RADIATION OF ELECTRIC, MAGNETIC AND TOROIDAL DIPOLES UNIFORMLY MOVING IN AN UNBOUNDED MEDIUM

G.N. Afanasiev [a], *Yu.P. Stepanovsky* [b]

[a] *Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research, Russia*
e-mail: afanasev@thsun1.jinr.ru
[b] *National Science Center "Kharkov Institute of Physics and Technology", Ukraine,*
e-mail: yustep@kipt.kharkov.ua

We considered the radiation of electric, magnetic and toroidal dipoles uniformly moving in unbounded medium. It turns out that the radiation intensity crucially depends on the mutual orientations of their symmetry axes and the velocity. The behavior of radiation intensities in the neighbourhood of the Cherenkov threshold $\beta = 1/n$ is investigated. The frequency and velocity regions are defined where radiation intensities are maximal. The comparison with previous attempts is given.

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1. INTRODUCTION

To our best knowledge, the electromagnetic field (EMF) arising from the motion of electric and magnetic dipoles in medium was first considered by Frank [1,2] who solved Maxwell's equations in the laboratory frame (LF) with electric and magnetic polarizations. Formulae describing the intensity radiation for a moving magnetic dipole did not satisfy Frank, as the intensity radiation did not disappear for the case when the dipole velocity coincided with the phase velocity in medium (the vanishing of the above radiation is intuitively expected and is satisfied, e.g., for a moving electric dipole). In 1952, another Frank's publication [3] on the same subject appeared. In it, he treated the magnetic dipole as consisting of two magnetic poles and obtained a correct expression (in the sense mentioned above) for the intensity radiation of a magnetic dipole moving in medium. To reconcile the results of [1,2] and [4], Frank suggested that transformation laws between the electric and magnetic moment moving in medium should differ from that in vacuum. This problem has been reconsidered by Ginzburg [4] who, starting with the Maxwell equations in a moving medium and writing the corresponding constitutive relations between the EMF strengths and inductions, obtained the correct vector of magnetic polarization. In 1984, two further publications by Frank [5] and Ginzburg [6] appeared. The difference between [1,2] and [3] was attributed to different definitions of magnetic dipoles used there: the electric current magnetic dipole and magnetic dipole composed of magnetic poles was used in [1,2] and [3], respectively. These two models of magnetic dipole possess different properties as to their interactions with magnetic medium and external EMF ([7]). At present, both experiment and theory definitely support that magnetic moments of elementary particles are of the electric-current type.

In Ref. [8], the radiation of toroidal moment (i.e., the elementary (infinitely small) toroidal solenoid (TS)) aligned along the velocity was considered. It was shown that the EMF of the TS moving in medium extends beyond its boundaries. This seemed to be surprising since the EMF of TS resting either in medium (or vacuum) or moving in vacuum is confined to its interior. In one of the latest lifetime publications [9], Frank returned to the initial idea ([1,2]) that the transformation laws between the dipole electric and magnetic moments moving in medium should be the same as in vacuum.

The goal of this consideration is to obtain EMF potentials and strengths for the point-like electric and magnetic dipoles and elementary TS moving in medium with an arbitrary velocity v, which may be greater or smaller than the light velocity in medium c_n. We postulate that in the reference frame attached to a moving source we have static distribution of charge and current densities. In the laboratory frame, relative to which the source moves with a constant velocity, the charge and current densities are obtained via the Lorentz transformations, the same as in vacuum. The further procedure is in a straightforward solution of the wave equations for the EMF potentials with the laboratory frame charge-current densities in their r.h.s. and in a subsequent evaluation of the EMF strengths. Formerly, in the time representation, this was done in [10]. The present consideration is just the translation of [10] into the frequency language, which is extensively used by experimentalists.

The question arises why not to use Frank's idea for the evaluation of EMF of the moving dipole. In our translation from Russian, it may be formulated as follows ([3], p. 190): It is suggested that a moving electric dipole p_1' is equivalent to some dipoles at rest, namely, to the electric p_1 and magnetic m_1 placed at the point coinciding with the instantaneous position of a moving dipole. The same is suggested for a magnetic dipole".

The reason for not using the transformation formulae for the electric and magnetic dipole moments moving in media is that there are different formulations of the moving media electrodynamics leading to different transformation laws for electric and magnetic polarizabilities and, therefore, for the electric and magnetic moments (which are the space integrals of polarizabilities). Even more confusing is the situation with toroidal moments for which the transformations formulae are not known.

Where the obtained exact expressions for EMF's can be applied to? First, any particle having either electric or magnetic dipole moments should radiate when its velocity exceeds the light velocity in medium. Then, exact results presented here give the frequency distributions of arising EMF's. Second, EMF's obtained in Sect. 2.2 can be observed in neutrino experiments. As far as we know, the neutrino possesses both dipole and toroidal magnetic moments. In the massless limit only the toroidal moment survives. This is valid, in particular, for Majorana neutrino.

2. UNBOUNDED MOTION OF MAGNETIC, TOROIDAL AND ELECTRIC DIPOLES IN MEDIUM

2.1 RADIATION OF MAGNETIC DIPOLE UNIFORMLY MOVING IN MEDIUM

2.1.1. Lorentz transformations of charge-current densities

In what follows, we need the Lorentz transformation formulae for the charge-current densities. They may be found in any textbook on electrodynamics. Let ρ'_{Ch} and j' be charge and current densities in the rest frame S', which moves with a constant velocity v relative to the laboratory frame (LF) S. Then,

$$\rho_{Ch} = \gamma(\rho'_{Ch} + \beta j'/c),$$
$$\vec{j} = \vec{j}' + \frac{\gamma}{\beta^2}\vec{\beta}(\vec{\beta}\vec{j}') + \gamma \vec{v}\rho'_{Ch} \tag{2.1}$$

Here $\gamma = (1-\beta^2)^{-1/2}$, $\beta = v/c$. If there is no charge density in S', then

$$\rho_{Ch} = \gamma\beta j'/c, \quad j_{\parallel} = \gamma j'_{\parallel}, \quad j_{\perp} = j'_{\perp}, \tag{2.2}$$

where $j_{\parallel}$ and $j_{\perp}$ are the components of j parallel and perpendicular to v. If there is no current density in S', then

$$\rho_{Ch} = \gamma\rho'_{Ch}, \quad j = \gamma v \rho'_{Ch} \tag{2.3}$$

2.1.2. The magnetic moment is parallel to the velocity

Consider a conducting loop $\mathcal{L}$ moving uniformly in a medium with the velocity v directed along the loop symmetry axis (coinciding with the z axis). Let in this loop a constant current I flows. In the reference frame attached to the moving loop, the current density is equal to

$$j = I\vec{n}_{\phi}\delta(\rho' - d)\delta(z'), \; \rho' = \sqrt{x'^2 + y'^2} \tag{2.4}$$

(x', y', z' are the coordinates in S'.) In accordance with (2.2), one gets in the LF

$$j = In_{\phi}\delta(\rho - d)\delta(\gamma(z - vt)) =$$
$$\frac{I}{\gamma}\vec{n}_{\phi}\delta(\rho - d)\delta(z - vt). \tag{2.5}$$

Here $\vec{n}_{\phi} = \vec{n}_y\cos\phi - \vec{n}_x\sin\phi$, $\gamma = 1/\sqrt{1-\beta^2}$. Since the current direction is perpendicular to the velocity, no charge density arises in the LF.

The current density j may be expressed through the magnetization

$j = curl\, M$.

The magnetization M is perpendicular to the plane of a current loop:

$$M_z = \frac{I_0}{\gamma}\Theta(d - \rho)\delta(z - vt).$$

Now, let the loop radius d tends to zero. Then,

$$\Theta(d - \rho) \to \pi d^2\delta(x)\delta(y),$$
$$M_z \to \frac{I\pi d^2}{\gamma}\delta(x)\delta(y)\delta(z - vt),$$
$$j_x = \partial M_z/\partial y, j_y = -\partial M_z/\partial x, j_z = 0.$$

The Fourier components of the current density are

$$j_x(\omega) = \partial M_z(\omega)/\partial y,$$
$$j_y(\omega) = -\partial M_z(\omega)/\partial x, \quad j_z(\omega) = 0. \tag{2.6}$$

where $M_z(\omega) = \frac{Id^2}{2\gamma v}\delta(x)\delta(y)\exp(i\psi)$ and $\psi = kz/\beta$.

The energy emitted in the radial direction per unit length per unit frequency equals zero for $v < c_n$ and

$$\sigma_{\rho} = \frac{\omega^3 m_d^2\mu}{v^2c^2\gamma^2\gamma_n^2}, \quad v > c_n. \tag{2.7}$$

Formerly, this equation was obtained by Frank [3], but without the factor γ^2 in the denominator. It is due to the factor γ in the denominator of (2.5). We suggested that the current density is equal to (2.4) in the reference frame attached to a moving current loop. The current density in the LF is obtained from (2.4) by the Lorentz transformation. On the other hand, Frank suggested that, in the LF, the charge density coincides with (2.5), but without the above γ factor. It follows from (2.7) that the intensity of radiation produced by the magnetic dipole parallel to the velocity differs from zero in the velocity window $c_n < v < c$. Therefore, v should not be too close either to c_n or c. For this, n should appreciably differ from unity. Probably, the best candidate to observe this radiation is a neutron moving in medium with large n. By comparing (2.7) with the radiation intensity of a moving charge $\left(\sigma_e = e^2\omega\mu/c^2\gamma_n^2\right)$, we see that there is a chance to observe neutron radiation only for very high frequencies.

2.1.3. The magnetic moment is perpendicular to the velocity

Let the current loop lies in the z=0 plane with its velocity along the *x-axis*. Then, in the rest frame S',

$$j'_x = -\frac{I_0}{d}\frac{y'}{\rho'}\delta(z')\delta(\rho' - d),$$

$$j'_y = \frac{I_0}{d}\frac{x'}{\rho'}\delta(z')\delta(\rho' - d),\quad j'_z = 0,\quad \rho'_{Ch} = 0.$$

Here $\rho' = (x'^2 + y'^2)^{1/2}$. According to (2.2), in the laboratory frame

$$j_x = -\frac{I_0}{d} y\gamma\delta(z)\delta(\rho - d),$$

$$j_y = \frac{I_0}{d}(x - vt)\gamma\delta(z)\delta(\rho - d),$$

$$\rho_{Ch} = -\frac{I_0}{c^2 d} yv\gamma\delta(z)\delta(\rho - d).$$

Here $\rho = \left[(x - vt)^2\gamma^2 + y^2\right]^{1/2}$. The charge density arises because on a part of the loop, the current has a non-zero projection on the direction of motion. It is easy to check that

$$j_x = I_0\gamma\delta(z)\frac{\partial}{\partial y}M_z,$$

$$j_y = -I_0\frac{1}{\gamma}\delta(z)\frac{\partial}{\partial x}M_z,$$

$$j_x = I_0\frac{v\gamma}{c^2}\delta(z)\frac{\partial}{\partial y}M_z,$$

where $M_z = \Theta(d - \rho)$. In the limit of an infinitesimal loop $(d \to 0)$,

$$M_z = \Theta(d - \rho) \to \delta(x - vt)\delta(y)\pi d^2/\gamma$$

and

$$j_x = I_0\pi d^2\delta(z)\delta(x - vt)\frac{\partial}{\partial y}\delta(y),$$

$$j_y = -\frac{1}{\gamma^2}I_0\pi d^2\delta(z)\delta(y)\frac{\partial}{\partial x}\delta(x - vt),$$

$$\rho_{Ch} = I_0\pi d^2\frac{v}{c^2}\delta(z)\delta(x - vt)\frac{\partial}{\partial y}\delta(y).$$

The Fourier components of these densities are

$$j_x(\omega) = \frac{I_0 d^2}{2v}\exp(i\psi_1)\delta(z)\frac{\partial}{\partial y}\delta(y),$$

$$j_y(\omega) = -\frac{iI_0 d^2\omega}{2v^2\gamma^2}\delta(z)\delta(y)\exp(i\psi_1), \tag{2.8}$$

$$\rho_{Ch}(\omega) = \frac{I_0 d^2}{2c^2}\delta(z)\exp(i\psi_1)\frac{\partial}{\partial y}\delta(y).$$

Here $\psi_1 = kx/\beta$. The energy flux through the cylindrical surface of the radius ρ_1 (co-axial with the motion axis), per unit length per unit frequency is equal to zero for $v < c_n$ and

$$\sigma(\omega,\phi) = \frac{m_d^2 k^3}{2\pi\varepsilon\beta v}\left[\frac{n^2}{\gamma^4\beta^2}\sin^2\phi + (n^2 - 1)^2\cos^2\phi\right] \tag{2.9}$$

for $v > c_n$. The integration over ϕ gives

$$\sigma(\omega,\phi) = \frac{m_d^2 k^3}{2\varepsilon\beta v}\left[\frac{n^2}{\gamma^4\beta^2} + (n^2 - 1)^2\right]. \tag{2.10}$$

Equations (2.9) and (2.10) coincide with ones obtained by Frank [3] who noted that in the limit $\beta \to 1/n$, these intensities do no vanish as it is intuitively expected. On these grounds, Frank declared them as to be incorrect. 30 years later, Frank returned to the same problem [5]. He attributed the non-vanishing of intensities (2.9) and (2.10) to the specific polarization of medium.

We analyze this question in some detail. Intensities (2.9) and (2.10) are non-zero for $\beta = 1/n + \varepsilon$ and zero for $\beta = 1/n - \varepsilon$, where $\varepsilon << 1$. Further examination shows that for $\beta_n = 1$, the radiation intensities are one half of (2.9) and (2.10). Again, neutron moving in dielectric medium with n appreciably different from unity, is the best candidate to observe this radiation. The absence of the overall $1/\gamma$ factor in (2.10) makes easier to observe radiation from the neutron with the spin perpendicular to the velocity than from the neutron with the spin directed along it.

2.2. ELECTROMAGNETIC FIELD OF THE POINT-LIKE TOROIDAL SOLENOID UNIFORMLY MOVING IN MEDIUM

Consider the poloidal current flowing on the surface of a torus equation of which in the rest frame is

$$(\rho' - d)^2 + z'^2 = R_0^2$$

(R_0 and d are the minor and large radii of torus).

It is convenient to introduce the coordinates $\rho' = d + R'\cos\psi$, $z' = R'\sin\psi$. In these coordinates, the poloidal current flowing on the torus surface is given by

$$\vec{j}' = j_0\frac{\delta(R_0 - R')}{d + R_0\cos\psi}\vec{n}_\psi .$$

Here is the vector $n_\psi = n_z\cos\psi - n_\rho\sin\psi$ lying on the torus surface in a particular $\phi = const.$ plane and defining the current direction, $R' = \sqrt{(\rho' - d)^2 + z'^2}$. The cylindrical components of j are

$$j_z = j_0\frac{\delta(R_0 - R')}{d + R_0\cos\psi}\cos\psi = j_0\delta(R_0 - R')\frac{\rho' - d}{R_0\rho'},$$

$$j_\rho = -j_0\frac{\delta(R_0 - R')}{d + R_0\cos\psi}\sin\psi = -j_0\delta(R_0 - R')\frac{z'}{R_0\rho'}.$$

2.2.1. The velocity is along the torus symmetry axis

Let this current distribution move uniformly along the z-axis (directed along the torus symmetry axis) with the velocity v. According to (2.3), in the laboratory frame, the no vanishing charge and current components are

$$\rho_{Ch} = j_0\gamma\beta\frac{\rho - d}{c\rho R_0}\delta(R_0 - R),$$

$$j_\rho = -j_0\gamma\frac{z - vt}{\rho R_0}\delta(R_0 - R),$$

$$j_z = j_0\gamma\frac{\rho - d}{\rho R_0}\delta(R_0 - R).$$

Here $R = \sqrt{(\rho - d)^2 + (z - vt)^2\gamma^2}$. These components may be represented in the form

$$j_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho M_\phi),\ j_\rho = -\frac{1}{\gamma^2}\frac{\partial M_\phi}{\partial z},\ \rho_{Ch} = \frac{\beta}{c\rho}\frac{\partial}{\partial\rho}(\rho M_\phi),$$

where

$$M_\phi = -j_0\gamma\frac{1}{\rho}\Theta(R_0 - R).$$

The Cartesian components of M are

$$M_x = j_0\gamma\frac{y}{\rho^2}\Theta(R_0 - R),\quad M_y = -j_0\gamma\frac{x}{\rho^2}\Theta(R_0 - R).$$

Then,

$$j_x = -\frac{1}{\gamma^2}\frac{\partial M_y}{\partial z},\quad j_y = \frac{\partial M_z}{\partial z}.$$

Let the minor torus radius R_0 tend to zero. Then,

$$\Theta(R_0 - R) \to \frac{\pi R_0^2}{\gamma}\delta(\rho - d)\delta(z - vt)$$

and

$$M_z = -\frac{j_0}{d}\pi R_0^2\frac{\partial}{\partial y}\Theta(\rho - d)\delta(z - vt),$$

$$M_y = \frac{j_0}{d}\pi R_0^2\frac{\partial}{\partial x}\Theta(\rho - d)\delta(z - vt).$$

Therefore,

$$j_x = -\frac{1}{\gamma^2}\frac{\partial M_y}{\partial z} = -\frac{j_0}{\gamma^2 d}\pi R_0^2\frac{\partial^2}{\partial z\partial x}\Theta(\rho - d)\delta(z - vt),$$

$$j_y = \frac{1}{\gamma^2}\frac{\partial M_x}{\partial z} = -\frac{j_0}{\gamma^2 d}\pi R_0^2\frac{\partial^2}{\partial z\partial y}\Theta(\rho - d)\delta(z - vt),$$

$$j_z = \frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y} = \frac{j_0}{d}\pi R_0^2\times$$

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2})\Theta(\rho - d)\delta(z - vt),$$

$$\rho_{Ch} = \frac{\beta}{c}(\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y}) = \frac{\beta j_0}{cd}\pi R_0^2\times$$

$$(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial x^2})\Theta(\rho - d)\delta(z - vt).$$

Let the major torus radius also tend to zero. Then,

$\Theta(d - \rho) = \pi d^2\delta(x)\delta(y)$ and

$$j_x = -\frac{j_0 d}{\gamma^2}\pi^2 R_0^2\frac{\partial^2}{\partial z\partial x}\delta(x)\delta(y)\delta(z - vt),$$

$$j_y = -\frac{j_0 d}{\gamma^2}\pi^2 R_0^2\frac{\partial^2}{\partial z\partial y}\delta(x)\delta(y)\delta(z - vt),$$

$$j_z = j_0\pi^2 R_0^2 d(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\delta(x)\delta(y)\delta(z - vt),$$

$$\rho_{Ch} = \frac{\beta j_0}{c}\pi^2 R_0^2 d(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\delta(x)\delta(y)\delta(z - vt).$$

Fourier transforms of these densities are

$$\rho_{Ch}(\omega) = \frac{m_t}{2c}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})D,\ j_z(\omega) = \frac{m_t}{2\beta}(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})D,$$

$$j_x = -\frac{ikm_t}{2\beta^2\gamma^2}\frac{\partial}{\partial x}D,\quad j_y = -\frac{ikm_t}{2\beta^2\gamma^2}\frac{\partial}{\partial y}D,$$

where

$$m_t = \frac{\pi^2 j_0 d R_0^2}{c},\quad D = \delta(x)\delta(y)\exp(i\psi),\quad \psi = \frac{kz}{\beta}.$$

The energy loss through the cylinder surface of the radius ρ coaxial with the motion axis per unit frequency, per unit length is zero for $v < c_n$ and

$$\sigma_\rho(\omega) = \frac{k^5 m_t^2}{\varepsilon v\beta^3}(\beta_n^2 - 1)(n^2 - 1) \tag{2.11}$$

for $v > c_n$. Formerly, this equation has been obtained in [8]. The absence of overall $1/\gamma$ factor in (2.11) and its proportionality to ω^5 show that the radiation intensity for the toroidal dipole directed along the velocity is maximal for large frequencies and $v \approx c$.

2.2.2. The velocity is normal to the torus axis

Let a toroidal solenoid move in medium with the velocity perpendicular to the torus symmetry axis. For definiteness, let the TS move along the *x-axis*. Then, in the LF

$$\rho_{Ch} = -\frac{j_0 v\gamma^2}{c^2 R_0}\frac{z(x - vt)}{\rho_1^2}\delta(R_1 - R_0),$$

$$j_x = -j_0\frac{\gamma^2}{R_0}\frac{z(x - vt)}{\rho_1^2}\delta(R_1 - R_0),$$

$$j_y = -j_0\frac{zy}{\rho_1^2}\frac{\delta(R_1 - R_0)}{R_0},\ j_z = j_0\frac{\rho_1 - d}{\rho_1}\frac{\delta(R_1 - R_0)}{R_0}.$$

Here $\rho_1 = \sqrt{(x - vt)^2\gamma^2 + y^2}$, $R_1 = \sqrt{(\rho_1 - d)^2 + z^2}$.

It is easy to check that

$$j_x = -\frac{\partial M_y}{\partial z},\ j_y = \frac{\partial M_x}{\partial z},$$

$$j_z = \frac{1}{\gamma^2}\frac{\partial M_y}{\partial x} - \frac{\partial M_x}{\partial y},\ \rho_{Ch} = -\frac{\beta}{c}\frac{\partial M_y}{\partial z},$$

where

$$M_y = -j_0\gamma^2 \frac{x - vt}{\rho_1^2}\Theta\left(R_0 - R_1\right),$$

$$M_x = j_0 \frac{y}{\rho_1^2}\Theta\left(R_0 - R_1\right), \quad M_z = 0.$$

Let the minor radius R_0 of a torus tend to zero. Then,

$$\Theta\left(R_0 - R_1\right) = \pi R_0^2 \delta\left(\rho_1 - d\right)\delta(z)$$

and

$$M_x = -j_0 \frac{\pi R_0^2}{d}\frac{\partial}{\partial y}\Theta\left(d - \rho_1\right)\delta(z),$$

$$M_y = j_0 \frac{\pi R_0^2}{d}\frac{\partial}{\partial x}\Theta\left(d - \rho_1\right)\delta(z).$$

Therefore,

$$\rho_{Ch} = -\frac{\beta j_0 \pi R_0^2}{cd}\frac{\partial^2}{\partial x \partial z}\Theta\left(d - \rho_1\right)\delta(z),$$

$$j_x = -\frac{j_0 \pi R_0^2}{d}\frac{\partial^2}{\partial x \partial z}\Theta\left(d - \rho_1\right)\delta(z),$$

$$j_y = -\frac{j_0 \pi R_0^2}{d}\frac{\partial^2}{\partial y \partial z}\Theta\left(d - \rho_1\right)\delta(z),$$

$$j_z = \frac{j_0 \pi R_0^2}{d}\left[\frac{1}{\gamma^2}\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\Theta\left(d - \rho_1\right)\delta(z).$$

Now we let the torus major radius d also tend to zero. Then

$$\Theta\left(d - \rho_1\right) = \frac{\pi d^2}{\gamma}\delta(x - vt)\delta(y),$$

$$\rho_{Ch} = -\frac{\beta j_0 \pi^2 d R_0^2}{c\gamma}\frac{\partial^2}{\partial x \partial y}\delta(x - vt)\delta(x)\delta(z),$$

$$j_x = -\frac{j_0 \pi^2 d R_0^2}{\gamma}\frac{\partial^2}{\partial x \partial z}\delta(x - vt)\delta(x)\delta(z),$$

$$j_y = -\frac{j_0 \pi^2 d R_0^2}{\gamma}\frac{\partial^2}{\partial y \partial z}\delta(x - vt)\delta(x)\delta(z),$$

$$j_z = \frac{j_0 \pi^2 d R_0^2}{\gamma}\left[\frac{1}{\gamma^2}\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\delta(x - vt)\delta(x)\delta(z).$$

The Fourier transforms of these densities are

$$\rho_{Ch} = -\frac{j_0 \pi d R_0^2}{2c^2\gamma}\frac{\partial^2}{\partial x \partial z}\exp(i\psi_1)\delta(y)\delta(z),$$

$$j_x = -\frac{j_0 \pi d R_0^2}{2c\gamma}\frac{\partial^2}{\partial x \partial z}\exp(i\psi_1)\delta(y)\delta(z),$$

$$j_y = -\frac{j_0 \pi d R_0^2}{2c\gamma}\frac{\partial^2}{\partial y \partial z}\exp(i\psi_1)\delta(y)\delta(z),$$

$$j_z = \frac{j_0 \pi d R_0^2}{2c\gamma}\left[\frac{1}{\gamma^2}\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right]\exp(i\psi_1)\delta(y)\delta(z).$$

Here $\psi_1 = kx/\beta$. The energy flux through the cylindrical surface of the radius ρ_1 per unit length per unit frequency is equal to zero for $v < c_n$ and

$$\sigma(\omega,\phi) = \frac{k^5 m_t^2}{2\varepsilon v\beta\pi\gamma^2}\left(n^2 - 1\right)^2\left(n^2\cos^2\phi + \frac{1}{\beta^2}\sin^2\phi\right) \quad (2.12)$$

for $v > c_n$. The integration over ϕ gives

$$\sigma(\omega) = \frac{k^5 m_t^2}{2\varepsilon v\beta\pi\gamma^2}\left(n^2 - 1\right)^2\left(n^2 + \frac{1}{\beta^2}\right). \quad (2.13)$$

As far as we know, radiation intensities (2.12) and (2.13) are obtained here for the first time. They are discontinuous: in fact, they fall from the values given by (2.12) and (2.13) for $\beta_n > 1$ to one-half of these values for $\beta = 1/n$ and to zero for $\beta < 1/n$. Also, we observe the appearance of the velocity window $c_n < v < c$ where the radiation differs from zero. Following to the Frank terminology, we conclude that the magnetic dipole parallel (perpendicular) to the velocity polarizes the medium in the same way as the toroidal dipole perpendicular (parallel) to the velocity.

2.3. UNBOUNDED MOTION OF A POINT-LIKE ELECTRIC DIPOLE

Consider an electric dipole consisting of point-like electric charges:

$$\rho_d = e\left[\delta^3(r + an) - \delta^3(r - an)\right]. \quad (2.14)$$

Here r defines the dipole center-of-mass, $2a$ is the distance between charges and the vector $n = \left(\sin\theta_0\cos\phi_0, \sin\theta_0\cos\phi_0, \cos\theta_0\right)$ defines the dipole orientation. Let the dipole move uniformly along the z axis. Then,

$$\begin{aligned}\rho_d = e\gamma\{&\delta(x + an_x)\delta(y + an_y)\delta((z - vt)\gamma + an_z) \\ &- \delta(x - an_x)\delta(y - an_y)\delta((z - vt)\gamma - an_z)\},\end{aligned} \quad (2.14')$$

and $j_z = v\rho_d$. Let the distance between charges tend to zero. Then,

$$\rho_d = 2ea\left(n\nabla\right)\delta(x)\delta(y)\delta(z - vt), \quad j_z = v\rho_d.$$

Here

$$\left(\vec{n}\vec{\nabla}\right) = \vec{n}_x\nabla_x + \vec{n}_y\nabla_y + \frac{1}{\gamma}\vec{n}_z\nabla_z, \quad \nabla_i = \frac{\partial}{\partial x_i}.$$

The Fourier components of these densities are

$$\rho_d(\omega) = \frac{ea}{\pi v}\left(\vec{n}\vec{\nabla}\right)\delta(x)\delta(y)\exp(i\psi), \quad j_z(\omega) = v\rho_d(\omega).$$

The radiation intensity per unit length of the cylindrical surface coaxial with the motion axis, per unit azimuthal angle and per unit frequency equals

$$\sigma_\rho(\phi,\omega)=\frac{m_d^2k^3n_z\tilde{n}_\rho}{\pi^2\varepsilon\beta^3v\gamma}\left(1-\beta_n^2\right)\times$$
$$\left[\frac{k\rho}{\beta}\left(1-\beta_n^2\right)\left(K_0^2+K_1^2\right)+\frac{1}{\gamma_n}K_0K_1\right] \qquad (2.15)$$

for $v<c_n$ and

$$\sigma_\rho(\phi,\omega)=\frac{m_d^2k^3}{2\pi\varepsilon\beta^3v}\left(\beta_n^2-1\right)\{\tilde{n}_\rho^2\left(\beta_n^2-1\right)+$$
$$n_z^2\left(1-\beta^2\right)+\tilde{n}_\rho n_z\frac{\pi}{2\gamma}[\frac{k\rho}{\beta}\left(\beta_n^2-1\right)(J_0^2+N_0^2+ \qquad (2.16)$$
$$J_1^2+N_1^2)-\frac{1}{\gamma_n}\left(N_0N_1+J_0J_1\right)]\}$$

for $v>c_n$. Here $m_d=2ea$ is the electric dipole moment in the reference frame attached to a moving electric dipole and

$$\tilde{n}_\rho=\sin\theta_0\cos(\phi-\phi_0),\quad \psi=kz/\beta.$$

The arguments of Bessel functions are $k\rho/\beta\gamma_n$. Integrating over the azimuthal angle ϕ one finds that $\sigma_\rho(\omega)=0$ for $v<c_n$ and

$$\sigma_\rho(\omega)=\frac{m_d^2a^2k^3}{\pi\varepsilon\beta^3v}\frac{1}{2}\left(\beta_n^2-1\right)\times$$
$$[\left(\beta_n^2-1\right)\sin^2\theta_0+\left(1-\beta^2\right)\cos^2\theta_0] \qquad (2.17)$$

for $v>c_n$. Here θ_0 is the angle between the symmetry axis of the electric dipole and its velocity. For the symmetry axis along the velocity $\left(\theta_0=0\right)$ and perpendicular to it $\left(\theta_0=\pi/2\right)$ one gets

$$\sigma_\rho(\omega,\theta=0)=\frac{m_d^2a^2k^3}{\varepsilon\beta^3v}\left(\beta_n^2-1\right)\left(1-\beta^2\right) \qquad (2.18)$$

and

$$\sigma_\rho(\omega,\theta=\pi/2)=\frac{m_d^2a^2k^3}{2\varepsilon\beta^3v}\left(\beta_n^2-1\right)^2, \qquad (2.19)$$

respectively.

It is rather surprisingly that for $\beta_n<1$, the non-averaged radiation intensities are equal to zero when the symmetry axis is either parallel or perpendicular to the velocity, but differs from zero for the intermediate inclination of the symmetry axis (see (2.21)). Integrating over the azimuthal angle one finds that

$$\sigma_\rho(\omega,\theta)=0 \text{ for } \beta_n<1.$$

Equations (2.22)-(2.24) coincide with those given by Frank [1-3], except for the factor $\left(1-\beta^2\right)$ in (2.17) and (2.18). This is due to the fact that we suggested that the charge distribution is given by (2.14) in the reference frame attached to a moving dipole while Frank suggested that (2.14) is valid in the laboratory frame. Recently, equations coinciding with those given by Frank were obtained in [11].

3. CONCLUSIONS

We briefly enumerate the main results obtained:

I. We investigated how radiate electric, magnetic and toroidal dipoles moving uniformly in unbounded medium. It turns out that radiation intensities crucially depend on the mutual orientation of the symmetry axis and velocity.

II. The behaviour of radiation intensities near the Cherenkov threshold $\beta=1/n$ is investigated in some detail.

III. The frequency and velocity domains where radiation intensities are maximal are defined.

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