delta \varepsilon}{\delta n_l} \right) + \\
&+ 2p(1-p) \frac{\partial \varepsilon}{\partial p} (n_i n_k - m_i m_k) + \bar{g}_{ikl} \left(\frac{\delta \varepsilon}{\delta m_l} \right).
\end{aligned} \tag{3.12}$$

4. SPECTRA OF COLLECTIVE EXCITATIONS

Let's consider the spectra of collective excitations based on the equations for biaxial nematic crystals. The equilibrium state of such a medium is believed to be homogeneous, strain-free ($p = 1/2$), while the medium as a whole is at rest ($v_k = 0$). From equations (4.2) we obtain the dispersion equation for determination of the collective excitation spectra:

$$\det \left| \omega^2 \delta_{ij} - k_i k_j \frac{\partial P}{\partial \rho} - \frac{1}{4\rho} \frac{\partial^2 \varepsilon}{\partial p^2} R_i(\vec{k}) R_j(\vec{k}) \right| = 0$$

where $R_i(k) \equiv (n_i(nk) - m_i(mk))$. Expanding the determinant, we obtain

$$\omega^2 (\omega^4 - \omega^2 L_4(k) + L_2(k)) = 0 \tag{4.1}$$

Here, the following notations are used:

$$\begin{aligned}
L_4(\vec{k}) &= k^2 c^2 + (\vec{k}(\vec{m} \times \vec{n}))^2 A \equiv k^2 c^2 + k_{\parallel}^2 A > 0, \\
L_2(k) &= c^2 A \left[4(\vec{k}n)^2 (\vec{k}m)^2 + k_{\parallel}^2 \left((\vec{k}n)^2 + (\vec{k}m)^2 \right) \right]
\end{aligned}$$

where

$$c^2 = \frac{\partial P}{\partial \rho} > 0, \quad A = \frac{1}{4\rho} \frac{\partial^2 \varepsilon}{\partial p^2} > 0.$$

Thus, we see that in the biaxial nematic crystals under consideration, propagation of two acoustic vibration branches is quite possible:

$$\omega_{\pm}^2(\vec{k}) = \frac{1}{2} \left(L_4(\vec{k}) \pm \sqrt{L_4^2(\vec{k}) - 4L_2(k)} \right) \equiv c_{\pm}^2 \left(\frac{k}{k} \right)^2 \tag{4.2}$$

corresponding to the 1st and 2nd sound, respectively. The solution of (4.2) with the positive sign complies with a branch similar to the 1st sound occurring in the normal liquid. The solution with the negative sign corresponds to a new excitation branch due to the presence of LC biaxiality; in general, the shape factor P appears for such LC states. For both solutions, the sound speed anisotropy is of significant importance. Introducing the unit vector $e \equiv k/k$, in the spherical coordinates $em = \sin\theta \cos\varphi$, $en = \sin\theta \sin\varphi$, $el = \cos\theta$, where θ, φ are the polar and azimuthal angles, respectively, defining the direction of the wave vector e . In terms of these variables, the speeds c_{\pm} (4.2) have the following form

$$\begin{aligned}
\frac{2c_{\pm}^2(\theta, \varphi)}{c^2} &= 1 + \lambda \sin^2 \theta \pm \left[(1 + \lambda \sin^2 \theta)^2 - \right. \\
&- 4\lambda \sin^2 \theta (\sin^2 \theta \sin^2 2\varphi + \cos^2 \theta) \left. \right]^{1/2}.
\end{aligned} \tag{4.3}$$

In the Figures, the character of anisotropy, as defined by (4.3), is presented (three vectors m, n, l form the Cartesian orthogonal coordinate system).

Comparing formulae (4.3) to the results presented in [14–16], it should be noted that in the latter, the additional modes related to the disturbed symmetry with respect to rotations in the configuration space are of dissipative character where the reactive component is absent. The accounting for the shape factor in the hydrodynamic equations for biaxial nematic crystals results in the reactive component being in the second sound spectrum that is already under adiabatic approximation.

The linearization of the hydrodynamic equations (3.11), (3.12) corresponding to the case of rod-shaped

molecules results in the same dispersion equation and, respectively, in the spectra defined by (4.3). This result is similar to the case of uniaxial nematic crystals considered before [8] where the distinction in hydrodynamic equations due to the molecular shape does not manifest itself in the spectra under the main approximation and appears only while accounting for the next approximation, namely, the dissipative one.

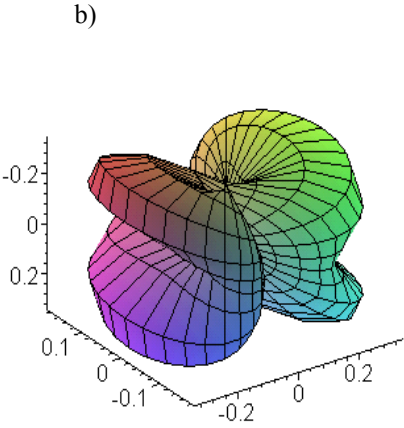
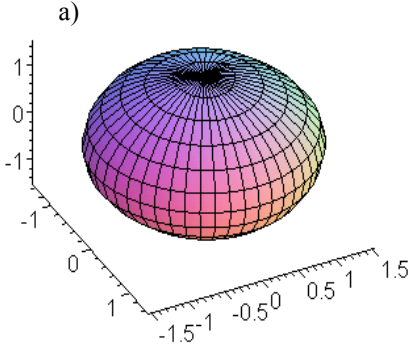
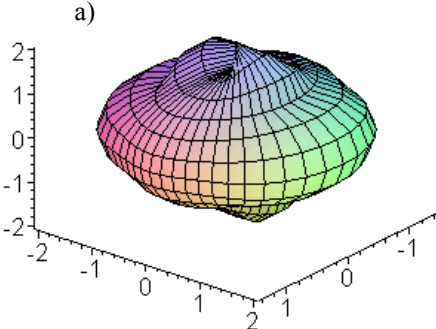


Fig. 1. The angular dependence of speeds c_+ (a) and c_- (b) at the parameter value $\lambda \equiv A/c^2 = 0.1$



b)

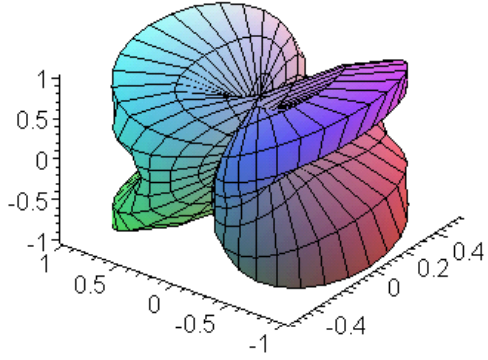


Fig. 2. The angular dependence of speeds c_+ (a) and c_- (b) at the parameter value $\lambda \equiv A/c^2 = 1$

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