

# EXCITATION OF LF ION OSCILLATIONS IN THE SYSTEMS WITH RELATIVISTIC ELECTRON BEAM

V.A. Balakirev, N.I. Onishchenko

*NSC Kharkov Institute of Physics and Technology, Kharkov, Ukraine*

The instability of LF wave in ion flow during acceleration by space charge wave in collective ion accelerator based on high current REB, which is modulated in time and in space, is investigated.

PACS: 41.75.Lx, 41.85.Ja, 41.60.Bq

## 1. INTRODUCTION

At acceleration of an ion stream in the field of space charge wave of high-current REB, for example, in the model of collective ion accelerator [1], alongside with forward motion ions make low-frequency (LF) transverse oscillations with frequency  $\omega_i = (2\pi n_{e0} e^2 / M)^{1/2}$ , where  $n_{e0}$  - density of REB,  $e$  - electron charge,  $M$  - ion mass (ions are single-charge). It is obvious, that such double-beam system is unstable concerning excitation of LF oscillations. As a result of instability development LF potential wave of electric field will be excited, which, on the one hand, will destroy the dynamics of resonant acceleration of ions, and on the another, can reduce in ejection of ions on the walls of the drift chamber and, accordingly, in current losses of the accelerated ions. It is necessary to note, that the study of LF ion processes in high-current REB of microsecond duration goes out far off frames of collective ion accelerator physics. LF ion processes may play also the important role, for example, in powerful microwave plasma filled generators on the basis of high-current REB (passotrons [2], vircators [3], Cherenkov generators [4], etc.)

In the present article outcomes of the theory of LF instability of ion stream propagated along high-current REB are presented.

## 2. THE MAIN PART

The theory of LF transverse ion instability is developed within the frames of the following model. In the flat drift chamber, consisting of two parallel ideally conducting planes, high-current REB and ion stream are propagated. The REB is homogeneous and completely fills in the drift chamber. The thickness of an ion stream is arbitrary with respect to the distance between walls of the drift chamber (the size of the drift chamber). The ion stream is symmetric concerning the plane of symmetry of the drift chamber. The system is located in exterior magnetic field, directional along the propagation of particle flows. The effect of magnetic field on ion movement is neglected.

On the initial stage of interaction of electronic and ion beams instability is developed and wave amplitude grows in time to exponential law. At the nonlinear stage phase displacement of ions in the field of wave reduces in stabilization of instability. For waves with antisymmetric distribution of a potential in the transverse direction the

requirement of synchronism between ions and LF wave is possible only for odd harmonics of the frequency of transverse ion oscillations

$$\omega = kv_{i0} + (2s + 1)\omega_i \quad s = 0, 1, 2, \dots,$$

where  $\omega$  - LF wave frequency,  $k_n$  - its longitudinal wave number,  $v_{0i}$  - longitudinal ion velocity. At  $s=0$  synchronism with first harmonic occurs.

The initial set of equations contains the equation of excitation for amplitude of LF wave

$$\frac{dC_n}{d\tau} = \sigma \frac{1}{2\pi} \int_0^{2\pi} J_s(a) e^{-i\vartheta} d\vartheta_0, \quad (1)$$

$\sigma = 1$  for fast charge density waves and  $\sigma = -1$  for slow waves, and also equation of ion motion in Lagrangian variables

$$\frac{d\vartheta}{d\tau} = \mu u - \left[ \frac{i}{2} C_n \frac{s}{a} J'_s(a) e^{i\vartheta} + \kappa.c. \right], \quad (2)$$

$$a \frac{da}{d\tau} = -\frac{1}{2} C_n s J_s(a) e^{i\vartheta} + \kappa.c., \quad (3)$$

$$\frac{du}{d\tau} = -\frac{1}{2} C_n J_s(a) e^{i\vartheta} + \kappa.c., \quad (4)$$

$\vartheta$  - phase coordinate of ion,  $u$  - the dimensionless longitudinal velocity,  $a$  - the dimensionless amplitude of transverse ion oscillations,

$$\mu = \omega_i \frac{k_{xn}^2}{k_{xn}^2} \sqrt{\frac{\omega_i b}{2\omega_{pi}^2 x_i} \frac{1}{k_{xn}^2 \chi}},$$

where for cherenkov branches  $\chi = \frac{\omega_b}{\gamma_0^{3/2}} \frac{k_n}{k_{xn}^3}$  and for

$$\text{cyclotron ones } \chi = \frac{\omega_b^2}{\omega_{He} \gamma_0} \frac{k_{xn}^2}{(k_{xn}^2 + k_n^2)^2},$$

$\omega_b, \omega_{He}$  - Langmuir and cyclotron frequencies of REB,  $\gamma_0$  - the relativistic factor of REB.

In the set of equations (1) - (4) the dimensionless variables are used.

$$C_n = \varphi_n / \varphi^*, \quad \tau = t / t^*, \quad u = (v_{iz} - v_{i0}) / v^* \dot{i},$$

where

$$\varphi * i \frac{M}{e} \omega_{pi} \sqrt{2 \omega_i \frac{x_i}{b} \frac{\chi}{k_{xn}^2}},$$

$$t * i \frac{1}{\omega_{pi}} \sqrt{\frac{\omega_i}{2} \frac{b}{x_i} \frac{1}{k_{xn}^2} \chi},$$

$$v * i \frac{\omega_i k_n}{k_{xn}^2} \omega_{pi} = \sqrt{\frac{4 \pi e^2 \bar{n}_i}{M}}$$

$x_i$  – half-thickness of ion stratum,  $k_{xn}$  – transverse wave number.

The set of equations (1) - (4) has integrals

$$\frac{a^2}{2} - su = Const, \quad (5)$$

$$|C_n|^2 + \frac{\sigma}{s} \frac{1}{2\pi} \int_0^{2\pi} a^2 d\vartheta_0 = Const. \quad (6)$$

From the first integral it follows, that reduction of the energy of transverse motion is accompanied by deceleration of ions and vice-versa. The second integral reflects the law of energy conservation in the system. In the case of fast charge density waves (Cherenkov and cyclotron waves,  $\sigma=1$ ), which energy is positive, reduction of the energy of transverse motion reduces in increase of wave energy. For a slow charge density wave  $\sigma=-1$ , which energy is negative, the increase of wave energy is accompanied by increase of the energy of transverse motion of ion oscillators.

The right member of the equation for phase coordinate of ions (2) contains two items, responsible for two mechanisms of ion grouping in the field of LF wave. The item proportional to parameter  $\mu$  takes into account ion grouping, stipulated by longitudinal motion, and second item – phase grouping, connected with transverse motion of ions in the field of LF wave. The indicated set of nonlinear equations has been solved by numerical methods for the basic resonance  $s=1$  and the lowest transverse harmonic  $n=1$  with transverse wave number  $k_{x1}=\pi/b$ . Initial value of LF wave amplitude is chosen equal to  $|C_1|=10^{-2}$ , number of ions on a period of LF wave is 300.

Let's analyze outcomes of numerical accounts for the case  $\mu=0$ . Initial value of  $a(\tau=0)=a_i=\pi/2$  that corresponds to initial thickness of ion stratum  $x_i=b/2$ . For value  $a_i=\pi/2$  the slow charge density wave which energy is negative is unstable.

On fig.1 the dependence of the amplitude module of excited slow charge density wave on time is presented. Wave amplitude on initial stage of instability exponentially rises with time, attains maximum value, and then makes damped oscillations. As it was already noted, growth of wave amplitude with the negative energy is accompanied by increase of transverse energy of ion oscillators  $a^2$  or increase of amplitude of transverse oscillations of ions.

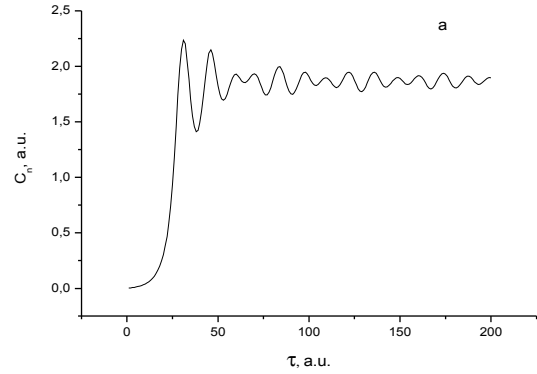


Fig. 1. The dependence of wave amplitude on time

On fig.2 phase portraits  $(a, \vartheta)$  in various moments of time are presented. On a phase portrait  $(a, \vartheta)$  the line  $a=\pi$  corresponds to the wall of the drift chamber. During numerical calculation ions which hit on walls of the drift chamber were output from the system and did not contribute to amplitude of LF wave. At the stage of exponential growth of amplitude on the indicated phase plane the figure such as an ellipse is formed.

In the moment of time  $\tau=31$  (fig. 2a) corresponding to maximum value of amplitude, all ions have increased the transverse energy in comparison with initial value. Thus, approximately, 30 % of ions will be rejected on the walls of the drift chamber. In the point of minimum of amplitude  $\tau=38$  transverse energy of ion oscillators according to integral (6) is minimal (fig. 2b). As a whole on a phase plane  $(a, \vartheta)$  two bunches are formed, which rotate in opposite directions. At major times (fig. 2c)  $\tau=114$ , bunches have complex multiflow structure. Partial phase mixing of ions reduces in reduction of phase oscillations of slow REB charge density wave amplitude.

Above the dynamics of LF slow charge density wave excitation with negative energy by ion oscillator stream was investigated. Let us consider the case  $a_i=3\pi/4$ . For such initial value of half-thickness of ion stratum the fast REB charge density wave is unstable, which energy is positive ( $\sigma>0$  in integral (5)).

At a nonlinear stage of instability the amplitude makes deep in high-scale regular phase oscillations. Maximum and especially average value of amplitude are much lower, than in the case of slow charge density wave. Though times  $\tau_m$ , during which amplitudes of fast and slow density charge waves attain the first maxima are close ( $\tau_m=31$  for slow and  $\tau_m=30$  - for fast charge density waves) the loss of ions on the walls of the drift chamber decreased up to 15 %.

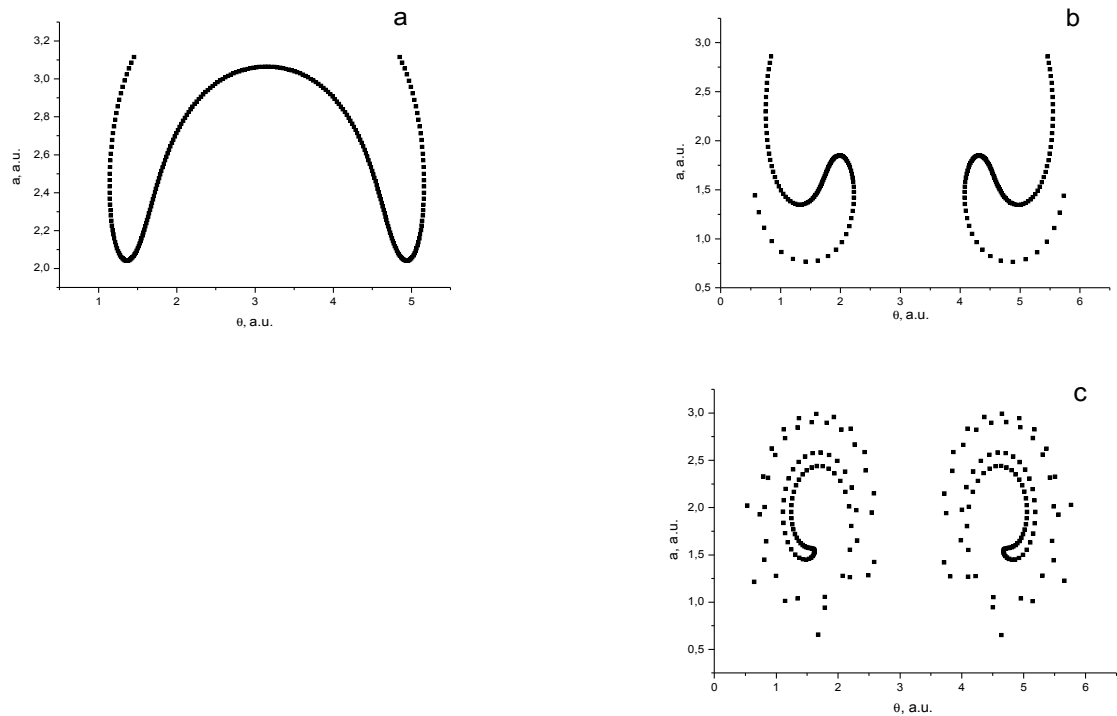


Fig. 2. Phase portraits of ions  $(a, \vartheta)$  at instants  $\tau = 31, 38, 114$ , correspondingly for  $a, b, c$

### 3. CONCLUSION

Thus, LF instability of ion oscillator stream concerning excitation of LF REB charge density waves is investigated. The self-consistent nonlinear set of equations is obtained, which describes nonlinear dynamics of ion oscillator instability, formed by ions that make transverse oscillations in a static electric field of REB. It is shown, that for slow and for fast charge density waves the patterns of instability development qualitatively differ. The slow charge density wave has negative energy. Due to this, growth of wave amplitude is accompanied by increase of transverse energy of ion oscillators and, accordingly, amplitudes of transverse LF oscillations of ions. The longitudinal velocity of ions also will increase. In the case of fast charge density wave instability develops under the traditional scenario. Increase of wave amplitude accompanies reduction of transverse energy of ion oscillators and their longitudinal velocity.

### REFERENCES

1. V.A.Balakirev, A.M.Gorban, I.I.Magda, V.E.Novikov, I.N.Onishchenko, S.S.Pushkarev, Collective acceleration of ions modulated by high-current REB // *Plasma Physics*. 1997, v23, N4, p.350-354.
2. Yu Bliokh, G. Nusinovich, J. Felsteiner, V. Granatstein // *Physical review E*. 2002, v.66, (056503).
3. A. Sabkin, A. Dubinov, V. Zhdanov, et al // *Plasma Phys. Reports*. v.23, №4, p.316-322.

4. V. Balakirev, N. Karbushev, A. Ostrovskij, Yu. Tkach  
*Theory of amplifiers and generators based on relativistic beams*. Kiev: "Naukova dumka", 1993.

**ВОЗБУЖДЕНИЕ НИЗКОЧАСТОТНЫХ ИОННЫХ КОЛЕБАНИЙ В СИСТЕМАХ  
С РЕЛЯТИВИСТСКИМ ЭЛЕКТРОННЫМ ПУЧКОМ**

*В.А. Балакирев, Н.И. Онищенко*

Исследована неустойчивость НЧ-волны в потоке ионов при ускорении их волнами плотности заряда в коллективном ионном ускорителе, базирующемся на сильноточном РЭП, промодулированном во времени и в пространстве.

**ЗБУДЖЕННЯ НИЗЬКОЧАСТОТНИХ ІОННИХ КОЛИВАНЬ У СИСТЕМАХ  
З РЕЛЯТИВИСТЬСКИМ ЕЛЕКТРОННИМ ПУЧКОМ**

*В.А. Балакірєв, М.І. Онищенко*

Досліджена нестійкість НЧ-хвилі у потоці іонів при прискоренні їх хвилями густини заряду у колективному прискорювачі, що базується на сильнострумовому РЕП, який промодульований у часі та просторі.