KNO AND POLYAKOV'S MULTIPLICITY SCALING IN INELASTIC pp - AND $\bar{p}p$ - COLLISIONS AT SUPERHIGH ENERGIES

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It is shown that the normalized Saleh-Teich's multiplicity charged particles distribution in inelastic pp- and pp-collisions describes the KNO scaling low in soft processes (the range of ISR energies) and can be exhibited as Polyakov's scaling low in hard processes ($\sqrt{s} >> 1800 \text{ GeV}$) but in any case the multiplicity scaling is always violated in the range of $S\overline{p} - pS$ energies. It is supposed that the increase of transverse momentuma (or, what is the same, increase of phase volume) is the reason of the violation of any type scaling in the range of $S\overline{p} - pS$ energies, when the part of semi-hard processes is essentially increased.

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1. INTRODUCTION

As its shown in [1], the multiplicity charged particles distributions in inelastic pp- and pp-collisions in all the range of ISR- and $S\overline{p} - pS$ energies up to energy of Fer-milab tevatron collider are well described by Salech-Teich's three-parametrical distribution. The STdistri-bution is obtained in the supposition that on the first stage of interaction (which is described by homogeneous Poisson process) the random number of primary particles < v> (partons, pomerons, strings, etc.) in the impact parameters b_t space¹ is produced, and on the secondary stage each of primary particles generates inhomogeneous Poisson process of a random number of secondary particles $\langle \varepsilon \rangle$ («grey» hadrons) production on characteristic length $\langle \Delta_b \rangle$ equal to average length of parton chain (or, in other words, characteristic length of hadron jet formation). Thus the decolouration of «grey» hadrons happens on the interval of impact parameters [0, R], where R is average so-called radius of interaction of primary particles [2].

We won't show here bulky expressions of Saleh-Teich's three-parametrical distribution [1]. Let us note that identification and quantitative analysis of appropriate parameters of experimental charged particles distributions in hadron-hadron collisions is reduced to the determination of parametrical triplet $\{R/<\Delta_b>, <\varepsilon>, <v>$ by solution of the system of the nonlinear equations for

¹ In impact parameters representation the naïve model [1] has such physical singularities. The Eq. (4) in [1] is the convolution of the function of hadron production on the interval [0, R] (in terms of present paper), which can be re-written as:

$$h_R(b_t') = \int_0^\infty H(b_t) \cdot h(b_t + b_t') db_t$$
, $H(b_t) = u(b_t) - u(b_t - R)$

where $u(b_t)$ is single Heaviside function. In this expression the physical interpretation of the «step» $H(b_t)$ consists in the fact that in impact parameters space it represents a model image of real function, i.e. overlap function G_{in} under condition of unitary limit $(G_{in} \le 1)$ at $s \to \infty$. Then the maximum value of b_t is meaningful of radius of primary particles interaction R.

ST-distribution moments, which are explicitly considered in Ref. [3].

As result of the analysis of fitting data (collected in Table) of ABCDHW [4], UA5 [5], CDF [6] Collaborations experimental multiplicity distribution the following empirical dependences of the ST-distribution parameters are obtained:

$$\langle n \rangle \sim \sqrt{s}^{2\Delta}$$
, (1)

$$\langle \varepsilon \rangle \sim 0.66 \left[\sqrt{s} + 100 \right]^{2\Delta} \cdot \exp \left[-\frac{2R}{\langle \Delta_b \rangle} \right] + (1.11 \ln s - 2.8) \times \left[1 - \exp \left[-\frac{2R}{\langle \Delta_b \rangle} \right] \right]$$
(2)

$$\frac{R}{\langle \Delta_b \rangle} \sim \left[-1.23 \gamma \left(\frac{s}{s_c} \right)^{2\gamma \Delta} + \sqrt{1.23 \left(\frac{s}{s_c} \right)^{4\gamma \Delta} + 0.41} \right]^{-1}, (3)$$

where $\Delta = 0.212$, $\gamma = (s - s_c)/|s - s_c|$, $\sqrt{s_c} = 770$ GeV.

In a way, the asymptotic expressions (1)-(3) make it possible to consider the three-parametrical ST-distribution as "one-parametrical" multiplicity distribution P(n,s), where the role of the parameter plays collision energy s. Hence naturally suggests the idea to verify the known hypothesis of the asymptotic scaling behavior of multiplicity distributions [7,8]:

$$P(n,s) = \frac{1}{\langle n(s) \rangle} \psi \left(\frac{n}{\langle n(s) \rangle} \right), \tag{4}$$

which is widely known as KNO scaling of the multiplicity distributions.

For the first time such form of P(n, s) was derived within the framework of the conformal theories in the paper by Polyakov [7] as consequence for the strong interactions of hadrons of the application of three ideas unitarity, analyticity and similarity. The similarity hypo-

thesis, i.e. hypothesis of asymptotic scale invariance (which was induced by "strong coupling" regime in the Pomeranchuk pole problem [9]) reduces both to scaling form of secondary particles distribution (4) and to the fact that the average multiplicity of hadrons in strong interactions at small distances is power-law function of collision energy s:

$$\langle n(s)\rangle \propto s^{2\Delta}$$
 (5)

The same form of P(n, s) was obtained by Koba, Nielsen and Olesen [8], but it was based on Feynman's scaling [10]. The Feynman's model notions about high-energy collisions of hadrons (including the suppositions of transverse momentuma limitation of produced particles and the independence of Feynman's functions on collision energy) led the authors [8] both to scaling form of secondary particles distribution (4) and to the fact that the average multiplicity of hadrons in strong interactions at large distances is logarithmic function of collision energy s:

$$\langle n(s) \rangle \propto \ln s$$
 (6)

Let us add that in strong interactions at large distances the dependence (5) was obtained earlier within frameworks of Regge phenomenology [11] and also in multiperipheral models [12].

The purpose of the given research are the study and designing of the properties of normalized multiplicity ST-distribution, which satisfy to condition of similarity in strong interactions at small distances (Polyakov's scaling) and at large distances (KNO scaling).

2. PHENOMENOLOGY OF ASYMPTOTICAL BEHAVIOR OF NAIVE MODEL PARAMETERS

Here we will consider in what degree qualitative and quantitative aspects of asymptotical behavior of expressions (1)-(3) at energies $\sqrt{s} \ge 900$ GeV meet to relevant physical mechanisms. We understand relevance as a such asymptotic properties of parameters (1)-(3), which ensure not only optimum fits of experimental date in the energy range $\sqrt{s} = 20 \div 1800$ GeV, but also make it possible physically grounded to simulate the probable behavior of multiplicity secondary particle distributions, which could be obtained in the future researches at various ultrahigh energies [1].

First of all it concerns the physical substantiation of evolution of the parameters $<\Delta_b>$, R and their ratio (3) with the energy \sqrt{s} growth (because an asymptotical behavior of other two parameters (1) and (2) is well known and are explicitly considered in Sec. 3 and 4 accordingly).

So, let us consider the phenomenology of the parameter (3) evolution at increasing of collision energy \sqrt{s} .

Now it is already indisputable that the semi-hard processes (i.e. production of hadron mini-jets with relative large characteristic transverse momenta but with small part of an initial energy x << 1 [2, 13, 14]) become the main source of secondary particles at increasing of the collision energy. Such picture of hadron high ener-

getic collisions was unusual for a long time because it was considered that dominating source of secondary particles were the soft processes. In particular, it concerns the $S\overline{p}pS$ energy range, where strong interactions at small distances become essential and where the QCD perturbation theory is true.

Levin and Ryskin have shown [15] that the estimation of radius of interaction R_0 calculated in frameworks of LLA QCD at certain value of scale of cutting (Q_0) on transverse momentuma p_{\perp} has following form:

$$R_0 \sim \frac{0.954}{Q_0} [2\langle n \rangle]^{1/4},$$
 (7)

where factor 2 takes into account the fact that in our model the average multiplicity of secondary particles $\langle n \rangle$ is considered not for all charged particles but for a number of meson pairs [1]. It follows that $R_0=2^{1/4}R$.

On the other hand, the ratio $R/<\Delta_b>$ jumping varies at energy 900 GeV (see Table 1), and at the subsequent increase of energy the area of indeterminacy $<\Delta_b>$ of hadron production (in one mini-jet) in the impact parameters space becomes essential small value in comparison with radius of interaction, i.e. $R>><\Delta_b>$. It means that just hard processes play decisive role at energy $\sqrt{s} \ge 900$ GeV. Then taking into account that, on the one hand, $p_\perp <\Delta_b> \sim 1$ and, on the other hand, that the consideration of the distribution of meson pairs numbers gives a value of effective transverse moment equal to $p_\perp \ge 2Q_0$, we have following an expression (with allowance for (7)) for hard processes:

$$\frac{R}{\langle \Delta_b \rangle} \ge 1.908 \cdot \langle n \rangle^{1/4} \quad for \quad \sqrt{s} \ge 900 GeV$$
 (8)

Thus the expression (8) qualitatively and quantitatively reflects velocity gradient of ratio $R/<\Delta_b>$ (3) at ultrahigh energies starting from $\sqrt{s}=900$ GeV and it's also completely determined by the value of average multiplicity < n > of secondary particles. It means, in it's turn, that multiplicity ST-distribution becomes practically two-parametrical at energies $\sqrt{s} \ge 900$ GeV, i.e. it depends only on average multiplicity < n > and average multiplicity $< \varepsilon >$ in one mini-jet.

3. POLYAKOV'S SCALING

In terms of moments the homogeneous form of distribution P(n,s) in (4) means the independence of normalized algebraic moments on collision energy s

$$C_q = \langle n^q \rangle / \langle n \rangle^q$$
, where $\langle n^q \rangle = \sum_{n=1}^{\infty} n^q P_n$. (9)

For clear understanding of the strategy of designing of the properties of normalized ST-distribution, which satisfies to conditions of similarity hypothesis in strong interactions at small distances, let's consider the asymptotic properties of second normalized algebraic moment of ST-distribution:

Table 1. Parameters of multiplicity ST-distributions obtained by fitting of experimental data and predicted by our naive model

Experiment	√s, GeV	<n></n>	<3>	$R/<\Delta_b>$	χ^2/NDF
	p	p – collisions	3		
ABCDHW Collaboration [4]	30.4	5.27	4.8	0.03	6/13
ABCDHW Collaboration [4]	44.5	6.04	5.5	0.05	6/15
ABCDHW Collaboration [4]	52.6	6.38	5.8	0.06	4/17
ABCDHW Collaboration [4]	62.2	6.81	6.2	0.07	3/16
	\bar{p}	p - collision	ıs		
UA5 Ansorge et.al. [5]	200	10.70	8.3	0.22	47/34
UA5 Alner et.al. [5]	546	14.55	10.4	0.37	63/56
UA5 Ansorge et.al. [5]	900	17.80	12.3	6.80	52/52
CDF [6]	1800	24.00	16.3	4.22	135/97
	Predict	ions of naïve	model		
-	2400	27.1	18.2	4.35	-
-	6000	40.0	26.6	4.80	-
-	14000	57.3	37.9	5.25	-
-	100000	131.8	53.7	6.47	-
-	200000	176.9	116.7	6.96	-

$$C_{2} = \frac{\left\langle n^{2} \right\rangle}{\left\langle n \right\rangle^{2}} = 1 + \frac{\operatorname{var}(n)}{\left\langle n \right\rangle^{2}} = \begin{cases} 1 + \frac{1 + \left\langle \varepsilon \right\rangle \frac{R}{\left\langle \Delta \right|_{b}} \right\rangle}{\left\langle n \right\rangle}, & \frac{R}{\left\langle \Delta \right|_{b}} \right\rangle < 1; \\ 1 + \frac{1 + \left\langle \varepsilon \right\rangle}{\left\langle n \right\rangle}, & \frac{R}{\left\langle \Delta \right|_{b}} \right\rangle > 1. \end{cases}$$
(10)

where as var (n) Eq. (11) from Ref. [1] is used.

As it show the analysis of fitting data (Table), the values of ratio $(R/<\Delta_b>)$ are practically identical in all the range of ISR energies and monotonically increase in all the range of $S\overline{p}pS$ -energies up to energy $\sqrt{s} = 1800$ GeV. The asymptotic tendency of second moment (10) to constant value at energies over $\sqrt{s} = 1800$ GeV is stipulated by an identical asymptotic of < n > and $< \varepsilon >$. It means that if average multiplicity in hadron mini-jet $< \varepsilon >$ and total average multiplicity of hadrons < n > behave uniformly (i.e., according to Polyakov [7], they are power-law functions of energy) the Polyakov's scaling *must be observed* in the range of ISR energies (where $R / < \Delta_b > < 1$) and *can be observed* at energies much above $\sqrt{s} = 1800$ GeV (where $R / < \Delta_b > > 1$).

The power-law dependence of average total hadron multiplicity on energy (4) is practically beyond doubt. This is confirmed by experimental data in the range \sqrt{s} = 20-1800 GeV presented in Table. At the same time the analogous statement, which concerning the average multiplicity in hadron mini-jet $<\varepsilon$, is rather ambiguous. Within the framework of existing experimental data, $<\varepsilon$ > can be fitted both by logarithmic and by power-law dependence on energy. The mixed variant of these dependences with specified "survival" one of them in the

limit of high energies is also possible. For instance, such mixed empirical dependence (which is almost «mirror» concerning Eq.(5)):

$$\langle \varepsilon \rangle \sim (1.11 \ln s - 2.8) \cdot \exp \left(-\frac{R}{\langle \Delta_b \rangle} \right) +$$

$$+ 0.66 \cdot \left(\sqrt{s} + 100 \right)^{2\Delta} \left[1 - \exp \left(-\frac{R}{\langle \Delta_b \rangle} \right) \right],$$
(11)

describes the appropriate values $<\varepsilon>$ as good as Eq. (5) (see Table), but this dependence asymptotically tends to power-law dependence at energies over $\sqrt{s} = 900$ Gev $(R/<\Delta_b>>>1)$, i.e., $<\varepsilon> \sim s^{\Delta}$ with $s \to \infty$.

Explanation of power-low behavior of average multiplicity in the jet $\langle \varepsilon \rangle$ is following. In 1968 Gribov and Migdal [9], considering the problem of the interaction of reggeons, showed that one possibility is "strong coupling" case in which the many-particle Green's functions are power-low, not logarithmic, functions of their arguments. On the other hand, as Polyakov has noted [7], the power-low asymptotic forms of the Green's functions at large momentuma mean that the theory has invariance with a respect to change of the space-time scale, which is manifested at very small distances. Thus the similarity hypothesis in the strong interaction at small distances reduces to power-low asymptotics of the average hadron multiplicity in the jet [7], i.e., $\langle \varepsilon \rangle \sim const \cdot s^{\Delta}$.

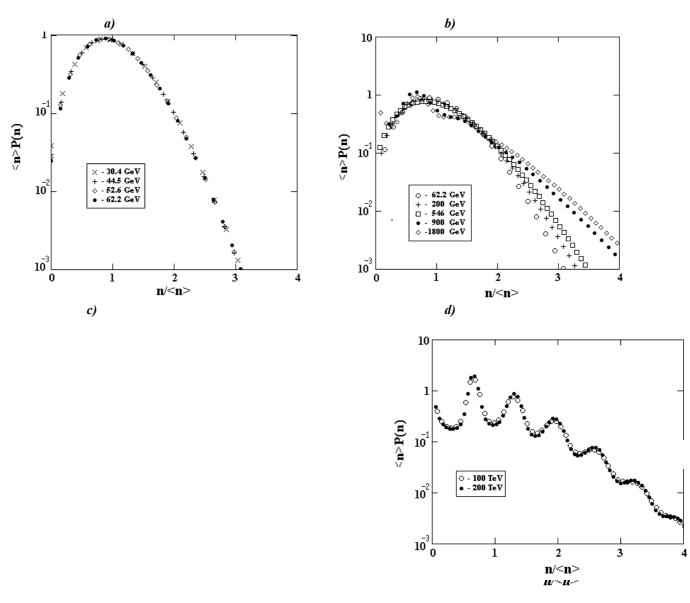


Fig. 1. Normalized multiplicity ST-distribution for the non single-diffractive p- and pp-collisions shows the validity of KNO scaling in soft processes (a) and predicts the approximate validity of Polyakov's scaling in semi-hard (c), hard processes (d) and strongly violation of any type scaling in the range of SppS -energies (b)

L ($(R/<\Delta_b>)<<1$), but in semi-hard and hard hadron-hadron collisions (i.e., in strong interactions at small distances) the power-law physics "survives "($(R/<\Delta_b>)>>1$).

et us consider the behavior of Polyakov's scaling in the energy range $\sqrt{s} = 0.03$ - 100 TeV. For this purpose we used the values obtained by fitting of experimental distribution of secondary particles on energy interval $\sqrt{s} = 0.03$ - 1.8 TeV and theoretically predicted by empirical expressions (1), (8) and (11) at asymptotically high energies (see Table) as the parameters of three-parametrical ST-distribution. The research of properties of such normalized ST-distribution (which satisfy to conditions of the similarity hypothesis in strong interactions at small distances) has shown an anomalous behavior of Polyakov's multiplicity scaling of charged particles produced in hadron collisions. Scaling is valid inthe range

of ISR energies (Fig.1a), it is approximately valid at superhigh energies starting from the energy $\sqrt{s} = 100 \text{ TeV}$ (Fig. 1d) and it is completely violated in $S\overline{p}pS$ energy range $\sqrt{s} = 200\text{-}1800 \text{ GeV}$ (Fig. 1b). In the energy range $\Box s = 2\text{-}100 \text{ TeV}$ slow but stable tendency to scaling is observed (Fig. 1c).

The analysis of multiplicity distributions parameters (Table 1) and expressions (1) and (11) shows that the parameters $\langle n \rangle$ and $\langle \varepsilon \rangle$ are monotone increasing power-law functions of energy, whereas the parameter $R/\langle \Delta_b \rangle$ jumping increase just in the energy range \sqrt{s} =200-1800 GeV. It is experimentally shown that an average radius of inelastic interactions R is increased insignificantly on energy in the range 200-1800 GeV and is on interval $R \sim \langle b_t^2 \rangle^{1/2} = 0.9-1.1$ Fm [16].

The theoretical estimation of average radius of inelastic interaction (7) is calculated in LLA of QCD for

large transverse momenta [15] and satisfactorily describes known experimental data for semi-hard processes [16]. Hence, the nature of $R/<\Delta_b>$ change on energy interval $\sqrt{s} = 200-1800$ GeV allows to conclude that the sharp decreasing of average length of parton chain $\langle \Delta_b \rangle$ is main reason of total violation of Polyakov's scaling. Such sharp decrease of $\langle \Delta_b \rangle$, which is accompanied by the simultaneous increase of average hadron multiplicity in mini-jet $\langle \varepsilon \rangle$, point out the sharp increase of parton density in the chain. This, in its turn, means that the increase of multiplicity in mini-jet happens only due to the increase of transverse momentum, i.e. directly due to increase of phase volume [2]. Thus, it is possible to assume that significant increase of phase volume in semi-hard processes is the reason of total violation of Polyakov's scaling.

Let's consider the mathematical interpretation of Polyakov's scaling violation in energy range \sqrt{s} =200-1800 GeV. It is easy to show that with $(<\varepsilon>R/<\Delta_b>)\to 0$ ST-distribution asymptotically transforms to Poisson distribution and with $(R/<\Delta_b>)>>1$ it transforms to Neyman type A distribution from two parameters < n> and $<\varepsilon>[3, 17]$:

$$P(n) = \sum_{k=0}^{\infty} \left[\frac{(k\langle \varepsilon \rangle)^n}{n!} \exp\left(-k\langle \varepsilon \rangle\right) \cdot \frac{1}{k!} \left(\frac{\langle n \rangle}{\langle \varepsilon \rangle}\right)^k \exp\left(-\frac{\langle n \rangle}{\langle \varepsilon \rangle}\right) \right], (12)$$

which exactly coincides with compound Poisson distribution [18]. Hence, the jumping increase of parameter $R/<\Delta_b>$ from 0.22 to 6.8 (see Table) in indicated energy range leads to the fact that the ST-distribution (by virtue of its asymptotic properties on parameter $R/<\Delta_b>$ [3, 17]) sharply changes its type and structure, i.e., it changes Poisson (one-humped) type at $R/<\Delta_b>>1$ [3, 17]. Thus, such an asymptotic transition causes the loss of self-similarity of distribution function, and this is the reason of Polyakov's scaling violation. Physically it usually means the change of phase states of researched system on parameter $R/<\Delta_b>$.

A test of the scaling hypothesis is provided by an examination of energy dependence of the moments of the distribution. The moments C_{2-5} for the non-single-diffractive events are shown in Fig. 2.

The behavior of moments confirms our expectations concerning to anomalous behavior of Polyakov's scaling (Fig. 1) and, what is especially important, total equivalence of Polyakov's scaling and normalized Neyman type A distribution (12) in the limit of high energies (over 100 TeV) (see Fig. 1d).

4. KNO SCALING

As it was noted above, KNO scaling grows out of the approach based on similarity hypothesis in strong interactions at large distances, and, in a way, it reflects the self-similar properties of logarithmic physics. The authors of Ref. [8] accent attention just to such properties. They written that theirs' result "... can be expressed by the saying that the normalized multiplicity distribution keeps its form independently of the beam energy and just scales up as lns" [8].

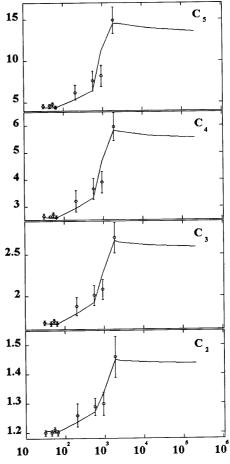


Fig. 2. Values of the first four C-moments for the non single-diffractive ST-distributions as a function of the center of mass energy. The values of C-moments at ISR-, SppS-energies and at \sqrt{s} =1800 GeV are given in Refs. [4], [5], [13], accordingly

The asymptotic analysis of algebraic moments (6) of ST-distributions (and, in particular, an example of an asymptotic dependence of normalized second moment (10) on collision energy s) presented in Sec. 3 allows to make the conclusion concerning to KNO scaling also. If the average multiplicity in hadron mini-jet $\langle \varepsilon \rangle$ and total average multiplicity of hadrons $\langle n \rangle$ behave uniformly (for instance, they both are logarithmic functions of energy), then KNO scaling must be observed in the ISR energy range (where $R/<\Delta_b><<1$) and can be observed at energies over $\sqrt{s} >> 1800 \text{ GeV}$ (where $R/\langle \Delta_b \rangle >> 1$). However, as it shown above (see Table), the average hadron multiplicity <n> is power-law function of collision energy s, i.e. $\langle n \rangle \sim s^{\Delta}$ at $s \to \infty$. Then, if the average hadron multiplicity in mini-jet < > is logarithmic dependence on collision energy (for example, such as Eq. (2)), KNO scaling must be observed in the range of ISR-energies and *always must be violated* at energies over ISRenergies. Such asymptotic of $\langle \varepsilon \rangle$ and $\langle n \rangle$ and also the reasons of KNO scaling violation are typical, for example, in different versions of dual parton model and quark-gluon strings model [19], which met with considerable success in describing and, in some cases, predicting the main feature of low- p_T physics at ISR and collider energies [20].

In our case, in spite of the fact that the parameters < n > and $< \varepsilon >$ have different functional forms on collision energy s, KNO scaling really satisfies to a good accuracy in the range of ISR-energies (and practically does not differ from Fig. 1a); however as energy increases KNO scaling is strongly violated. The validity of KNO scaling in the ISR energy range is explained by the fact that the asymptotic expressions for < n > and $< \varepsilon >$ (though they have power-law and logarithmic forms of the dependence on collision energy s) are close in this energy range and insignificantly vary with energy growth.

5. CONCLUSIONS

The basic result of the present work consists in that the normalized multiplicity ST-distribution describes KNO scaling in soft processes and can be exhibited as Polyakov's scaling in hard processes. We assume that the increase of transverse momentuma (or what is the same, increase of phase volume) is the reason of the violation of any type scaling low in the range of SppS - energies, where the part of semi-hard processes is essentially increased.

If further collider pp-experiments at energies $\square s >> 1800$ will prove the Polyakov's scaling existence, then, firstly, the importance of true reasons of any type scaling violation in area of $S\overline{p}pS$ -energies can be scarcely exaggerated, and, secondly, the very important answer to very old question: «Could it be that as in the theory of critical phenomena, while the basic physics (Hamiltonians) between e^+e^- annigilation and hadronic multiparticle production could be entirely different, multiplicity scaling may yet still be a universal feature shared by both ?» [21] can be obtained.

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