THE CHARGED PARTICLES MOVEMENT ON THE CLOSED TRAJECTORIES IN THE CROSSED CONSTANT ELECTRICAL AND MAGNETIC FIELDS

***Yu.A. Kirochkin*[1], *A.Yu. Kirochkin*[2]**
[1]*V.N. Karazin Kharkov National University, Kharkov, Ukraine, kononenko@htuni.kharkov.ua;*
[2]*Civil Protection Academy of Ukraine, Kharkov, Ukraine, canc@apbu.edu.ua*

Charges motion trajectories peculiarity in the crossed electric and magnetic fields which strengths are inversely proportional to distance from axis of symmetry is turning points coordinates' exponential dependence on movement parameters. It leads to the isotopes ions trajectories remarkable distinction. The analytic forms for ions trajectories based on non-relativistic movement equations and trajectories closure conditions are obtained. As an example the trajectories of ^{6}Li (dotted closed line) and ^{7}Li (solid open line) ions are built. The separation possibility of these isotopes is discussed.
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In the vacuum chamber between two coaxial metal non-magnetic cylinders at presence of constant axisymmetric radial-electric and azimuthal-magnetic fields which strengths are inversely proportional to distance r from a cylinders longitudinal axis z, the charges movement trajectories in a plane rOz under certain conditions can be closed. These trajectories are described by simple integral which depends only on two parameters chosen beforehand — values of cross coordinates of the turning point nearest to an axis $r_\uparrow = a$ and a second turning point $r_\downarrow > r_\uparrow$. Conveniently the source of ions to have so that their starting point is coincided with a turning point $r_\uparrow = a$.

On preset values $r_\uparrow$ and $r_\downarrow$ the fields strengths and ions initial velocities are determined.

The electric field is created due to a constant potential difference of cylinders ($R_B > R_A$ — their radiuses), and magnetic — by the rectilinear direct current along an axis z at $r \le R_A$. Let's present fields strengths as

$$\vec{E}=\vec{e}_r E_r\ ,\ E_r=-\frac{\psi}{r}\ ,\ \psi=\frac{\psi_0}{\ln\left(R_B/R_A\right)}>0\ ,$$
$$\varphi\left(r\right)=-\psi_0\left(1-\frac{\ln\left(r/R_A\right)}{\ln\left(R_B/R_A\right)}\right)\ , \qquad (1)$$

$$\vec{H}=\vec{e}_\alpha H_\alpha\ ,\ H_\alpha=-\frac{2J}{cr}\ , \text{ current strength } J>0, \qquad (2)$$

where

r, α, z, $\vec{e}_r$, $\vec{e}_\alpha$ and $\vec{e}_z$ — coordinates and orthovectors of cylindrical system of axes,

φ — potential of an electrostatic field $\vec{E}$ at $R_A \le r \le R_B$.

Let the particle with a charge $q > 0$ and mass m has velocity $\vec{u}_0=\vec{e}_r\delta u_r+\vec{e}_\alpha\delta u_\alpha+\vec{e}_z\left(u_0+\delta u_z\right)$ in a point with coordinates ($r_\uparrow = a > R_A$, α_0, $z_\uparrow$) (values of fluctuations of velocity δu_r, δu_α and δu_z are small in comparison with $u_0 > 0$. From movement non-relativistic equations of a particle in fields (1, 2) at the set entry conditions by simple transformations we find the trajectory section equation in a plane rOz, describing charge movement from a turning point $\xi_\uparrow$ up to a turning point $\xi_\downarrow$ ($\xi = r/a$ — cross, $\zeta = z/a$ — longitudinal dimensionless coordinates of a charge)

$$\zeta^{(+)}\left(\xi\right)=\zeta^{(+)}\left(\xi_\uparrow\right)+$$
$$+ve^{v}\int\limits_{-\pi/2}^{\theta(\xi,v)}d\theta\left(\frac{\eta}{1-\eta}-\sin\theta\right)\exp\left(v\sin\theta\right)\ , \qquad (3)$$

then from $\xi_\downarrow$ up to $\xi_\uparrow$

$$\zeta^{(-)}\left(\xi\right)=\zeta^{(+)}\left(\xi_\downarrow\right)-$$
$$-ve^{v}\int\limits_{\theta\left(\xi_\downarrow,v\right)}^{\theta(\xi,v)}d\theta\left(\frac{\eta}{1-\eta}-\sin\theta\right)\exp\left(v\sin\theta\right)\ , \qquad (4)$$

where value of a charge longitudinal coordinate $\zeta^{(+)}\left(\xi_\uparrow\right)$ in a turning point $\xi_\uparrow = 1$ is determined by a choice of the coordinates beginning,

$1 = \xi_\uparrow \le \xi \le \xi_\downarrow = e^{2v}$,
$$\theta\left(\xi,v\right)=\arcsin\left(\frac{1}{v}\ln\xi-1\right)\ , \qquad (5)$$

$$\eta=\frac{c^2\psi}{2\ Ju_0}=\frac{cE_r}{H_\alpha}\frac{1}{u_0}=\frac{u_d}{u_0}>0\ , \qquad (6)$$

u_d — drift charge velocity,

$$v=\kappa\left(1-\eta\right)\ , \qquad (7)$$

$$\kappa=\frac{mc^2u_0}{2\ qJ}=\frac{u_0}{\frac{q}{mc}\left|H_\alpha\right|}\frac{1}{r}=\frac{r_L\left(r\right)}{r}=\frac{r_L\left(a\right)}{a}\ \text{—} \qquad (8)$$

constant relation of Larmor radius of a charge to its distance from an axis along a trajectory.

From calculations follows that the turning point at movement along an axis z has cross dimensionless coordinate

$$\xi_z = \exp\kappa \ . \tag{9}$$

At performance of an inequality

$$\xi_\uparrow < \xi_z < \xi_\downarrow \tag{10}$$

the curve (3) is convex. From (10) with the account (5), (7) and (9) we find

$$\delta << \eta < 0{,}5 \ , \tag{11}$$

where $\delta << 1$ — greatest of values $\delta_r = \frac{|\delta u_r|}{u_0}$, $\delta_\alpha = \frac{|\delta u_\alpha|}{u_0}$ и $\delta_z = \frac{|\delta u_z|}{u_0}$.
The left hand side of an inequality (11) enables to neglect charge velocity fluctuations as in (3) and (4).

Let's show that at performance of parity

$$\frac{\eta}{1-\eta} = \frac{I_1(\nu)}{I_0(\nu)} \ , \tag{12}$$

(I_0 and I_1 — modified Bessel functions) the charge trajectory in a plane rOz is closed. In this case (3) becomes

$$\zeta^{(+)}(\xi) = \zeta^{(+)}(\xi_\uparrow) + \\ + \nu e^\nu \int_{-\pi/2}^{\theta(\xi,\nu)} d\theta \left(\frac{I_1(\nu)}{I_0(\nu)} - \sin\theta \right) \exp(\nu \sin\theta) \ , \\ 1 \le \xi \le e^{2\nu}, \tag{13}$$

and the right hand side depends really only on one parameter ν, which value is set by the value chosen beforehand $\xi_\downarrow$ (see (5)). It is easy to see that $\zeta^{(+)}(\xi_\downarrow) = \zeta^{(+)}(\xi_\uparrow)$ (charge z-coordinates at performance (12) are equal at turning points). At the further charge movement from a turning point $\xi_\downarrow$ to $\xi_\uparrow$ (see (4)) we have

$$\zeta^{(-)}(\xi) = \zeta^{(+)}(\xi_\downarrow) + \\ + \nu e^\nu \int_{\theta(\xi,\nu)}^{\pi/2} d\theta \left(\frac{I_1(\nu)}{I_0(\nu)} - \sin\theta \right) \exp(\nu \sin\theta) \ , \\ 1 \le \xi \le e^{2\nu}, \tag{14}$$

whence

$$\zeta^{(-)}(\xi_\uparrow) = \zeta^{(+)}(\xi_\downarrow) = \zeta^{(+)}(\xi_\uparrow) \tag{15}$$

and a trajectory is really closed.

From (13)-(15) we find

$$\zeta^{(+)}(\xi) - \zeta^{(+)}(\xi_\uparrow) = -\left\{ \zeta^{(-)}(\xi) - \zeta^{(+)}(\xi_\uparrow) \right\} . \tag{16}$$

It specifies mirror symmetry of the closed charge trajectory in a plane rOz concerning a straight line $\zeta = \zeta^{(+)}(\xi_\uparrow)$.

On beforehand chosen value $\xi_\downarrow < R_B/a$ from (5) we find ν, η is determined from (12), then κ is determined from (7) and from (8) we find the relation u_0/J. It is necessary to set one of these values. For example let's set J (at the set sizes of a chamber current strength J should be about 10^4 A) then the magnetic field (2) and charge velocity u_0 will be determined. After that from (6) we find stressness of an electric field (1). Apparently it is possible to increase a quantity of the same charges on the closed trajectory right up to disintegration of this beam because of Coulomb force.

For the open-ended convex trajectory in a plane rOz when (12) is not carried out, charge z-coordinate in a turning point $\xi_\downarrow$ is determined by expression (see (3))

$$\zeta^{(+)}(\xi_\downarrow) = \zeta^{(+)}(\xi_\uparrow) + \pi\nu e^\nu I_0(\nu) \left(\frac{\eta}{1-\eta} - \frac{I_1(\nu)}{I_0(\nu)} \right) . \tag{17}$$

At the further charge movement from $\xi_\downarrow$ to $\xi_\uparrow$ its z-coordinate in a turning point $\xi_\uparrow$ will be equal (see (4), (17))

$$\zeta^{(-)}(\xi_\uparrow) = \zeta^{(+)}(\xi_\uparrow) + 2\pi\nu e^\nu I_0(\nu) \left(\frac{\eta}{1-\eta} - \frac{I_1(\nu)}{I_0(\nu)} \right) , \tag{18}$$

that differs from $\zeta^{(+)}(\xi_\uparrow)$, and the trajectory is open-ended.

Now in view of (12) not fulfilled it is necessary to set value η in an interval (11) and then to determine other values.

As an example let's consider movement trajectories of monocharged positive ions ^{6}Li and ^{7}Li in a plane rOz. Let $R_A = 3$ cm, $R_B = 20$ cm. Isotopes ions take off from a source in a point with coordinates $r_\uparrow = a = 4{,}6$ cm, $z_\uparrow = 0$ и $\alpha_\uparrow = 0$ ($\xi_\uparrow = 1$, $\zeta^{(+)}(\xi_\uparrow) = 0$). Let's construct the closed trajectory for an ion of easier isotope ^{6}Li, designating its parameters an index «0» (parameters of a trajectory of an ion ^{7}Li we designate an index «1»). Value of dimensionless cross coordinate $\xi_{0\downarrow}$ of more distant turning point it is natural to choose more but so that the inequality was carried out

$$r_{0\downarrow} < R_B \ . \tag{19}$$

(Note that for heavier ion ^{7}Li value $r_{1\downarrow}$ is more than $r_{0\downarrow}$ and this ion can collide with a wall of the chamber.)

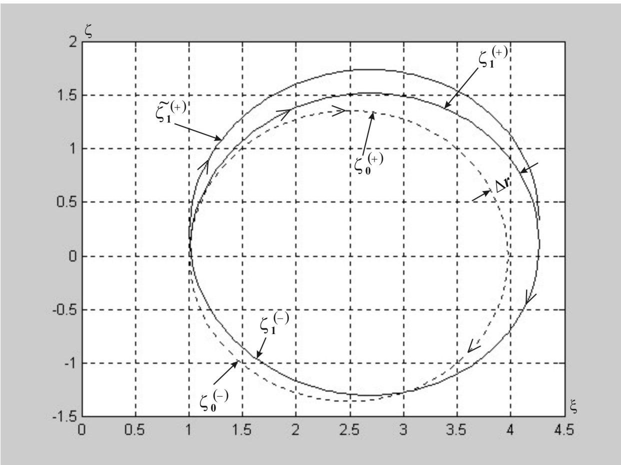
Let's choose $\xi_{0\downarrow} = 3{,}975$ (thus $r_{0\downarrow} \approx 18{,}3$ cm < $R_B = 20$ cm). Then mirror symmetric closed trajectory of an ion ^{6}Li in a plane *rOz* is determined from (13) at $\nu_0 = 0{,}5\cdot\ln\xi_{0\downarrow} = 0{,}69$, $\xi_{0\uparrow} = 1$, and $\zeta_0^{(+)}\left(\xi_\uparrow\right) = 0$ (sections of this trajectory $\zeta_0^{(+)}\left(\xi\right)$ and $\zeta_0^{(-)}\left(\xi\right)$ are shown in figure as a dotted curve). Using the procedure described above, we find $\eta_0 = 0{,}246$; $\kappa_0 = 0{,}915$; $u_{00} = 2{,}93\cdot10^2\,J$. At $J = 10^4$ A we receive $u_{00} = 2{,}93\cdot10^6$ cm/s and $\varphi_A \equiv -\psi_0 = -27{,}33$ V, $\varphi_B = 0$. The accelerating potential corresponding to velocity u_{00} of an ion ^{6}Li is equal 26,8 V.

We find parameters of a trajectory for ions ^{7}Li in a plane *rOz*: $\kappa_1 = \kappa_0\sqrt{\dfrac{m_1}{m_0}} \approx 0{,}988$;

$$u_{01} = u_{00}\sqrt{\frac{m_0}{m_1}} = 2{,}7\cdot10^6 \text{ cm/s};$$

$\eta_1 = \eta_0\sqrt{\dfrac{m_1}{m_0}} \approx 0{,}2655$; $\nu_1 = \kappa_1(1-\eta_1) \approx 0{,}726$ (see (7)); $\xi_{1\downarrow} = e^{2\nu_1} \approx 4{,}27$ and $r_{1\downarrow} \approx 19{,}64$ cm < $R_B = 20$ cm.

In figure the trajectory of an ion ^{7}Li in a plane *rOz* is shown with solid line consisting of sections (by way of sequence of motion) $\zeta_1^{(+)}\left(\xi\right)$ (at $\xi_{1\uparrow} = 1$, $\zeta_1^{(+)}\left(\xi_{1\uparrow}\right) = 0$), $\zeta_1^{(-)}\left(\xi\right)$ and $\tilde{\zeta}_1^{(+)}\left(\xi\right)$ where the first and third sections are calculated under the formula (3), and the second – under the formula (4). The maximal distance Δr between points of the trajectories specified by arrows is about 1,4 cm. This distance is enough to place the device increasing a distance between trajectories and to receive a good method of easy isotopes separation.



In the similar device the isotopes separation for other ions trajectories kinds was considered in [1, 2].

It is necessary to take into account that calculations are carried out in monoparticle approximation therefore beams density should be small enough. It is possible to compensate this lack in part using instead of one dot source the ring one of radius *a* (or several dot sources located on a ring). Simplicity of manufacturing and exploitation of device, its compactness seduces. From figure follows that ions move in volume which longitudinal size is about 16 cm, and cross radius is about 20 cm. Presence of the closed ions trajectory in such small space (length of the closed trajectory in figure is about 85 cm) is also of interest at presence of the crossed fields. At ion movement the longitudinal component of moment $M_z = rp_\alpha$ is constant. For a ring ions source these closed trajectories form tore. It would be interesting to investigate stability of this formation (smokers know about stability of the smoke rings similar on such tore).

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ДВИЖЕНИЕ ЗАРЯЖЕННЫХ ЧАСТИЦ ПО ЗАМКНУТЫМ ТРАЕКТОРИЯМ В СКРЕЩЕННЫХ ПОСТОЯННЫХ ЭЛЕКТРИЧЕСКОМ И МАГНИТНОМ ПОЛЯХ

Ю.А. Кирочкин, А.Ю. Кирочкин

Характерной особенностью траекторий движения зарядов в скрещенных электрическом и магнитном полях, напряженности которых обратно пропорциональны расстоянию от оси симметрии, является экспоненциальная зависимость координат точек поворота от параметров движения. Это приводит к заметному различию траекторий ионов изотопов. На основе нерелятивистских уравнений движения найдены аналитические выражения для траекторий ионов, условия их замкнутости. Для примера построены траектории ионов ^{6}Li (пунктирная замкнутая линия) и ^{7}Li (сплошная незамкнутая линия). Обсуждается возможность разделения этих изотопов.

РУХ ЗАРЯДЖЕНИХ ЧАСТИНОК ПО ЗАМКНУТИМ ТРАЄКТОРІЯМ У СХРЕЩЕНИХ ПОСТІЙНИХ ЕЛЕКТРИЧНОМУ ТА МАГНІТНОМУ ПОЛЯХ

Ю.О. Кірочкін, О.Ю. Кірочкін

Характерною особливістю траєкторій руху зарядів у схрещених електричному та магнітному полях, напруженості яких зворотно пропорційні відстані від осі симетрії, є експоненціальна залежність координат точок повороту від параметрів руху. Це приводить до помітної різниці між траєкторіями іонів ізотопів. На основі нерелятивістських рівнянь руху знайдені аналітичні вирази для траєкторій іонів, умови їх замкнутості. Для прикладу побудовані траєкторії іонів ^{6}Li (пунктирна замкнута лінія) та ^{7}Li (суцільна незамкнута лінія). Обговорюється можливість розділення цих ізотопів.