THE PECULIARITIES OF PARTICLES TRANSITION THROUGH SPLIT SEPARATRIX

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The dynamics of charged particle transition through a stochastic layer generated by a split separatrix is investigated on an example of mathematical pendulum motion with damping under acting of external periodic disturbance. It is shown that such layer has properties of an effective potential barrier, which can prevent to transition of charged particles through it. The dwelling time of particle in such layer can be anomalous large.

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1. INTRODUCTION

Under development of dynamic chaos in a Hamiltonian system all phase space is broken to regions with random and regular behavior. The investigation of boundaries of the regions, which separate random behavior from regular, demonstrates that these boundaries are fractal and have property of "sticky". The result of which one can be the apparent violation of second law of thermodynamics [1].

The interaction of a wave-particle type plays the main role for beam problems. Most important point of such interaction is the point of a qualitative change of particles motion nature. At this in the phase space the trajectory of particle passes through separatrix. In real situations the separatrix is split, more often, while as generating the stochastic layer. The nature of particle motion inside region of split separatrix and outside it differs essentially.

In the present paper the features of charged particle transition through a split separatrix are investigated analytically and numerically on the model of mathematical pendulum with damping and with external high frequency disturbance.

Besides it is considered the case of excitation by the beam of monochromatic wave at the case of Cherenkov's effect; and the features of beam particles' dynamics are analyzed at such excitation.

2. PROBLEM STATEMENT. THE BASIC EQUATIONS

The problem about interaction of charged particles with a field of electromagnetic waves is the primary goal in accelerator theory and in plasma theory. So, at charged particle capturing by the field of electromagnetic waves, and also at escaping from the capture the particle should pass separatrix region, which separates captured particles from transient-time. The separatrix is, practically always, split. For analysis of dynamics of particles motion, if there is a split separatrix, it is convenient to use the model of mathematical pendulum with damping ν , on which the external periodic force of amplitude h acts

$$\ddot{x}+\nu\ \dot{x}+\sin x=h\sin\Omega\ t\ ,\quad (\Omega=\omega/\omega_0)\ . \tag{1}$$

In absence of damping and disturbance the Hamiltonian of mathematical pendulum is:

$H_0(\dot{x},x)=1/2\ \dot{x}^2-\omega_0^2\cos x$, where ω_0 - frequency of small oscillations. On a phase plane of such pendulum the separatrix separates infinite invariant curves for transient-time particles from closed curves for captured particles.

The presence of external force ($h\neq0$) at ν =0 leads to splitting of separatrix and appearance in its neighborhood a stochastic layer. Width of this layer is proportional to disturbance [2]. In the case *h=0* at ν> 0 point $\dot{x}=0$, $x=0$ is stable focus. If $h=0$, ν < 0 the particle motion should become infinite. The damping ($\nu\neq0$) at $h\neq0$ can lead to disappearance of splitting. Using Melnikov's method (see for example, [3]) it is possible to show, that the splitting of separatrix is saved, if the value of damping fulfills to inequality

$$\nu<\pi\ h/4\ ch\left(\pi\ \Omega/2\right)\ . \tag{2}$$

From (2) follows, that the stochastic layer generated by an external disturbance, is collapsed much faster at disturbance with large frequency ($\pi\,\Omega$ >>1), than stochastic layer generated by low frequency disturbance. This result is in full compliance with results of papers on modulation diffusion [3-6], and also with results of paper [7]. We shall compare the passing time for separatrix region when there is an external disturbance and when it isn't. At external disturbance (h =0) the change of maximum velocity of particle on small times ($\nu\ t\ll1$) can be evaluated by formula

$$\Delta\ \dot{x}_{\max}=-\dot{y}_{\max}\cdot\left(\nu/2\right)\ t\ , \tag{3}$$

where $y(t)$ satisfies the equation of mathematical pendulum $\ddot{y}+\sin y=0$.

3. THE RESULTS OF NUMERICAL ANALYSIS

The transition of particles through separatrix was investigated by numerical solution of equation (1) for different values of parameters h , ν , Ω and various initial conditions. The initial conditions for $\nu>0$ were selected on invariant curves outside of separatrix, for $\nu<0$ s - inside separatrix (see Fig. 1).

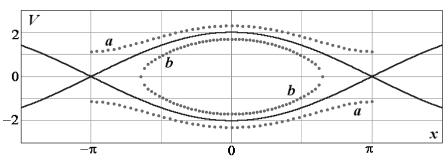

Fig.1. Initial conditions for particles ($V_{0\,\min}$ =–

$V_{0\max}$):

a) $\nu > 0$, $V_{0\max}$ =2.3; b) $\nu < 0$, $V_{0\max}$ =1.7

On the Figs. 2,3 the typical dependencies of particle motion velocity on time, and the time of passing through separatrix for different values of damping ν without disturbance $(h=0)$ are shown.

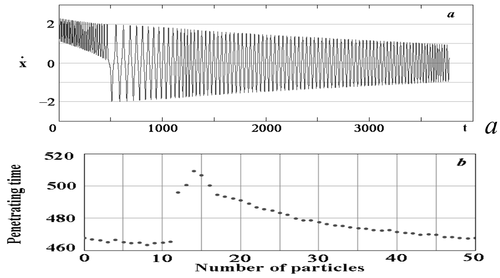

Fig. 2. a) dependence of particle velocity on time ;
b) distribution of trapping time for transient-time particles;
at values ν =0.0025, $V_{0\max}$ =2.3, h=0

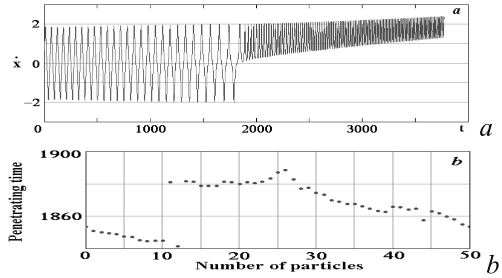

Fig. 3. a)dependence of particle velocity on time ;
b) distribution of time for particles exit from trapping;
at values ν = -0.0025, $V_{0\max}$ =1.7

The change of maximum particle velocity, as it is visible from these plots, is well described by the formula (3).

If there is an external periodic disturbance one can see the splitting of separatrix and dynamics of particle motion changes essentially. At first, the particle can move chaotically, becoming whether trapped or transient-time (see Fig. 4).

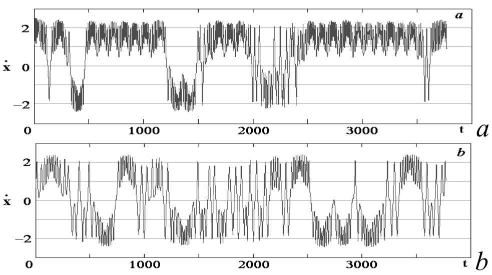

Fig. 4. Dependence of particle velocity on time:
a) ν =0.0025, $V_{0\max}$=2.3, h=0.23, Ω=1.5*;*
b) ν =-0.00075, $V_{0\max}$=1.7, h=0.23, Ω=1.5

The character of its motion, and also the values of velocities getting by a particle, do not depend on the dissipation sign; and for all run time (t ~ 4000) the maximum values of velocities, accessible by particle, practically did not vary. Spectrum of its motion is wide, and the correlation function droops rapidly (see Fig. 5).

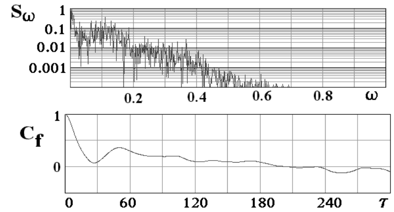

Fig.5. Spectrum and correlation function of particle velocity;
ν = -0.00075, $V_{0\max}$ =1.7, h =0.23, Ω =1.5

Secondly, except for such "long-lived" chaotic motion, one can see regimes, when the motion of particle is regular, though has multifrequency character (see Fig. 6).

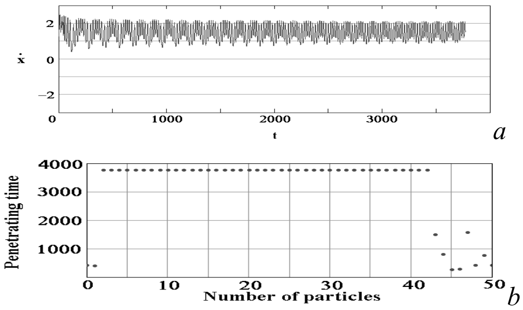

Fig. 6. a) dependence of particle velocity on time ;
b) distribution of trapping time for transient-time particles;
at values ν=0.0025, $V_{0\max}$=2.3,h=0.23, Ω=1.8

Most important peculiarity of such motion is the fact, that despite of the availability of damping particle for all run time remains transient-time and does not fall into stochastic layer, while without an external disturbance the particle becomes trapped during the time that is smaller almost on the order. The motion of such particles has line spectrum, and the correlation function oscillates with slowly drooping amplitude.

3. BEAM-PLASMA INTERACTION

The dynamics of motion of thin electron beam with radius *b* interacting in conditions of Cherenkov's resonance with the magnetized plasma waveguide with radius *a*, under acting of the external periodic electrical field with given amplitude $E_z=E_0\cos(\omega_0 t)$, is investigated. For thin beam it is possible to neglect the effect of beam stratification in the field of wave. The system is placed into a strong magnetic field, so that the motion of beam and plasma particles is one-dimensional. Besides, let's consider that plasma is linear.

The universal non-linear set of equations, describing the dynamics of excitation of plasma waves by electron beam, is well-known [8]. It is easily possible to take into account the presence of the external monochromatic field of given amplitude, by introducing additional addend into equation of motion of beam electrons [9].

The numerical solution of a non-linear set of equations was carried out at fixed density of beam and plasma. Linear increment was equal δ=0.05, initial velocity of beam $v0=V0/V_{ph}=1.0$. As far as δ<< ΔΩ (ΔΩ=1.0 – distance between harmonics), is possible to use only single-mode approximation for a field. The initial values of field amplitude are: Re *E* = Im *E* = 2 10^{-4}.

The results of numerical solution of the set of equations are shown on Figs. 7, 8 for various values of external field amplitude $\varepsilon_0 = eE_0 / m\omega V_{ph}$. In these figures one can see the dependencies of field amplitude on time, power spectrum $S\omega$, and correlation function C_f of oscillations at $\varepsilon_0=0$ - Fig. 7, at $\varepsilon_0=0.063$, $\Omega=0.0015$ - Fig. 8, accordingly.

How one can see from the plots (see Fig. 7) the exponential increasing of amplitude of the field with linear increment δ, change into oscillations conditioned by phase oscillations of bunch of beam particles, trapped by wave. The frequency of oscillations is about δ, modulation depth of amplitude is, approximately, half of maximal [9].

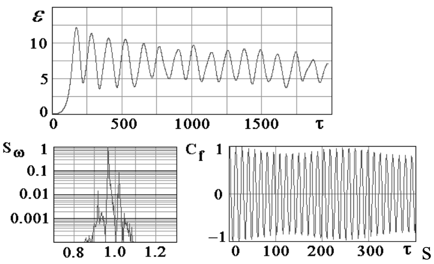

Fig.7. Amplitude of external field $\varepsilon_0=0$

Spectrum of oscillations has narrow peak on the base frequency and two satellites with shifting of frequency ~δ=0.05.

The correlation function, oscillating on the base frequency, droops slowly during the time. Physically such behavior of correlation function can be explained by such a way.

At the beam motion in the field of wave with oscillating amplitude, the stochastic instability of motion develops [10]. The stochastic instability acts first of all on particles located in a stochastic layer in vicinity of separatrix. While the basic group of particles which are generating a bunch, is far from separatrix in islands of stability. Therefore, the influence of neighboring non-linear resonances on motion of bunch is not enough, and also the smoothing of amplitude oscillations occurs with characteristic time, which is much more than the time of splitting of particles' motion correlations.

The presence of the external signal (Ω=0.0015, ε0=0.063) significantly changes the dynamics of instability. At first (see Fig. 8), as well as in the previous case, the exponential increasing of field amplitude, limits by trapping of beam particles into potential well of excited wave. The level of this field exceeds the level, necessary for overlapping non-linear resonances between field of wave and external field. Under the influence of the external field there is a misalignment of motion of beam bunch and of fundamental wave. It leads to more fast chaotization of beam particles motion and, in turn, to chaotic modulation of field amplitude (300<τ). Spectrum of the field, though has a maximum on a base frequency, is notably widened. The correlation function of this field droops during the time fast enough.

4. CONCLUSIONS

Thus, the stochastic layer, which is generated in vicinity of separatrix, can have the property of a potential barrier, which resists the passing of particles through it.

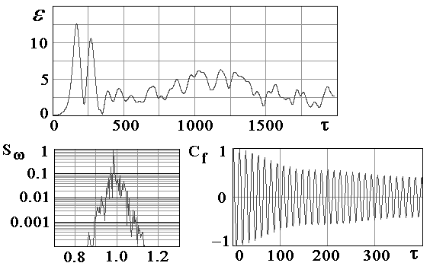

Fig.8. Amplitude of external field $\varepsilon_0=0.063$, $\Omega=0.0015$

The properties of this stochastic layer essentially depend on moving direction of particles (transient-time - trapped, entrapped - transient-time). Moreover, the stochastic layer is less transparent for the passing transient-time - trapped and is more transparent for the passing trapped - transient-time. The lifetime in the stochastic layer can be anomalously big. This result is in the good agreement with results of work [1].

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ОСОБЕННОСТИ ПРОХОЖДЕНИЯ ЧАСТИЦ ЧЕРЕЗ РАСЩЕПЛЕННУЮ СЕПАРАТРИСУ

В.А. Буц, А.П. Толстолужский

На примере движения математического маятника с затуханием под действием внешнего периодического возмущения исследована динамика прохождения заряженной частицы через стохастический слой, образованный расщепленной сепаратрисой. Показано, что такой слой обладает свойствами эффективного потенциального барьера, который может препятствовать прохождению заряженных частиц через него. Время нахождения частицы в таком слое может быть аномально большим.

ОСОБЛИВОСТІ ПРОХОДЖЕННЯ ЧАСТИНОК ЧЕРЕЗ РОЗЩЕПЛЕНУ СЕПАРАТРИСУ

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На прикладі руху математичного маятника із загасанням під дією зовнішнього періодичного збурення досліджена динаміка проходження зарядженої частинки через стохастичний шар, утворений розщепленою сепаратрисою. Показано, що такий шар має властивості ефективного потенційного бар'єру, який може перешкоджати проходженню заряджених частинок через нього. Час знаходження частинки в такому шарі може бути аномально великим.