POLARIZATION EFFECTS IN SYNCHROTRON RADIATION IN STRONG MAGNETIC FIELD

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The electron self-polarization effect is found to be more essential in comparison with quasi-classical approximation. The results of ultra-quantum and quasi-classical calculations for the photon polarization coincide with each other. The rigid correlation between the photon linear polarization and the electron spin is found.

PACS: 42.25-Ja; 41.60-Ap

1. INTRODUCTION

Study of the quantum-electrodynamic processes in the presence of strong magnetic field close to the critical value of about 10^{13} Gs is important for exploration of astronomical objects such as pulsars. These sources of radiation, it is well known, are connected to neutron stars, where near to stars' surface, the magnetic field has value of such an order.

Quantum-electrodynamic processes with magnetic field occur in heavy ion collision too. The magnetic field produced by colliding nuclei in the region between them at the moment of the closest approach has order of magnitude about 10^{12} Gs in the case, when that region has the size of Compton wavelength of electron (see Fig. 1). The electric fields of nuclei mutually compensate one another in that region.

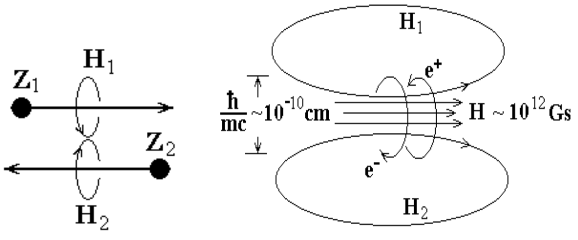

Fig. 1. Magnetic field of colliding nuclei and e^+e^- pair in that field

We consider, that the series of quasi-equidistant narrow peaks in the electron-positron distribution of total energy, observed in heavy ions collision at GSI, Darmstadt [1,2], is a result of movement of an electron-positron pair in such magnetic field in that region (see Fig. 2). Narrow lines are the resonant pair production on the Landau levels. The resonant pair production by two equivalent photons is the second order quantum-electodynamic process. But in resonant conditions, as is known, this process breaks up to two independent first order processes.

Therefore it is meaningful to study the most elementary processes in detail, with magnetic field, which has order of magnitude about 10^{12} Gs.

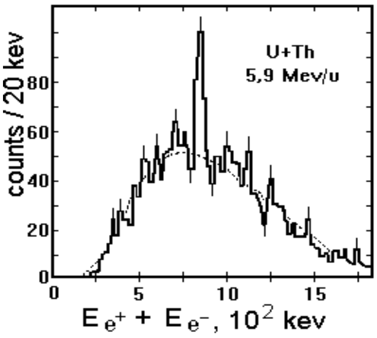

Fig. 2. Narrow peaks in heavy ions collision experiments at GSI, Darmstadt [1, 2] are the resonant pair production on the Landau levels: $E_n = E_{e^+} + E_{e^-} = 2\sqrt{m^2 + 2neH + \pi^2 / L_z^2}$

2. ELECTRON SELF POLARIZATION EFFECT

Now we pass to consideration of the synchrotron radiation process. Certainly this process has been well investigated. Its general relativistic theory for a long time had been constructed [3,4]. But only ultrarelativistic quasi-classical approximation was studied in detail [5,6]. Other approximations were studied too [7,8], but there were some questions.

The quantum effects in the process have dual character. The quantum, that is the single photon, is radiated. And movement of the electron is also quantized. The dispersion law of the electron has the form ($\hbar$=1, c=1):

$$\varepsilon_n = m^2 \cdot \sqrt{1 + 2nh + p_z^2 / m^2}\,, \qquad (1)$$

$$h = H / H_0, \quad H_0 = m^2 / e\,.$$

The movement is characterized by the principle quantum number n that is number of the Landau level. H is magnetic field in units of critical one. The case of the big value of Landau level is the case of quasi-classical electron behavior.

But we examine a case of strong field, when the individual energy levels are experimentally distinguishable. Thus the number of final states of electron is equal to one. This approximation is called ultra-quantum approximation. However, the magnetic field that is 10^{12} Gs in units critical field is a good small parameter of the problem. Due to the presence of the small parameter all calculations can be made analytically and it is possible to receive in Furry picture the simple expressions for probabilities of the process.

Formulas (2-5) describe the probabilities of the process with transition of electron on the next level:

$$\frac{dW^{++}}{du} = \frac{1}{2}\alpha\, mh^2 \cdot (n-1)\cdot(1+u^2), \quad (2)$$

$$\frac{dW^{--}}{du} = \frac{1}{2}\alpha\, mh^2 \cdot n\cdot(1+u^2), \quad (3)$$

$$\frac{dW^{+-}}{du} = \frac{1}{4}\alpha\, mh^3 \cdot (1+u^2), \quad (4)$$

$$\frac{dW^{-+}}{du} = \frac{1}{64}\alpha\, mh^5 n(n-1)(1+11u^2-5u^4+u^6)\,. \quad (5)$$

The signs «+» and «-» designate the direction of initial and final electron spins along and against field respectively, a is the fine structure constant, *u* is the cosine of the angle q between directions of the magnetic field and the photon motion.

The formulas (2) and (3). It is so called spin-up state and spin-down state of electron.

The formulas (4) and (5) describe probabilities of spin reorientation. These probabilities are not equal, what is the essence of the electron self-polarization effect, which had been found by Sokolov and Ternov in quasi-classical approximation.

You can see, the probabilities have different powers of magnetic field [8,9]. It is influence of quantum character of the particles' movement. For example: In field about 10^{12} Gs one probability is three order of magnitude greater then another. The dependence on an angle of photon radiation in these formulas (4), (5) is also different, what is shown in Fig. 3.

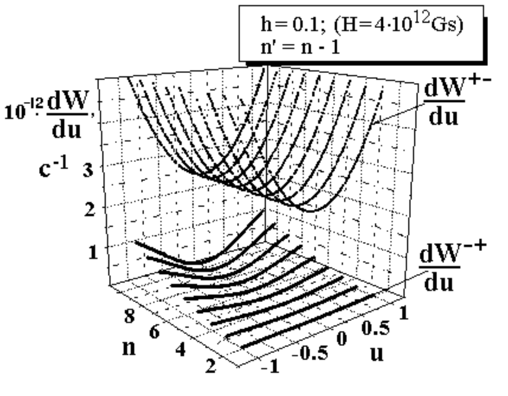

Fig. 3. *Probability of spin-flip process versus both angle of photon radiation $u = \cos\theta$ and electron energy level n*

So, the self-polarization effect is more essential in comparison with quasi-classical approximation.

3. PHOTON POLARIZATION

The following question is the photon polarization. The polarization vector has the form:

$$e_\lambda^\nu = (0, \vec{e}_\lambda), \quad \vec{e}_{\lambda=1} = (\cos A,\ \sin A\cdot e^{iB},\ 0),$$
$$\vec{e}_{\lambda=2} = \vec{e}_{\lambda=1}(A \to A+\pi/2) \quad (6)$$

in the case, when the photon is radiated along the axis z (along magnetic field). If the photon's wave vector *k* is directed arbitrarily it is necessary to perform the corresponding rotation of polarization vector. A and B are the parameters, which determine the type of polarization. The Stokes parameters are more known. The connection of these parameters is defined by formula:

$$\xi_1 = \sin 2A\cdot\cos B\,, \quad \xi_2 = \sin 2A\cdot\sin B\,,$$
$$\xi_3 = \cos 2A\,. \quad (7)$$

The probabilities of polarized radiation have the form shown in formulas (2-4), where it is necessary to make a replacement of $(1 + u^2)$ by the following expression:

$$(1+u^2) \to \left(1-\frac{1}{2}(1-u^2)(1+\xi_3)+u\xi_2\right), \quad (8)$$

if $\mu = \mu'$; and

$$(1+u^2) \to \left(1-\frac{1}{2}(1-u^2)(1-\xi_3)+u\xi_2\right), \quad (9)$$

if $\mu = -\mu'$.

You can see, that the probabilities don't depend on parameter ξ_1. It is an obvious result, as this parameter characterizes polarization on the angle + 45 and -45 degrees relatively to the plane (*k*, *H*), and these are symmetrical situations. The degree of polarization is equal to ratio of the difference to the sum of probabilities with the opposite polarizations:

$$\beta = \frac{dW_{\lambda=1} - dW_{\lambda=2}}{dW_{\lambda=1} + dW_{\lambda=2}} = \frac{2u\xi_2 - (1-u^2)\xi_3}{1+u^2}\,. \quad (10)$$

The degree values as function of parameters ξ_2 and ξ_3 lie in some plane, as shown in Fig. 4.

The type of polarization is defined by ξ_2 and ξ_3, at which modulo degree of polarization $|\beta|$ has the maximal value. For example: In the case of forward radiation (along field) (u=1, ξ_2=1, ξ_3=0) we have the right circular polarization. In the case sideways radiation (u=0, ξ_2=0, ξ_3=-1) we have the linear polarization, when the vector of electric field of the photon is perpendicular to the plane (*k*, *H*). So we have received the result known in classical consideration of the problem. Ultra-quantum and quasi-classical results coincide with each other. The reason is in following: the type of photon polarization does not depend on the Landau level numbers of initial and final electrons. From every energy level the radiation has the similar polarization, which depends only on a direction of photon motion.

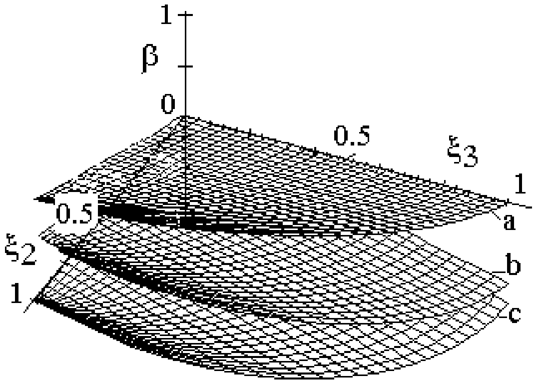

Fig. 4. Degree of photon polarization versus Stokes parameters, a) u = 1, b) u = 1/3, c) u = 0

However, the rigid correlation between the photon linear polarization and the electron spin is found. You can see in formulas (8) and (9), that in a spin-flip process (9) the sign of linear polarization (the sign of ξ_3) is opposite in comparison with a process without spin reorientation (8).

4. CONCLUSION

In conclusion the idea to measure polarization of the X-ray pulsar radiation is proposed. There are two types of the directional radiation pattern of pulsar (see Fig. 5): pencil-shaped, when radiation is along the field direction and knife-shaped, when radiation is perpendicular to the field. In a radiation spectrum of an X-ray pulsar the cyclotron lines were found (see Fig. 6). They are hyrolines, predicted by Gnedin. The cyclotron radiation is principle in vicinity of these lines. We propose to select one of lines, to measure its polarization and to define the Stokes parameters. Then it is not difficult to calculate the angle between directions of magnetic field and radiation. It will allow confirming the type of the directional radiation pattern of pulsars (pencil or knife), and may allow proposing a new radiation scheme.

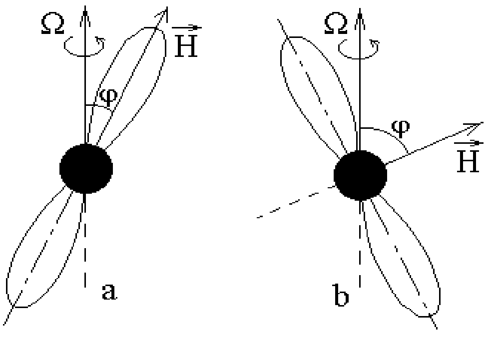

Fig. 5. Directional radiation pattern of X-ray pulsar (cross-section), a) pencil-shaped, b) knife-shaped

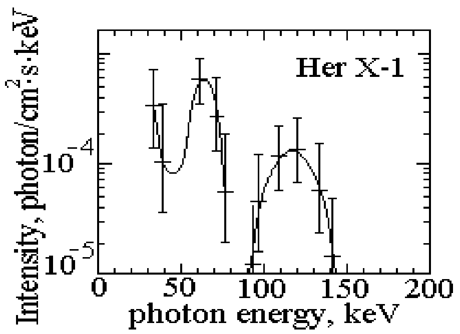

Fig. 6. Spectrum of X-ray pulsar,Hercules X-1

ACKNOWLEDGMENT

The authors are grateful to V.E. Storizhko for his constant help in holding this work and also to S.P. Roshchupkin for useful discussions.

The work was partly supported by the Grant for the young researchers of the National Academy of Sciences of Ukraine.

REFERENCES

1. I. Koenig, E. Berdermann, F. Bosch et al. Investigations of correlated e^+e^- emission in heavy-ion collisions near the Coulomb barrier // *Z. Phys. A.* 1993, v. 346, p. 153-164.
2. R. Bar, A. Balanda, J. Baumann et al. Experiments on e^+e^- - line emission in HI collisions // *Nuclear Phys. A.* 1995, v. 583, p. 237-145.
3. N.P. Klepikov. Photons and electron-positron pairs emission in magnetic field // *Zh. Eksp. Teor. Fiz.* 1954, v. 26, p. 19-34 (in Russian).
4. A.A. Sokolov, I.M. Ternov. *Relativistic electron*. Moskow: ''Nauka'', 1974, 392 p. (in Russian).
5. V.N. Baier, A.I. Milstein. Radiation effects near cyclotron resonance // *Zh. Eksp. Teor. Fiz.* 1978, v. 75, p. 390-401 (in Russian).
6. W. Tsai, A. Yildiz. Motion of electron in a homogeneous magnetic fields - modified propagation function and synchrotron radiation // *Phys. Rev. D.* 1973, v. 8, № 10, p. 3446-3460.
7. V.G. Bagrov, D.M. Gitman, V.N. Rodionov et al. Influence of strong electromagnetic wave on emission of the weak excited electrons moving in magnetic field // *Zh. Eksp. Teor. Fiz.* 1976, v. 71, p. 433-439 (in Russian).
8. I.G. Mitrofanov, A.S. Pozanenko. Generation of radiation in quantum transitions of electrons in a strong magnetic field // *Zh. Eksp. Teor. Fiz.* 1987, v. 93, p. 1951-1962 (in Russian).
9. R.I. Kholodov, P.V. Baturin. Polarization effects in synchrotron radiation in ultra-quantum approximation // *Ukr. Jour. of Phys.* 2001, v. 46, №5-6, p. 621-626.