ON THE THEORY OF THE INITIAL STAGE OF SLOW NUCLEAR BURNING

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Scientific principles of linear nuclear power systems of slow burning are elaborated. Two concepts of a slow burning reactor, one using U-Pu cycle and the other Th-U cycle are proposed. In the first concept the reactor is approximated by a homogeneous media and operates with fast neutrons. In the second concept the reactor is a heterogeneous assembly using both fast and thermal neutrons. The fast neutron flux at the initial stage after the reactor was ignited was determined for both concepts.

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In the present paper• the scientific fundamentals of linear nuclear power systems of slow burning are developed.

The nuclear disarmament has resulted in accumulation of significant stocks of fissile materials ^{235}U and ^{239}Pu, which are expedient to burn in nuclear power plants. On the other hand, the long-term production of military fissile materials has resulted in accumulation (on a scale of hundreds thousand tons) of the fertile uranium (uranium-238 with density of the uranium-235 isotope about 0.2... 0.3 %, instead of 0.7 % in natural uranium). Feoktistov [1] has proposed to use these stocks in power nuclear reactors of a new type, in which the initial critical mass of ^{239}Pu is enclosed by fertile uranium. An active zone neutron exposure of this uranium will convert uranium-238 to a plutonium-239 and the active zone sequentially displaces itself to a new position, leaving behind itself nuclear "ashes" consisting of fission fragments and a not burnt out fissile material. Analogous linear reactor (TIW reactor — E. Teller, M. Ishikawa, and L. Wood) was proposed and calculated in USA [2]. Such nuclear burning process can be realized on the basis of both ^{232}Th and ^{233}U fuel cycles. In this paper an attempt of the systematic analysis of reactors of a Feoktistov and TIW types is undertaken.

Let us consider a reactor in a form of cylinder of radius R and length L. Fig. 1 shows the burning process in a such prolate reactor. Let us identify the principal processes and the corresponding zones within a reactor.

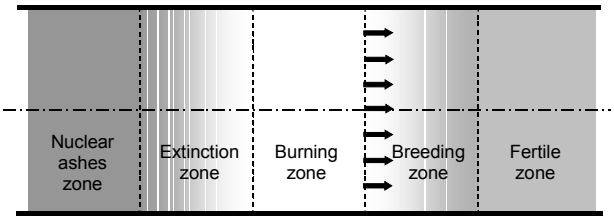

Fig. 1

Zone of burning where the fissile ^{239}Pu burns, producing energy.

Breeding zone, where ^{238}U catches a neutron, diffusing from the burning zone, and transforms finally into ^{239}Pu.

Fertile material zone is a zone containing fertile ^{232}Th or ^{238}U which can be transformed under neutron irradiation into fissile nuclear fuel. In the case of Th this zone should be heterogeneous interlacing fertile ^{232}Th isotope with a moderator, because the capture cross section is higher for thermal neutron than for fast ones. Heterogeneity is, thus, another parameter which can be used to control the rate of burning.

Zone of extinction of burning is not precisely defined. Here, due to the burn-out of the fissile fuel, to the poisoning of the zone with fission products and to other factors, the condition of a self-sustaining nuclear reaction is not fulfilled any more, but the heat release is still high. This zone plays an important role in determination of the criticality conditions of the active or burning zone.

Nuclear ashes zone, where products of the radioactive decay of the nuclear fuel are concentrated together with the unburned material.

From the physical point of view the formulated problem has much in common with a known problem of the slow burning due to chemical reactions, therefore, we shall speak about the reactor with slow nuclear burning.

The following problems can be formulated:

1. To determine velocity of propagation of the zone of slow nuclear burning depending on parameters of fertile material and other parameters of the problem (radius of assembly, structure and composition of the fertile material and moderator).
2. To calculate the multiplication factor k_{eff} in the burning zone and to find conditions of stationarity and stability of the burning zone, elemental composition and its modification lengthwise of assembly from a beginning up to its extremity.
3. To evaluate total heat release in a burning zone and the role of an additional neutron illumination from the zone of extinction.
4. To examine by the numerical methods an elemental composition in the zone of extinction as well as the

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dynamics of accumulation and disintegration of different elements in this zone.

5. To determine the elemental composition of nuclear ashes.
6. To compare the results to obtain with the results of Feoktistov[1] and Teller[2].

To begin with let us write down the balance equation for scalar neutron flux $\Phi = nv$ (n is the neutron density, v is their velocity)

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = \mathrm{div}\left(D\,\mathrm{grad}\,\Phi\right) - \Sigma_\alpha \Phi + \nu_{\mathrm{eff}}\Sigma_f \Phi \qquad (1)$$

Here D is the neutron diffusivity, Σ_α and Σ_f are the macroscopic cross sections of neutron absorption and nuclear fuel fission, ν_{eff} is the effective number of surplus neutrons. The number of nucleus of fissile element N_f, bred by neutrons, is governed by the burn-out equations

$$\frac{dN_f}{dt} = \Sigma_{\alpha f}\Phi\ , \qquad (2)$$

where $\Sigma_{\alpha f}$ is a macroscopic cross-section of neutron capture resulting in breeding of fissile element.

Eq. (1) is the principal equation describing the neutron dynamics. It is a nonlinear equation respective to Φ, because of the last term in the RHS of equation.

At the first phase of the burning process, while the density of the fissile nuclei in a breeding zone is small, this term can be neglected and we can consider, at first, a linear stage of the process development.

Two scenarios of the breeding process differing in initial and boundary conditions of the problem are conceivable.

A) ***Uranium – Plutonium assembly***. Fertile material is uranium-238. Nuclear fuel in the active zone consists of plutonium-239. The entire system operates at fast neutrons. There is no necessity of heterogeneous structure of the breeding zone; therefore, hereinafter we will examine this scenario in the supposition, that only fast neutrons are of importance (one-group approximation).

B) ***Thorium – uranium assembly***. Fertile material is thorium-232. Nuclear fuel in the active zone consists of uranium-233. Essentially, that both nuclear reactor and breeding zone operate with thermal neutrons, while the fission process generates fast neutrons. In this scenario it is necessary either to employ a complete kinetic equation of neutron dynamics, or as it is used in the theory of reactors, to resort to a multigroup approximation (in the simplest case, two systems of connected equations, for fast and thermal neutrons).

Let us consider first the problem (A). The truncated Eq. (1) is linear

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = \mathrm{div}\left(D\,\mathrm{grad}\,\Phi\right) - \Sigma_\alpha \Phi \qquad (3)$$

with the following initial and boundary conditions:

$$\begin{aligned} \Phi &= \psi_0 \delta(z) J_0(kr) && \text{for } t = 0, \\ -D\frac{\partial \Phi}{\partial z} &= \frac{v_z}{v}\Phi && \text{for } z = 0, \\ \Phi &= 0 && \text{for } r = R + \lambda_{\mathrm{extr}}, \end{aligned} \qquad (4)$$

where ψ_0 is a constant characterizing the strength of the neutron flux produced in the active zone, J_0 is a zero-order Bessel function (see below), R is the radius of the cylinder reactor, λ_{extr} is the length of the extrapolation of the neutron flux into the moderator (Fig. 2).

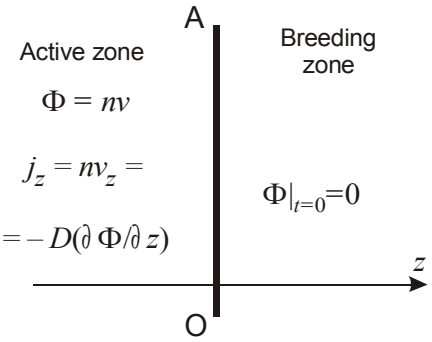

Fig. 2

In Fig. 2. AO is an impermeable membrane. At the moment $t = 0$ the membrane is removed and the breeding process starts. Here j_z is a flux of the fast neutron in the reactor directed to the breeding zone, $j_z = \Phi\, v_z/v$, where v_z is z-component of the velocity. Apparently, $(v_z/v) \le 1$.

The equation for neutron flux takes the form

$$\frac{1}{v}\frac{\partial \Phi}{\partial t} = D\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial z^2}\right) - \Sigma_\alpha \Phi\ .$$

Separating the variables

$$\Phi(r,z,t) = R(r)Z(z)T(t)\ ,$$

after some calculation we obtain the solution for $z > 0$ and $t > 0$ in the form

$$\Phi(r,z,t) = \int_{''} \Phi_\alpha e^{-\alpha^2 vt} J_0(kr)\left(\cos k_3 z - \frac{v_z}{Dvk_3}\sin k_3 z\right) d\alpha\ ,$$

where $\alpha = \Sigma_\alpha + D\left(k^2 + k_z^2\right)$ is the separation constant, k_z is the Fourier transform variable for $Z(z)$ and $k\left(R + \lambda_{\mathrm{extr}}\right) = 2.405$ is the first root of the zero-order Bessel function J_0. The above solution satisfies all boundary conditions at $z = 0$ and $r = R + \lambda_{\mathrm{extr}}$. However, path of integration C and function Φ_α has to be determined to satisfy the initial condition at $t = 0$.

Below, for convenience, we shall use k_z as an integration variable,

$$\Phi(r,z,t) = \int_C \Phi_{k_z} e^{-vt\left[\Sigma_a + Dk^2 + k_z^2\right]} J_0(kr)\left(\cos k_z z - \right.$$

$$\left. - \frac{\nu_z}{Dv k_z} \square\square\square k_z z\right) dk_z\ . \qquad (5)$$

Here the integration path C is all real values of k_z in the interval $(0,\infty)$. However, one can see that the complex value $k_z = i\chi$, where $\chi = \frac{v_z}{Dv}$, also satisfies the equation and conditions of the problem. Then

$$\Phi(r,z,t) = J_0(kr)e^{-v t\left[\Sigma_\alpha + Dk^2\right]}\left\{ Ae^{t\frac{v_z^2}{Dv} - \frac{v_z}{Dv}z} + \right.$$
$$\left. + \int_0^\infty \Phi_{k_z} e^{-vtDk_z^2}\left(\cos k_z z - \frac{v_z}{Dvk_z}\sin k_z z \right) \mathrm{d}k_z \right\}. \quad (6)$$

The constant A and function Φ_{k_z} should be determined from the initial condition.

Let us analyze the physical meaning of each term in Eq. (6). The factor $e^{-vt\left[\Sigma_\alpha + Dk^2\right]}$ testifies, that the neutrons coming from zone $z < 0$ into zone $z > 0$ are consumed in the processes of absorption and diffusion into the reflector. Therefore, the main concern is connected with the dynamics of neutrons in variables z,t. The term

$$Ae^{t\frac{v_z^2}{Dv} - \frac{v_z}{Dv}z}$$

describes the neutron "pumping" through the AO section (Fig. 2). The neutron density increases exponentially with a characteristic rise time

$$T = \frac{Dv}{v_z^2}$$

and exponentially goes down with length. In order to displace the equilibrium to the right (Fig. 2) the condition

$$\frac{v_z^2}{Dv} = v\left(\Sigma_\alpha + Dk^2\right) + \varepsilon,$$

should be, evidently, satisfied. Here ε is a certain small quantity. The term

$$\int_0^\infty \Phi_{k_z} e^{-vtDk_z^2}\left(\cos k_z z - \frac{v_z}{Dvk_z}\sin k_z z \right) \mathrm{d}k_z \quad (7)$$

describes ordinary diffusion broadening of neutron density. Indeed, the integral

$$\int_0^\infty e^{-vtDk_z^2}\cos(k_z z)dk_z = \sqrt{\frac{\pi}{4vtD}}\, e^{\frac{z^2}{4vDt}} \quad (8)$$

is responsible for diffusion spreading of an initially given (at $z = 0$) flat distribution of neutron density in the form of $\delta(z)$, because

$$\lim_{t\to\infty}\int_0^\infty e^{-vtDk_z^2}\cos(k_z z)dk_z = \delta(z), \quad \int_0^\infty \delta(z)dk_z = \frac{1}{2}. \quad (9)$$

To satisfy the remaining initial condition at $t = 0$ let us investigate the behavior of the integral Eq. (7) for $t \to 0$. Integrating Eq. (8) over z from 0 to z, one finds

$$\int_0^\infty e^{-vtDk_z^2}\frac{\sin k_z z}{k_z}dk_z = \sqrt{\pi}\int_0^{\frac{z}{\sqrt{4vDt}}} e^{-\xi^2}d\xi,$$

therefore, in the limit of small $t \to 0$

$$\lim_{t\to 0}\int_0^\infty e^{-vtDk_3^2}\left(\cos k_z z - \frac{v_z}{Dvk_z}\sin k_z z \right)dk_z = \delta(z) - \frac{\pi v_z}{2Dv}.$$

Assuming next that

$$\Phi_{k_z} = \psi_0 + g_{k_z} \quad (10)$$

we can write down the initial condition at $t = 0$ for the desired solution Φ in the form

$$Ae^{-\frac{v_z}{Dv}z} + \int_0^\infty g_{k_z}\left(\cos k_z z - \frac{v_z}{Dvk_z}\sin k_z z \right)dk_z - \psi_0\frac{\pi v_z}{2Dv} = 0 \quad (11)$$

for all z values. Using the well-known integral representation of the exponential function

$$e^{-\frac{v_z}{Dv}z} = \frac{2}{\pi}\cdot\frac{v_z}{Dv}\int_0^\infty \frac{\cos k_z z}{k_z^2 + \left(\frac{v_z}{Dv}\right)^2}\mathrm{d}k_z,$$

we find

$$2e^{-\frac{v_z}{Dv}z} - 1 = \frac{2v_z}{\pi Dv}\int_0^\infty \frac{dk_z}{k_z^2 + \left(\frac{v_z}{Dv}\right)^2}\left(\cos k_z z - \frac{v_z}{Dvk_z}\sin k_z z \right)$$

Substituting this equation into Eq. (11), we see that all initial and boundary conditions (4) would be satisfied if

$$A = \psi_0\frac{\pi v_z}{Dv},$$
$$g_{k_z} = -\psi_0\left(\frac{v_z}{Dv}\right)^2\frac{1}{k_z^2 + \left(\frac{v_z}{Dv}\right)^2}. \quad (12)$$

Then the neutron flux in the region $z > 0$ for $t > 0$ will be given by

$$\Phi(r,z,t) = \psi_0 J_0(kr)e^{-vt\left[\Sigma_\alpha + Dk^2\right]}\left\{ \frac{\pi v_z}{Dv}e^{t\frac{v_z^2}{Dv} - \frac{v_z}{Dv}z} + \right.$$
$$\left. + \int_0^\infty \frac{k_3^2\,\mathrm{d}k_3}{k_3^2 + \left(\frac{v_z}{Dv}\right)^2}e^{-vtDk_3^2}\left(\cos k_3 z - \frac{v_z}{Dvk_3}\sin k_3 z \right) \right\}. \quad (13)$$

The asymptotics of Eq. (13) for large enough argument $v_z t - z > 0$ takes the form

$$\Phi(r,z,t) \propto \psi_0 J_0(kr)e^{-vt\left(\Sigma_a + Dk^2\right)} \times$$
$$\times\left\{ \frac{1}{2}\sqrt{\frac{\pi}{vtD}}e^{-\frac{z^2}{4vtD}} + \frac{\pi v_z}{Dv}e^{\frac{v_z}{Dv}(v_z t - z)} \right\} \quad (14)$$

One can see that the first exponential describes the losses due to absorption and escape through the cylinder reactor surface. The first term in the curly brackets relates to the usual diffusion and the second one is the source due to the neutron flux incoming from the reactor active zone.

Once the neutron flux is found, the newly produced fuel can be determined as $\Sigma_f \Phi$.

Considering the problem (B), i.e. the Th-U heterogeneous assembly (see Fig. 3), we shall use a two-group approach. The fast neutrons of the first group diffuse into the breeding zone and slow down to the thermal velocities. Thus they pass to the second group of thermal neutrons.

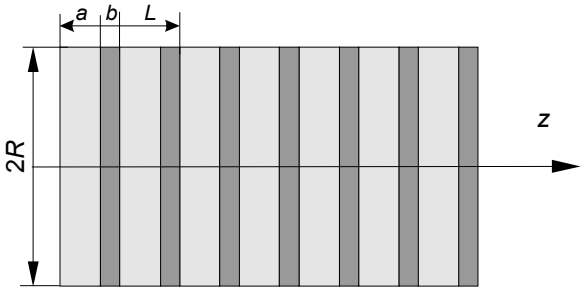

Fig. 3

The dynamics of the process is described by the balance equations

$$\frac{1}{v}\frac{\partial \Phi^{\mathrm{I}}}{\partial t} = \operatorname{div}\left(D^{\mathrm{I}} \operatorname{grad} \Phi^{\mathrm{I}}\right) - \Sigma_a \Phi^{\mathrm{I}} - \Sigma_s \Phi^{\mathrm{I}}, \qquad (15)$$

$$\frac{1}{v}\frac{\partial \Phi^{\mathrm{II}}}{\partial t} = \operatorname{div}\left(D^{\mathrm{II}} \operatorname{grad} \Phi^{\mathrm{II}}\right) - \Sigma_a \Phi^{\mathrm{II}} + \Sigma_s \Phi^{\mathrm{I}}, \qquad (16)$$

where Σ_s is the macroscopic cross section of the neutron transition from the first group into the second one. Evidently, $\Sigma_s \sim D_s^{-1}$, where D_s^{-1} is the characteristic length of neutron slowing-down. The problem should be solved with the following initial and boundary conditions for $z > 0$

$$\Phi^{\mathrm{I}} = \psi_0^{\mathrm{I}} \delta(z) J_0(kr)$$

$$\Phi^{\mathrm{II}} = \psi_0^{\mathrm{II}} \delta(z) J_0(kr) + \Sigma_s \Phi^{\mathrm{I}}$$

for $t = 0$ and for any $z \geq 0$

$$-D^{\mathrm{I,II}} \frac{\partial \Phi^{\mathrm{I,II}}(0)}{\partial z} = j_0^{\mathrm{I,II}} \quad \text{for } z = 0, \qquad (17)$$

$$\Phi^{\mathrm{I,II}} = 0 \quad \text{for } r = \left(R + \lambda_{\mathrm{extr}}\right).$$

We study, as before, the initial stage of the process when the number of the surplus neutrons is insignificant. But, contrary to the previous problem, the zone of preparation to burning is heterogeneous. We suppose that it consists of the periodically placed layers, or discs, of two different materials (fertile fuel and moderator) with the corresponding width a and b, diffusivities D_1 and D_2, and the absorption cross sections Σ_{a1} and Σ_{a2}.

Separating the variables for the neutron flux of the first group

$$\Phi^{\mathrm{I}}(r,z,t) = R(r) Z(z) T(t)$$

and searching for the solution in the first period of the assembly, one can find

$$Z(z) = \left(u_1(z) + \frac{e^{\pm i\varphi} - u_1(L)}{u_2(L)} u_2(z) \right) \qquad (18)$$

where $u_1(z)$ and $u_2(z)$ are the fundamental solutions and φ is a parameter which determines the evolution of the solution at the length of the period, $L = a + b$, i.e., for first layer

$$u_1(0) = 1, \quad D_1 u_1'(0) = 0, \qquad u_2(0) = 0, \quad D_1 u_2'(0) = 1 \qquad (19)$$

and

$$\cos\varphi = \frac{1}{2}\left[u_1(L) + D_1 u_2'(L)\right]. \qquad (20)$$

At the second period of the assembly the solution will differ from the solution of the first period by a factor $e^{\pm i\varphi}$, at the third period by a factor $e^{\pm 2i\varphi}$, and at $(n+1)$ -period by a factor $e^{\pm in\varphi}$. With the use of Eq. (18) it is possible to construct the solutions satisfying the boundary conditions (17).

Next, for simplicity, we shall consider a physically interesting case of closely spaced disks, when $p_1 a << 1$, $p_2 b << 1$, and, therefore, $\varphi = k_z L << 1$. After somewhat tedious calculations the problem can be reduced to that of the homogeneous medium but with somewhat different parameters,

$$\Phi^{\mathrm{I}}(r,z,t) = \psi_0^{\mathrm{I}} J_0(kr) e^{-vt\left[\bar{\Sigma}_a + D_\perp k^2\right]} \left\{ \frac{\pi v_z}{D_{\mathrm{II}} v} e^{t\frac{v_z^2}{D_{\parallel} v} - \frac{vlz}{D_{\parallel} v} z} + \right.$$

$$\left. + \int_0^\infty \frac{k_z^2 \, \mathrm{d}k_z}{k_z^2 + \left(\frac{v_z}{D_{\parallel} v}\right)^2} e^{-vtD_{\parallel} k_z^2} \left(\cos k_z z - \frac{v_z}{D_{\parallel} v k_z} \sin k_z z \right) \right\}. \qquad (21)$$

Namely, the mean macroscopic absorption cross section of the periodic assembly is equal to one averaged over the period.

$$\bar{\Sigma}_a = \frac{a\Sigma_{a1} + b\Sigma_{a1}}{a + b}, \qquad (22)$$

The diffusivity becomes anisotropic. The transversal diffusivity is averaged over the period, but as for the longitudinal diffusivity, averaged are the reciprocal quantities, or diffusional times,

$$D_\perp = \frac{aD_1 + bD_2}{a + b}, \qquad (23)$$

$$D_\lambda = \frac{(a+b) D_2 D_1}{aD_2 + bD_1}. \qquad (24)$$

The product $\Sigma_a \Phi(r,z,t)$ gives the thermal neutron source strength (in $cm^{-3}s^{-1}$) due to the slowing-down of fast neutrons in the volume with $z > 0$.

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