# MOTION OF THE EARTH'S CENTRE OF MASS. PHYSICAL PRINCIPLES

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It has been found that the residual (Chandler) motion of the Earth's rotation pole results from the forced translational motion of the Earth's rotation axis relative to the geographic axis. The Earth's rotation axis moves parallel to itself without changing the angle of inclination to the ecliptic plane. The translational motion of the Earth's axis of rotation is caused by the motion of the Earth's center of mass in the Earth's body in the range from 1 to 30 meters relative to the Earth's surface. The motion of the Earth's center of mass in space is due to the motion of the consistent inner core of the Earth in the liquid outer core under the action of the total (internal and external) gravitational field. Formulae for calculation of the trajectory of the Earth's centre of mass from astronomical observations are suggested. The comparison of our calculations and observational data on variations in the latitudes of places and acceleration of gravity has shown that they are in good agreement. Our model has been shown to adequately describe the physical process of motion of the Earth's centre of mass inside the Earth's body.

## INTRODUCTION

In the middle of the 18th century the Russian scientist M. Lomonosov made an attempt to reveal experimentally the motion of the Earth's centre of mass. In 1883, G. Schiaparelli put forward the hypothesis that there is a redistribution of masses inside the Earth that leads to noticeable movements of the axis of inertia and the Earth's axis of rotation. However, this idea was neither supported nor developed. The goal of our investigations of the motion of the Earth's centre of mass and axes of rotation was to reveal the physical laws governing the motion of the Earth's axes of rotation in space and to suggest the method for taking into account this motion in astronomical observations and solution of problems of gravimetry, satellite navigation, metrology, *etc.*

## EARTH'S CENTRE OF MASS AND CENTRE OF FIGURE

***What does the trajectory of the intermediate pole mean?*** The coordinates of the intermediate Earth's rotation pole published by IERS are calculated from observations of coordinates of known stars with corrections for nutation and precession. Therefore, primary observations of the motion of *the instantaneous Earth's rotation axis* (Fig. 1) in space are converted to numerical data on the motion of the *Earth's axis of rotation.* $O_1$ and $\vec{\omega}_1$ are the rotation axis and the proper Earth's angular velocity; $O_i$ and $\vec{\omega}_i$ are the instantaneous rotation axis and instantaneous angular velocity of the Earth's rotation; $O_{nut}$ and $\vec{\omega}_{nut}$ are the instantaneous position in space of the nutational axis and the angular velocity of the nutational rotation, $O_{pr}$ and $\vec{\omega}_{pr}$ are the instantaneous position in space of the precessional axis and the angular velocity of the precessional rotation of the Earth's axis; $O^*$ is the Earth's centre of mass; and $O$ is the centre of the figure of the Earth. Figure 2 shows, on a virtual plane parallel to the equator, the residual motion of the Earth's rotation pole (axis) corrected for the secular motion. Without introducing new notions, it can be concluded from the aforesaid that the residual motion of the Earth's rotation axis is translational, *i.e.*, the Earth's rotation axis moves in the space of objects parallel to itself. However, the rotation axis is an imaginary line passing through the Earth's centre of mass. Therefore, it is the Earth's centre of mass that moves in the space of objects.

***Variations in latitude.*** Without discussing the essence of the physical processes responsible for appearance and motion of the instantaneous Earth's centre of mass, we consider the latitude variations taken from [1] (Pulkovo, Johannesburg, Onkativo, Basewather). Schematically, the set of observed latitude variations is shown in Fig. 3 and Fig. 4. In constructing the schemes, the property of the plumb-line to indicate direction toward the Earth's centre of mass was used. The points in the schemes correspond to those given in Table 1. It is evident from Fig. 3 and Fig. 4 that the latitudes in the North and South change synchronously, and the nature of latitude changes is the same (motion of the centre of mass). In the consideration of the nature of motion of

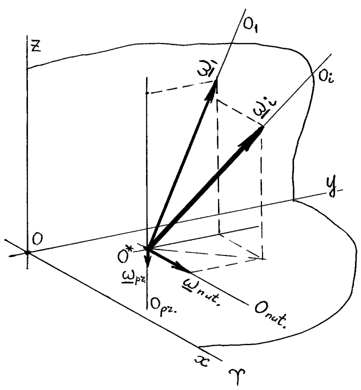


Figure 1. Axes of the Earth's rotation $\vec{\omega}_i = \vec{\omega}_1 + \vec{\omega}_{nut} + \vec{\omega}_{pr}$

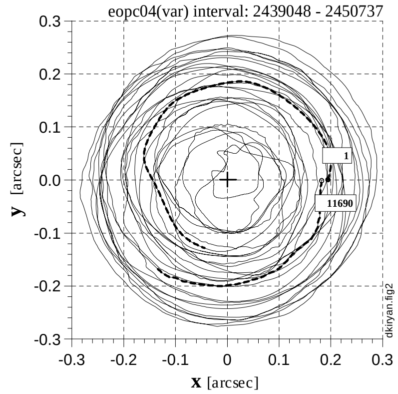


Figure 2. Trajectory of the Earth's pole of rotation

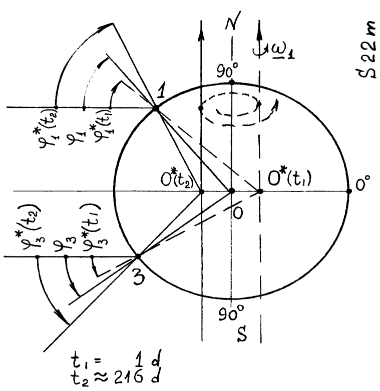


Figure 3. Scheme of variations in latitudes of the points at the same meridian in the N and S hemispheres

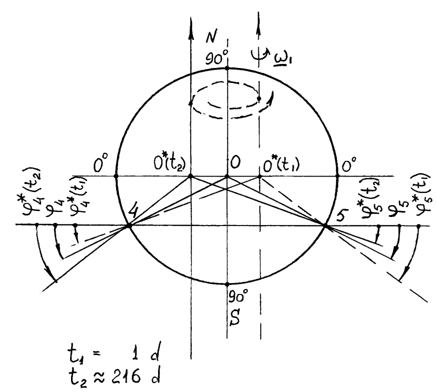


Figure 4. Scheme of latitude variations of the points with a difference in longitude of 180°

Table 1.

| | $\varphi_i$ | $\lambda_i$ |
|---|---|---|
| 1. Pulkovo | $+59^\circ\ 46'\ 18''$ | $30^\circ\ 19'\ 45''$ |
| 2. Johannesburg | $-26^\circ\ 10'\ 55''$ | $28^\circ\ 04'\ 30''$ |
| 3. Onkativo | $-31^\circ\ 55'\ 10''$ | $296^\circ\ 18'\ 00''$ |
| 4. Basewather | $-31^\circ\ 55'\ 13''$ | $115^\circ\ 55'\ 00''$ |

the Earth's pole of rotation (trajectory of the intermediate pole `eopc04`, given by `IERS`), *angular* movements of the rotation axis relative to the Earth's centre of mass are excluded. If we assume that the Earth's centre of mass moves along the axis of maximum moment of inertia (the $z$ axis of the geocentric equatorial coordinate system), the observed simultaneous changes in the latitudes of Pulkovo, Johannesburg, Onkativo, and Basewather cannot occur. If the centre of mass moves in the equatorial plane (or close to it), and the Earth's rotation axis executes the translational movement, the whole set (without any exception) of the observed simultaneous latitude variations receives a natural explanation. Taking into consideration the structure of variations in the latitudes of the observation points, physical laws, and following the Newton's rule

> *"We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances"*,

we can state that *latitude variation results from spatial variation in the orientation of the plumb-line*. Thus, variations in the latitudes and longitudes of the points of the Earth's surface observed by astronomers are caused by the natural phenomenon, *i.e.*, motion of the centre of mass. The astronomer determines the current latitude of the place $\varphi_i^*$ relative to the local normal in the astronomic (moving) coordinate system rather than in the geocentric (fixed) system. Every measurement of the angle occurs under the conditions of constantly changing spatial position of the reference system origin $O^*$ and position of the plane of the measured angle $\varphi_i^*$. The dependence of the latitude of a place on the position of the Earth's centre of mass in the Earth's body is illustrated by comparison of the model and observed corrections to the latitude of Pulkovo (Fig. 5). The observational data and our calculations of latitude variations for other observatories in the Southern and Northern hemispheres of the Earth also demonstrate a convincing agreement.

## MOTION OF THE CENTRE OF MASS

Thus, the residual (Chandler) motion of the Earth's rotation axis is governed by the motion of the Earth's centre of mass in the Earth's body. The physical and mechanical characteristics of the inner and outer structural shells of the Earth are such that the central Newton forces of interaction between the mass of the Earth's inner consistent core and gravitating masses in the surrounding space (mainly those of the Moon and the Sun) cause movement of the inner core in the liquid, low-viscosity medium of the outer core relative to the Earth's surface. The direction and velocity of motion of the Earth's centre of mass are determined by the continually changing summary vector of the impulse of interaction between the inner consistent core mass and gravitating masses in the Universe. The forced anharmonic motion of the centre of mass exhibits a regular pattern. The cyclicity of motion relative to the centre of the figure of the Earth in the geocentric equatorial system is determined by the half-difference between the frequency of variation in the distance from the ascending node to the perigee point of the Moon's orbit and the frequency of the Earth's revolution around the Sun. As analysis has shown, the duration of the cycle of the $2\pi$-rotation of the centre of mass (the axis of the maximum moment of inertia, rotation axis) with respect to the centre of the figure of the Earth ranges from 350 to 550 days (Fig. 6). The distance between the Earth's centre of mass and centre of the figure of the Earth continually changes from 1 to 20 meters. In more detail, the physical processes responsible for appearance of the moving Earth's centre of mass and kinematics and dynamics of its motion are described in [2].

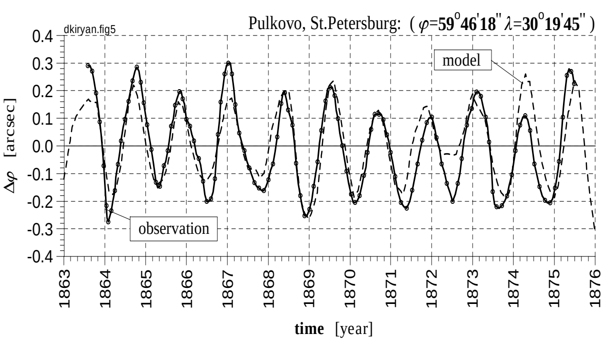


Figure 5. Variation in the latitude of Pulkovo

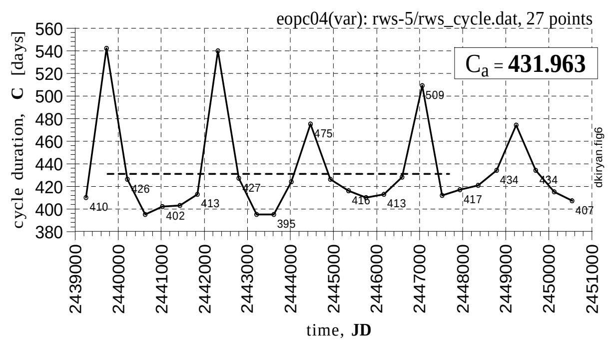


Figure 6. Cycles of the Earth's centre of mass

## MONITORING OF THE CENTRE OF MASS

The residual motion of the Earth's rotation axis $O_1$ (axis of maximum moment of inertia) is the consequence of motion of the Earth's centre of mass $O^*$. To have a clear idea of the motion of the Earth's centre of mass $O^*$ in the space of objects as a natural phenomenon, systematic observations of the motion of the Earth's centre of mass in the regime of monitoring and calculation of its current coordinates with a prescribed accuracy are necessary. Monitoring of the Earth's centre of mass will allow studies of the effect of external and internal forces on the gravitating mass of the consistent inner core of the Earth, of the spatial and temporal structure of the gravitational field on the Earth's surface, and also of the geodynamic parameters of the Earth. Coordinates of stars (with an accuracy to proper motion) and geocentric coordinates of the point of astronomical observations on the Earth's surface (with the accuracy to displacement of continental plates) do not vary in time, and therefore the observed changes in the orientation of the local normal relative to stars characterize the motion of the instantaneous Earth's centre of mass $O^*$ in the Earth's body. In observations of the changes in the orientation of the local normal relative to known stars, the observers detect angular values $\varphi_i^*$ and $\lambda_i^*$ and variations in these angular values $\Delta\varphi_i$ and $\Delta\lambda_i$, *i.e.*, they determine the current orientation of the local normal in space.

$$\varphi_i^*(t) - \varphi_i = \Delta\varphi_i(t)\,, \quad \lambda_i^*(t) - \lambda_i = \Delta\lambda_i(t)\,. \tag{1}$$

Then, by using the expression relating the coordinates of the moving astronomical system and the coordinates of the fixed geocentric system, it is possible to calculate the geocentric coordinates of the Earth's centre of mass $O^*$. The expressions for transformation of coordinates for the general case when $z(t) \neq 0$, *i.e.*, the Earth's centre of mass goes out of the equatorial plane, were given in [2]. In this case $z(t) = 0$, the geocentric coordinates of the centre of mass $x(t)$ and $y(t)$ are given by

$$x(t) = x_i - z_i \frac{1}{\tan \varphi_i^*(t) \sqrt{1 + \tan^2 \lambda_i^*(t)}}\,, \qquad y(t) = y_i - z_i \frac{\tan \lambda_i^*(t)}{\tan \varphi_i^*(t) \sqrt{1 + \tan^2 \lambda_i^*(t)}}\,, \tag{2}$$

where

$$x_i = R_i \cos \varphi_i \cos \lambda_i\,, \quad y_i = R_i \cos \varphi_i \sin \lambda_i\,, \quad z_i = R_i \sin \varphi_i\,. \tag{3}$$

Current coordinates of the Earth's centre of mass $O^*$ can be calculated from observations of known stars performed from one or several points of the Earth's surface. In the case observations are performed from several points of the Earth's surface with different latitudes and longitudes, it is possible to obtain data on the component $z(t)$ of the motion of the Earth's centre of mass.

## CENTRE OF MASS AND GRAVIMETRY

As the spatial position of the Earth's centre of mass changes, the distance between the observation point and instantaneous centre of mass varies. Acceleration of gravity **g** at the observation point for the current moment of time is calculated from the difference between radii-vectors in terms of the hypothesis on the linear variation in the acceleration of gravity in the direction of the Earth's centre. It is generally believed that the motion of the Earth's centre of mass does not exceed several centimeters and does not exert a considerable influence on variations in **g** on the Earth's surface. In reality, the Earth's centre of mass systematically moves from 1 to 30 meters relative to the points of the Earth's surface. Fig. 7 shows calculated variations in the acceleration of gravity at Potsdam. Our calculations were performed for a particular time interval, the actual trajectory of motion of the Earth's centre of mass was taken into account. For other points of the Earth's surface, the structure of variations in **g** is similar. The maximum amplitude of variation in **g** (here, **g** is a function of motion of the Earth's centre of mass) must be observed in the equatorial regions of the Earth's surface, and the minimum amplitude must be observed in polar regions. In 1981–1984, a Space Geodesy Research Group (FGS, Germany) detected simultaneously temporal variations in acceleration of gravity **g** and variations in the latitude by using a superconducting gravimeter and a zenith tube, respectively [3]. We calculated variations in the acceleration of gravity $\mathbf{g}(\mathbf{t})$ from the observational data on coordinates of the instantaneous Earth's centre of mass during the period 1981–1984. For the same time interval, variations in the latitude $\varphi(t)$ of Bad Homburg were calculated (Fig. 8). Comparison of the observational data obtained by the FGS group and our calculations leads to the conclusion that the suggested physical model of motion of the Earth's centre of mass and its mathematical description yield calculated $\Delta\varphi(t)$ and $\Delta\mathbf{g}(\mathbf{t})$ consistent with the observational data on the motion of the instantaneous Earth's centre of mass. Fig. 8 shows variations in $\mathbf{g}(\mathbf{t})$ and $\varphi(t)$ during the same time interval. It can be seen that the correlation is good. All the curves have common nodal points with a periodicity of $\approx 1.2$ years. A good agreement between experimental and calculated $\Delta\mathbf{g}(\mathbf{t})$ and $\Delta\varphi(t)$ indicates that *a superconducting gravimeter and zenith tube (tiltmeter) independently detect the same geodynamic natural phenomenon, i.e., motion of the Earth's centre of mass in its different manifestations.*

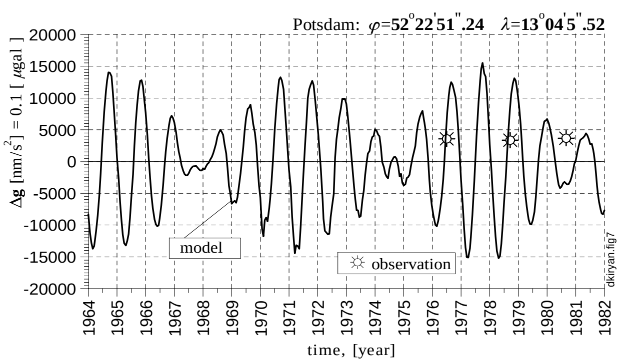


Figure 7. $\Delta\mathbf{g}(\mathbf{t})$ at Potsdam

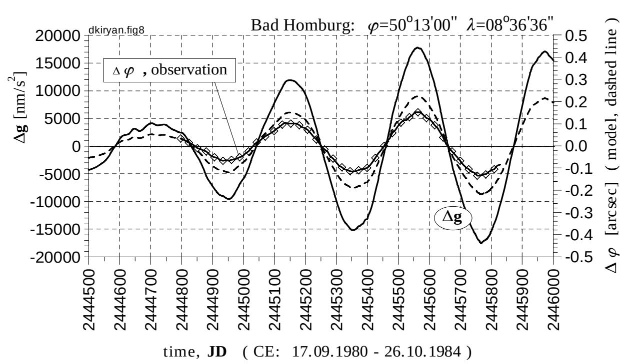


Figure 8. $\Delta\mathbf{g}(\mathbf{t})$ and $\Delta\varphi(t)$ at Bad Homburg

## CONCLUSIONS

Analysis of the IERS and observational data on variations in the latitudes of observatories has shown that

a) free nutation as an interpretation of the residual (Chandler) motion of the Earth's rotation axis has no physical sense and cause-effect relations with the surrounding material world;
b) calculations of variations in latitudes are performed without taking into account the principle of invariance with respect to the latitudes of the Northern and Southern hemispheres of the Earth.

Analysis of the suggested corrections to latitudes and longitudes taking into account the principle of invariance has shown that

1. The residual motion of the Earth's rotation pole is the translational motion of the Earth's rotation axis;
2. The translational motion of the Earth's rotation axis (parallel to itself) is caused by the motion of the Earth's centre of mass in the Earth's body;
3. The motion of the Earth's centre of mass in the space (in the Earth's body) results from the motion of the consistent inner core of the Earth in the liquid outer core under the action of the total (internal and external) gravitational field that varies in time;
4. The orientation of the plumb-line is determined by the motion of the Earth's centre of mass relative to the Earth's surface;
5. There exists the spatial-temporal structure of the gravitational field on the Earth's surface caused by the moving Earth's centre of mass;


[1] *Kulikov K. A.* Variability of Latitudes and Longitudes.–Moscow: Fizmatgiz, 1962.

[2] *Kiryan D. G., Kiryan G. V.* Motion of the Earth's Center of Mass. Physical Principles.–St.-Petersburg: St.-Petersburg State Polytechnical University, 2003. [http://www.wplus.net/pp/Diki/index.html]

[3] *Earth rotation*, Research group for space geodesy.–Frankfurt am Main: Druckerei Heinrich GmbH, 1998.