ROTATED CHARGED BLACK HOLES
IN EINSTEIN-BORN-INFELD THEORIES

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In this work the solution of the Einstein equations for slowly rotating black hole with Born-Infeld charge is obtained. Geometrical properties, singularities, horizons of this solution are analyzed. There are considered the conditions when the black hole modifies its mass (like in the non-linear monopole cases) and angular momentum for the same non-linear electromagnetic field what produces the black hole.
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INTRODUCTION

The four dimensional solutions with spherical symmetry of the Einstein equations coupled to Born-Infeld fields have been well studied in the literature [1]. In particular the Born-Infeld monopole, in contrast to Maxwell, contributes to the mass of the compact object source of the field. B. Hoffmann was the first who studied such static solutions in the context of the general relativity with the idea to obtain a consistent particle-like model. Unfortunately, these static Einstein-Born-Infeld models generate conical singularities at the origin [1,9]. This type of singularities cannot be removed like global monopoles or other non-localized topological defects of the space-time [6,7]. In this report the solution for slowly rotating black hole with Born-Infeld charge is obtained. This solution presents coupled terms of charge and angular momentum what modifies asymptotically the mass and the angular momentum of the rotating source. Families of solutions are obtained varying b (Born-Infeld parameter). The fundamental feature of this metric is the lack of conical singularities at the origin (the rotation produces a repulsive potential what protects the singular zone), and for particular values of b there are solutions with $M^2 < a^2 + Q^2$ without violation of cosmic censorship hypothesis.

ROTATED EINSTEIN-BORN-INFELD MODEL

In this case, one expects found a metric with an asymptotic behaviour like the well-known Kerr-Newman's metric for the rotating and charged black hole. But there exist few difficulties: the metric is non-diagonal (rotating frame) and the energy-momentum tensor of Born-Infeld includes the invariant pseudoscalar [2,3], because of the rotation of the compact object (it is appear magnetic field). The used convention is the *spatial* of Landau and Lifshitz (1974), with signatures of the metric, Riemann and Einstein all positives (+++) [4,5].

STATEMENT OF THE PROBLEM

One proposes a line element like Kerr's geometry in the Boyer and Lindquist generalization [8,9], with the expected asymptotic behaviour:

$$ds^2 = -\frac{\Delta}{\rho^2}\left[dt - a\,Sin^2\theta\,d\phi\right]^2 +$$

$$+ \frac{Sin^2\theta}{\rho^2}\left[\left(r^2 + a^2\right)d\phi - a\,dt\right]^2 + \frac{\rho^2}{\Delta}dr^2 + \rho^2 d\theta$$

where the functions Δ and ρ are in principle depending of r and θ and are to be determined. For obtaining the Einstein equations, the most powerful is the Cartan's method [5,8,9]. This method applies differential forms and is based on two fundamental geometric equations (structure equations). In the 1-forms orthonormal basis the line element is written:

$$ds^2 = -\left(\omega^0\right)^2 + \left(\omega^1\right)^2 + \left(\omega^2\right)^2 + \left(\omega^3\right)^2,$$

where the association between coordinate and orthonormal frame is not trivial in the case of an axially rotating symmetry and requires to solve an equation system. Because of the geometrical symmetries of the Riemann tensor, ρ immediately can be determined:

$$\rho^2 = r^2 + a^2 Cos^2\theta.$$

One can see, the ρ function is the same what the ρ of Boyer and Lindquist and does not depend of the axially symmetric source considered. With the function ρ founded, only Δ left to be find.

The calculation of the energy-momentum tensor components in the rotating system will give all information to determine the function Δ. For this we shall use the metric symmetrized expression of $T^a{}_b$:

$$T^a{}_b = \delta^a{}_b L_{BI} - \frac{\partial L_{BI}}{\partial S}F^a{}_l F^l{}_b - \frac{\partial L_{BI}}{\partial P}F^a{}_l \widetilde{F}^l{}_b$$

where:

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$$L_{BI} = \frac{b^2}{4\pi}\left(1 - \sqrt{1 - \frac{2S}{b^2} - \frac{P^2}{b^4}}\right)$$

and

$$S \equiv -\frac{1}{4}F_{ab}F^{ab} = L_{Maxwell};$$

$$P \equiv -\frac{1}{4}F_{ab}\widetilde{F}^{ab} \quad \left(\widetilde{F}^{ab} \equiv \frac{1}{2}\varepsilon^{abcd}F_{cd}\right)$$

For the electromagnetic tensor F, one proposes the form similar to the Boyer and Lindquist generalization for a Kerr-Newman problem [8,9]:

$$F = F_{20}dr \wedge \left[dt - aSin^2\theta\, d\phi\right]$$
$$+ F_{31}Sin\theta\, d\theta \wedge \left[\left(r^2 + a^2\right)d\phi - adt\right] =$$
$$= F_{20}\omega^2 \wedge \omega^0 + F_{31}\omega^3 \wedge \omega^1.$$

where F_{20} and F_{31} are to be determined. One can see, F_{20} and F_{31} are the only field components in the tetrad and the energy-momentum tensor takes the diagonal form:

$$-T_{00} = T_{22} = \frac{b^2}{4\pi}\left(1 - u\right),$$

$$T_{11} = T_{33} = \frac{b^2}{4\pi}\left(1 - u^{-1}\right),$$

where:

$$u \equiv \sqrt{\frac{\left(\overline{F}_{31}\right)^2 + 1}{1 - \left(\overline{F}_{20}\right)^2}}\left(\overline{F}_{ab} \equiv \frac{F_{ab}}{b}\right).$$

The fields are to be obtained from the dynamical (Eulerian) equations in the tetrad form. Let us solve these equations with the following boundary condition: the fields asymptotically have the same behavior like the electromagnetic fields of the Kerr-Newman model. Then one obtains ($r_0^4 = bQ^2$):

$$\left(\overline{F}_{20}\right)^2 = \frac{r_0^4 r^2}{\rho^8\left(r_0^4 + r^4\right)}\left[r^2\left(r^2 - a^2Cos^2\theta\right)^2 - 4a^2Cos^2\theta\, r_0^4\right]$$
,

$$\left(\overline{F}_{31}\right)^2 = \frac{\left(r_0^4 + r^4\right)}{r_0^4 r^2}\left[\frac{4Cos^2\theta\, r^2 a^2 r_0^8\left(r^2 - a^2Cos^2\theta\right)^2}{\left[r^2\left(r^2 - a^2Cos^2\theta\right)^2 - 4a^2Cos^2\theta\, r_0^4\right]}\right]\frac{1}{\rho^8}.$$

Putting all the ingredients in the Einstein equations, one obtains the next expression:

$$4\rho^2 b^2 - 2\rho^2 b^2\left[\sqrt{\frac{\left(r_0^4 + r^4\right)}{\rho^2 r^2 - 4a^2Cos^2\theta\left(r_0^4 + r^4\right)}}\frac{\left(r^2 - a^2Cos^2\theta\right)}{r}\right] -$$
$$- 2\rho^2 b^2\left[\sqrt{\frac{\rho^2 r^2 - 4a^2Cos^2\theta\left(r_0^4 + r^4\right)}{\left(r_0^4 + r^4\right)}}\frac{r}{\left(r^2 - a^2Cos^2\theta\right)}\right] + 2 = \partial_r \partial_r \Delta.$$

This expression, although exact, is not integrable by transcendental functions like in the static cases. One must to make an expansion in power series for a/r small (slowly rotating). Looking the last equation, one can see that Δ depends of the radial coordinate r and the angular coordinate θ. The integrals are calculated in indefinite form and the values of the two constants $A(\theta)$ and $B(\theta)$ of the problem are selected according to the asymptotical behavior of Δ and the metric. The obtained solution takes the form:

$$\Delta(r,\theta) = r^2 + P_{ST} + \left\{2a^2\frac{Q^2}{r_0^3}\left\{\frac{9}{5}\sqrt{i}E\left[\frac{\pi}{4},2\right] + 1.525Sin^2\theta\right\} - 2M + 2\frac{Q^2}{r_0^5}a^4\left(2Sin^2\theta - Sin^4\theta\right)2.8653\right\}r +$$

$$+ a_{KN}^2 + \left(2Sin^2\theta - Sin^4\theta\right)2\frac{Q^2}{r_0^4}a^4 0.8576 +$$

$$+ 2a^2\frac{Q^2}{r_0^4}\cos^2\theta\left\{r^2 + \frac{4}{5}\sqrt{r_0^4 + r^4} - \frac{1}{10}\frac{r_0^4}{r^4}\sqrt{r_0^4 + r^4} - rr_0\frac{9}{5}\sqrt{i}E\left[\frac{1}{2}ArcCos\left(i\frac{r^2}{r_0^2}\right),2\right]\right\} +$$

$$+ 2\frac{Q^2}{r_0^4}a^4\cos^2\theta\left\{-2ArcSinh\left(\frac{r^2}{r_0^2}\right) + \sqrt{1 + \frac{r^4}{r_0^4}}\left[-\frac{r_0^2}{r^2}\frac{163}{770} - \frac{r_0^6}{r^6}\frac{313}{385} - \frac{r_0^{10}}{r^{10}}\frac{21}{385}\right]\right\} -$$

$$- 2\frac{Q^2}{r_0^4}a^4\cos^2\theta\left\{\frac{17}{11}(-1)^{\frac{1}{4}}\frac{r}{r_0}F\left[iArcSinh\left[(-1)^{\frac{1}{4}}\frac{r}{r_0}\right],-1\right]\right\},$$

where the constants have been selected to obtain asymptotically Kerr-Newman metric and P_{ST} is identical to the similar quantity in the static case:

$$P_{ST} = \frac{3}{2}Q^2\left\{\bar{r}^4 + \bar{r}^2\sqrt{\bar{r}^4 + 1} + 2\bar{r}(-1)^{1/4}F\left[ArcSin\left[(-1)^{\frac{3}{4}}\bar{r}\right],-1\right]\right\}\left(b^2 \equiv \frac{Q^2}{r_0^4};\bar{r} \equiv \frac{r}{r_0}\right)$$

One can see that this solution contains new terms that do not appear in the Kerr-Newman model, like Reissner-Nordstrom and the static Born-Infeld model. There are coupled terms of charge and angular momentum. The expansion is for $\frac{a}{r} << 1$ and $r_0 \leq r$.

ANALYSIS OF THE METRIC
IN THE BORN-INFELD ROTATING CASE

The general behavior of the metric is similar to the Kerr-Newman's model. How one can see from the last expression for the Δ, the metric has two horizons and depends strongly of r_0 (related with Born-Infeld parameter: $b \equiv Q^2/(r_0)^4$) and its quotient to a^2 . The asymptotical behavior of the $\Delta(r)$ is:

$$\Delta(r) \cong r^2 - \left[2M + \frac{Q^2}{r_0}\left(\frac{4}{3}1.854 - 1.525\frac{2a^2}{r_0^2} - \frac{2a^4}{r_0^4}2.8653\right)\right]r$$
$$+ Q^2 + a_{KN}{}^2 + a^2\left(0.8576\frac{a^2Q^2}{r_0^4}\right)$$

what corresponds to have asymptotically an effective mass:

$$M_{Eff} = M + \frac{Q^2}{r_0}\left[\frac{2}{3}1.854 - 1.525\frac{a^2}{r_0^4} - 2.8653\frac{a^4}{r_0^4}\right]$$

and an effective angular momentum:

$$a_{Eff} = \sqrt{a_{KN}{}^2 + 2\frac{Q^2}{r_0^4}a^4 0.8576} \;,$$

where M is Schwarzschild mass and $a_{KN} \equiv$ Kerr-Newman's model angular momentum (this asymptotic term is due **only** to the rotation of the spherical body and is characteristic of the Kerr's model [8])

CONCLUSIONS

In this report a solution of the Einstein equation for slowly rotating black hole is presented. The general behavior of the geometry is strongly modified according to the value what take r_0 (Born-Infeld radius [1,2]) relative to a value. This metric has not the problem of the conical singularities at the origin of the static Born-Infeld models and permits solutions with $M^2 < a^2 + Q^2$ without violation of the cosmic censorship hypothesis. Solutions with $M = 0$ and $a = 0$ are possible, too; the Born-Infeld-rotating model gives mass and angular momentum to the source of the non-linear fields. In next papers the particle-like models with Born-Infeld rotating black holes will be studied and analyzed, and also the possibility of supersymmetrical extensions (see recent papers [10,11,12]) for such model will be considered.

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