VARIATIONS OF THE ENERGY SPECTRA OF SOLAR X-RAY FLARES: INTERCONNECTION WITH THE PHOTOSPHERIC, CHROMOSPHERIC, AND MAGNETIC ACTIVITY OF THE SUN (1972–2001)

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The problem of flare energy distribution of the Sun is strongly related to the power-law energy spectra of flares on the UV Ceti-type stars. In a preliminary study (1972–1993), the X-ray flare integral energy spectrum (IES) was represented by a unique power law of $N \sim E^{-b}$ with index $b = 0.76$. Later it was shown that the exponent index b varies with the cycle phase, that is with the Wolf number. The purpose of this paper is to investigate the temporal variations in the power-law spectrum of soft X-ray flares over the three solar cycles (1972–2001). A more extensive statistic (56 000 flares) allows us to revise earlier results and to derive new ones. Not only b-index undergoes variations associated with a 11-year cycle but the limiting energy of flares ($\log E_m$) as well. The three-cycle-averaged b is 0.666, with variations in the range $0.50 < b < 0.80$. The results may be useful for study of flare activity on red dwarf stars.

INTRODUCTION

A statistical study of the active UV Ceti-type flare stars showed that time-integrated flare energy at the optical wavelength can be represented by a power function $\sim E^{-b}$ with a wide variety of b-indices $0.5 < b < 0.9$ [2]. The Sun behaves as an UV Ceti star whose exponent of the energy spectrum for flare optical radiation in all spectral lines of hydrogen series is $b = 0.80$ [5].

Based on X-ray (1–8 Å) flux data for 1972–1993, the flare integral energy spectrum was calculated with mean $< b >= 0.76 \pm 0.03$ (37 000 flares) [4]. The IES approximation gives the linear dependence of $\log E = 30.9 - 1.35 \log N$, where N is the accumulated number of flares with energy $E > E_m$. The study of IES of flares shows clear variations in the spectral index b with the phase of the 11-year cycle [4]. The correlation coefficient between the Wolf number W and the b-index is 0.6–0.8. These results need to be supported by a new data set related to the end of the 22nd cycle and to the beginning of the 23rd cycle (1994–2001).

DATA REDUCTION AND RESULTS

To obtain the flare energy E, the X-ray flux F_X (erg cm^{-2} s^{-1}) have to be integrated over the duration of the individual flare T and over the hemisphere from the flare site to the Earth by the formula

$$E = \simeq 2\pi R_{au}^2 \int_0^T F_X dt, \tag{1}$$

where R_{au} is the distance from the Sun to the Earth. The X-ray flux F_X data were taken from the site of National Oceanic and Atmospheric Administration – US Space Enviroment Center (1972–2001). In the earlier work the above integral was calculated by the triangle approximation of the flare profile, $1/2F_{max}T$, which was sufficient for the massive calculations for more than 1000 flares per year. This study somewhat takes into account the light curves of $F(t)$. To estimate the flare energy E, a new technique was adopted by dividing the phase of rise and decay of $F(t)$. There exists a wide scope of flare light curves, but their main feature is a sharp increase in $F(t)$ from the time of onset t_0 to the maximum of flare t_{max} and a farther exponential decay of F from t_{max} to t_e to the end of flare. Therefore, the flux $F(t)$ was time integrated by taking into account the beginning, the phase of maximum and the end of flare. Next step, the amount of flux was spatially integrated over the hemisphere $2\pi R_{au}^2$, where $R_{au} = 1$ AU. Assuming for the raise phase the "triangle" approximation of

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Eq. (1) and for decay part the exponential approximation that is t_e corresponds to the e times decay of $F(t)$ we have the empirical estimation formula for E:

$$E = 10^{28} F_{max}[\,4.22\,(t_{max} - t_0) + 2.53\,(t_e - t_{max})], \tag{2}$$

where F is in ergs per square centimetre per second, t is in seconds, and E is in ergs. The individual E values from Eq. (2) fits the range of energy $10^{24} < E < 10^{32}$ in accordance with the previous data.

Thereupon, the accumulated number of flares with energy $E > E_m$ was approximated by the power-like function:

$$N(E_m) = \int_{E_m} n(E)dE \sim CE^{-b}. \tag{3}$$

A dependence similar to Eq. (2) means that, if E is plotted versus N, one obtains on a "log–log" scale a linear dependence:

$$\log E = \log E_m - \frac{1}{b} \log N. \tag{4}$$

Table 1 presents the example of energy spectra parameters for 1978–2001. The data for 1972–1977 are approximate since the values of b-index were calculated using small statistics. The high limiting energy of one flare for 30 years is estimated as $\log E_m = 32.5$ (1989) while the mean energy for one flare is $<\log E_m>= 31.01$ in the "log" scale. It is smaller than the upper limit of the radiant energy of $5 \cdot 10^{32}$ erg, reported by some authors [5].

Table 1. The parameters of the power-law energy spectrum in X-ray solar flares

Year	$\log E_m$	b	N/year	W	$\log N_{25}$
1978	31.6	0.60 ± 0.03	1134	92	4.24
1979	31.6	0.64 ± 0.03	1469	155	4.28
1980	31.6	0.69 ± 0.01	2463	156	4.60
1981	31.9	0.684 ± 0.005	4005	140	4.84
1982	32.3	0.631 ± 0.005	3852	116	4.64
1983	31.8	0.71 ± 0.01	2583	67	3.95
1984	31.8	0.58 ± 0.01	2176	46	3.95
1985	30.1	0.68 ± 0.03	1065	18	3.45
1986	30.3	0.62 ± 0.04	916	13	3.39
1987	30.4	0.68 ± 0.03	1389	29	3.96
1988	31.6	0.64 ± 0.02	2367	100	4.24
1989	32.5	0.60 ± 0.01	2610	158	4.56
1990	31.5	0.72 ± 0.01	2630	142	4.68
1991	32.2	0.659 ± 0.005	3324	145	4.08
1992	31.6	0.67 ± 0.01	2816	90	4.40
1993	31.1	0.69 ± 0.01	2429	56	4.04
1994	30.3	0.69 ± 0.02	1612	22	3.63
1995	29.2	0.72 ± 0.03	1124	16	3.45
1996	29.2	0.63 ± 0.07	510	9	3.03
1997	30.4	0.66 ± 0.03	1138	22	3.57
1998	31.1	0.68 ± 0.01	2244	62	4.32
1999	30.9	0.76 ± 0.01	2421	95	4.52
2000	30.9	0.80 ± 0.01	2260	130	4.76
2001	31.4	0.72 ± 0.01	2730	134	4.60

As an example, Figure 1 shows the power spectrum in X-ray flares for the 1984 year, $N = 4005$ flares. As follows from Fig. 1, the flares of intermediate energies give a linear relation between $\log E$ and $\log N$. As regards the fainter flares (approximately 10^{26} erg), however, a sharp "break" appears in the spectrum. The energy E_{break} is near the flare detection threshold, and the break, therefore, is due to observational selection [2].

The IES parameters b and $\log E_m$ are mutually poorly correlated, $r \leq 0.37$. They can be taken, therefore, as two independent parameters of the power spectrum. Their physical meanings are different. While the former may be defined as the "slope of spectrum", the latter defines the "limit of energy" intersected by the line on the $\log E$ axis.

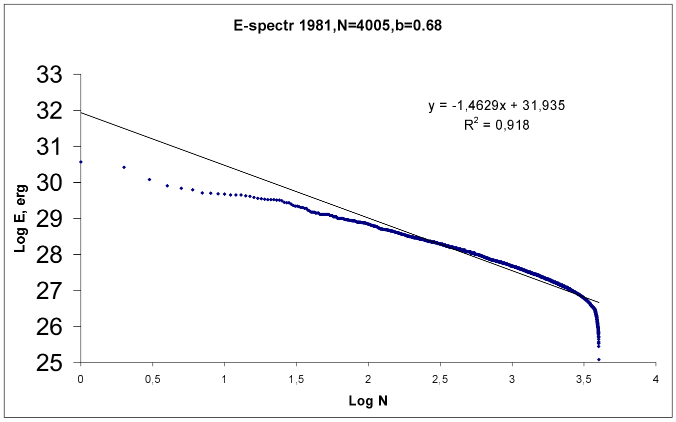

Figure 1. The power energy spectrum of flares in X-rays ($N = 4\,005$). The slope of the straight line gives b value of 0.684

As it is seen from Table 1, besides b and $\log E_m$ there is another important parameter, that is $\log N_{25}$. The latter defines the "number of flares" with energy level 10^{25} erg intersected by the line (4) on the $\log N$ axis. Energy level 10^{25} defines the number of microflares that could be observed every year if the power-law spectrum model (3) is correct. Microflares have excited considerable interest in different areas of solar physics. In particular, the existence of small, frequent energy releases suggests mechanism for coronal and chromospheric heating. Microscopic magnetic reconnection processes are suspected to exist virtually anywhere on solar surface, and this could reveal them as tiny X-ray flares [3]. As it can be seen from Table 1 (last column), the number of microflares correlated well with the Wolf number.

The importance of microflares parameter $\log N_{25}$ becomes more clear if we take it along with energy parameter $\log E_m$ as two independent parameters of power-law IES. Their product reflects the square of IES in the logarithmic scale:

$$I_E = \frac{1}{2}\,(\log E_m \cdot \log N_{25}). \tag{5}$$

The Eq. (7) defines the "integral" of IES that is the proliferation of energy and numbers of flares over the double logarithmic scale.

The summary cross-correlations for the five parameters of IES with the Wolf number (W) at the zero, one, and 11 years are given in Table 2. As it is expected, W and number of flares (N) are well correlated at zero and 11-year time-lag. More interesting is the fact that mutual $W{-}N$ correlation is higher at the lag +1 year. This lag of +1 year is also observed in the correlation maximum of index b with respect to Wolf number (Table 2, last column). An analogous phase shift between the spectral index b and an average flare energy release have been found at red dwarf star EV Lac by the authors [1]. This fact comprises the problem and needs further investigation.

Table 2. Cross-correlations of IES parameters with W in X-ray solar flares

Wolf	N, flares	$\log E \cdot \log N$	$\log N_{25}$	$\log E_m$	b
$W(0)$	0.70	0.88 ± 0.10	0.87	0.80	0.30 ± 0.20
$W(1)$	0.92	0.870± 0.10	0.83	0.63	0.51 ± 0.20
$W(11)$	0.79	0.75 ± 0.15	0.60	0.73	0.48 ± 0.24

Table 2 also shows that the spectral index b is the lowermost correlated parameter with the sunspot (W) and flare (N) activity. Therefore, it is reasonable to assume that the logarithmic integral of linear IES (5) is more important physical parameter than the index b. It is significant that all the parameters mentioned in

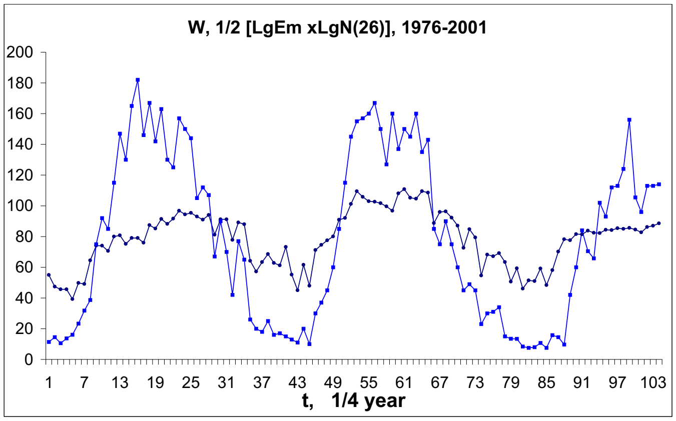

Figure 2. The modulation of the logarithmic integral of IES (circles) by the Wolf numbers (squares) for 1976–2001 with the 1/4 of a year resolution

Table 2 have shown rather high correlation with the Wolf number at the lag +11 years – $W(11)$. This fact reflects the 11-year modulation of all activity processes on the Sun.

A large amount of flares per year (≥ 1000) allows us to calculate the IES with more time resolution of 1/4 of a year. The comparison of $W(t)$ with logarithmic integral of IES are presented in Fig. 2 with time resolution of one quarter of a year. Both curves show a good correlation, revealing a significant 11-year modulation at a level of 0.87 (see Table 2).

RELATION OF IES PARAMETERS WITH OTHER SOLAR INDICES

So far we have considered the relation of IES parameters and Wolf number which may be considered as the main index of sunspots 11-year activity cycle. The other indices, in one way or another, related to the Wolf number (W). The other commonly used index strongly connected with W is the radio flux at $\nu = 2800$ MHz (wavelength $\lambda = 10.7$ cm) or F10.7. For completeness of approach from the Solar Geophysical Data (SGD) we have taken the yearly rows of F10.7 cm, number of active regions N_{AO}, and number of optical flares $N_{H\alpha}$. They may be considered as the coronal, photospheric, and chromospheric indices of solar activity. The correlation of these indices with power index b was done. The correlation coefficient maximum of $r \sim 0.65$–0.55 falls on +1 year lag related to the time of F10.7, N_{AO}, and $N_{H\alpha}$. A significant 11-year modulation of b by the indices is also revealed. It is remarkable that there is no observed time-lag difference between three indices and $b(t)$. They are all synchronized in time with respect to power index b. Therefore, F10.7, N_{AO}, and $N_{H\alpha}$ indices may be called “Wolf-similar” despite a physical difference between them. At the same time it underlines the fact that the time-lagging of 1 year of b relative to all three indices is as important as it is for EV Lac star [1].

In the recent time a new specific index – NOAA Mg II – has appeared. Index NOAA Mg II gives the ratio of intensity “center-to-limb” h and k lines of Mg II (280 nm). Therefore, the Mg II-index represents an essential measure of chromospheric activity. In Fig. 3 a smoothed correlation of Mg II-index and logarithmic integral of IES (5) are displayed. Data time resolution is three months. One can see that Mg II-index is positively correlated with the integral of IES. Particularly, near sinusoidal 11-year correlation with maximum $r = 0.84$ (point 42) takes place. The maximum correlation does not seem to be lagging with respect to Mg II-index in the beginning of the period.

The main conclusions are the following.

1. The integral energy spectrum of solar flares has a power-law form. The three-cycle-averaged index of power spectrum in soft X-rays is $b = 0.666 \pm 0.005$. The range of variations of b; $0.509 < b < 0.805$ is in reasonable agreement with the corresponding index of red dwarf stars [2].
2. The exponent b undergoes variations with the 11-year cycle phase, and correlates with the Wolf number W, number of flares N, chromospheric Mg II-index, radio flux index F10.7, and other indices. The exponent b increases from the epoch of minimum (0.637) to the epoch of maximum $b = 0.715 \pm 0.005$.

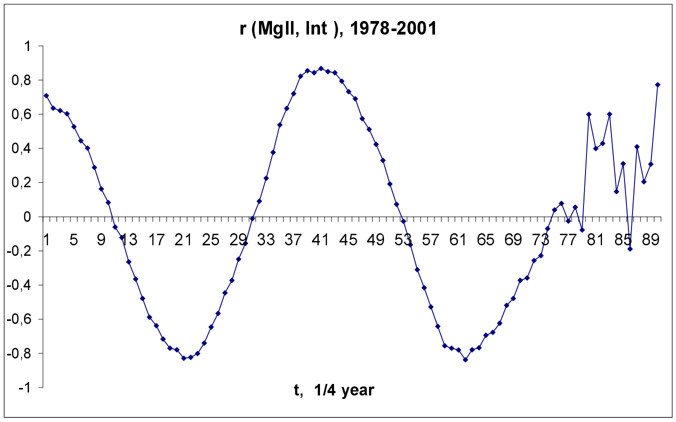

Figure 3. The correlation of chromospheric Mg II-index and logarithmic integral of IES with a time resolution of 1/4 of a year. A strong 11-year correlation (0.84) can be seen at point 42

3. The 11-year cyclic variation of IES parameters, the b-index, limiting energy and of flares $\log E_m$ and logarithmic integral of IES ($\log E_m \cdot \log N_{25}$) may serve as a fundamental dependence for revealing a similar activity on the red dwarf (UV Cet) stars.

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