TWISTORIAL SUPERPARTICLE WITH TENSORIAL CENTRAL CHARGES

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A twistorial formulation of the N=1 D=4 superparticle with tensorial central charges describing massive and massless cases in uniform manner is given. The twistors resolve energy-momentum vector whereas the tensorial central charges are written in term of spinor Lorentz harmonics. The model makes possible to describe states preserving all allowed fractions of target-space supersymmetry. The full analysis of the number of conserved supersymmetries in models with N=1 D=4 superalgebra with tensorial central charges has been carried out.

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1. INTRODUCTION

Some interesting supersymmetric theories admit as scalar central charges, which are presented in the conventional Poincare supersymmetry, as well as the nonscallar ones [1]. In the supersymmetry algebra tensorial central charges are usually associated with topological contributions of extended objects. It is attractive to consider the pure superparticle models having symmetry of this kind. Such a model was firstly obtained in massless case with two or three local κ-symmetries [2]. We have constructed the model of massive non-extended superparticle with central charges [3] having single κ-symmetry, which is equivalent to conventional spinning (spin 1/2) particle. In a certain sense the commuting spinor variables of the model play the role of index spinor variables [4], [5]. Analogous model of massive superparticle has been formulated in [6,7] without Lorentz invariance. In fact the model [6,7] is obtained in particular case of model [3] with constant index spinor fixed in non-Lorentz invariant way.

In recent work [8] we proposed twistorial formulation of superparticle with tensorial central charges in which massive and massless cases are described uniformly. The model uses both central charge coordinates and auxiliary bosonic spinor variables simultaneously. In term of the last variables the energy-momentum vector and the tensorial central charges are resolved. In the massive case of the proposed model we have twistorial formulation of massive superparticle with tensorial central charges preserving 1/4 or 1/2 of target-space supersymmetries. For zero mass this model turns into twistorial formulation of the massless superparticle with tensorial central charges [2] in which one or two of target-space supersymmetries are broken. But the case of massless superparticle with only one κ-symmetry is impossible in this formulation.

Our purpose here will be to give twistorial formulation of *N*=1 *D*=4 superparticle with tensorial central charges which is able to describe massive and massless cases in a uniform manner with all allowed possibilities of the target-space supersymmetry violation. In addition to twistors we use pair of harmonic spinors by means of which the tensorial central charges are resolved in Lorentz-covariant way. But in some cases the choice of corresponding gauges makes possible to remain only with one type of spinors, for example, with twistors.

We will in section 2 investigate the *D*=4 *N*=1 superalgebra with tensorial central charges with respect to all allowed parts of unbroken target-space supersymmetry depending on the value of the momentum square (particle mass). Section 3 presents the twistor formulation of *D*=4 *N*=1 superparticle with tensorial central charges. Excepting twistor spinors we use Lorentz harmonics also. Section 4 describes possible sets of tensor central charges coefficients in particle action and corresponding interconnection of harmonic and twistor spinors. Section 5 contains some comments.

2. SUPERALGEBRA WITH TENSORIAL CENTRAL CHARGES

Generalized central extension of nonextended 4-dimensional supersymmetry algebra with Majorana supercharges Q can be written in the form

$$\{Q,Q\} = 2\mathcal{Z}\ .$$

Here $\mathcal{Z}^T = \mathcal{Z}$ is the matrix of central charges with a total of ten real entries. We have tensorial central charges as coefficients in decomposition of this matrix

$$\mathcal{Z}C = (\gamma^{\mu})P_{\mu} + \frac{i}{2}(\gamma^{\mu\nu})Z_{\mu\nu}$$

on the basis defined by products of γ-matrices. The vector P_{μ} is (in general) a sum of the energy-momentum vector and a vectorial 'string charge'. Namely this vector plays the role of particle energy-momentum in considering theory. In follows we suppose what the vector P_μ satisfies spectral property, i.e. it is positive time-like or light-like four-vector; $P^2 = -m^2$, where m is mass. Six real charges $Z_{\mu\nu} = -Z_{\nu\mu}$ are related to the symmetric complex Weyl spin-tensor $Z_{\alpha\beta} = Z_{\beta\alpha}$ in standard way. The spin-tensors $Z_{\alpha\beta}$ and $\bar{Z}_{\dot\alpha\dot\beta} = (\overline{Z_{\alpha\beta}})$ represent the self-dual and anti-self-dual parts of the central charge matrix.

The l.h.s. of the equation for eigenvalues of the matrix Z,

$$\Pi(\lambda) \equiv \det(\mathrm{Z} - \lambda) = 0,$$

can be written as a polynomial [1] in λ,

$$\Pi(\lambda) = \sum_{k=0}^{4} \Pi_k \lambda^k,$$

with

$$\Pi_0 = [(P_0)^2 - a]^2 + 8bP_0 - 4c,$$

$$\Pi_1 = 4P_0[(P_0)^2 - a] + 8b,$$

$$\Pi_2 = 2[3(P_0)^2 - a],$$

$$\Pi_3 = 4P_0,$$

$$\Pi_4 = 1.$$

Here

$$a = \vec{E}^2 + \vec{H}^2 + \vec{P}^2,$$

$$b = \vec{P}(\vec{E}\times\vec{H}),$$

$$c = |\vec{P}\times\vec{E}|^2 + |\vec{P}\times\vec{H}|^2 + |\vec{E}\times\vec{H}|^2$$

and

$$E_i = Z^{0i},\quad H_i = \frac{1}{2}\varepsilon_{ijk}Z^{jk}$$

are electric and magnetic vectors of tensorial central charges.

For the case of massive particle, in the rest frame where $\vec{P} = 0$ and $P^0 = m$, we can choose the first axis along the vector $\vec{E}$ if it is nonzero. The second axis is embedded in a half-plane defined by the vector $\vec{H}$ with respect to $\vec{E}$. In the massless case if $\vec{E}\vec{H} \neq 0$ one can take $\vec{E} \parallel \vec{H}$ without loss of generality. If $\vec{E}\vec{H} = 0$ one can choose $\vec{H} = 0$ for $\vec{E}^2 - \vec{H}^2 > 0$ and $\vec{E} = 0$ for $\vec{E}^2 - \vec{H}^2 < 0$.

The massive superparticle with unique preserved SUSY ($\Pi_0 = 0$, $\Pi_1 \neq 0$) is obtained only if

$$(m^2 - \vec{E}^2 - \vec{H}^2)^2 = 4|\vec{E}\times\vec{H}|^2 \neq 0.$$

Up to rotations the variety of such configurations is characterised by positive modulus of non-collinear vectors $\vec{E}$ and $\vec{H}$ and the angle ϑ between them. At the boundary of this region of parameters when $m^2 = \vec{E}^2 + \vec{H}^2 \neq 0$ with collinear $\vec{E}$ and $\vec{H}$ we have two preserved SUSYs ($\Pi_0 = \Pi_1 = 0$, $\Pi_2 \neq 0$). Conditions for preserving more than two SUSYs ($\Pi_0 = \Pi_1 = \Pi_2 = 0$) in massive case are contradictory.

In massless case there are two types of configurations preserving unique SUSY. One of them takes place for the collinear $\vec{E}$ and $\vec{H}$ with non-collinear to them $\vec{P}$ if $\vec{E}^2 + \vec{H}^2 = 4\vec{P}^2\sin^2\varphi \neq 0$ where φ is the angle between $\vec{P}$ and $\vec{E}$. Another one consists in mutual orthogonality of three vectors $\vec{E}$, $\vec{H}$ and $\vec{P}$ forming right triple, $\vec{P}\vec{E}\vec{H} > 0$, and equality of two modulus, $|\vec{E}| = |\vec{H}| \neq P^0$. At the boundary of the first of these regions we have the odd sector of conventional massless superparticle without central charges ($\vec{E} = \vec{H} = 0$) preserving two SUSYs. We have two preserved SUSYs also if three mutually orthogonal vectors $\vec{E}$, $\vec{H}$ and $\vec{P}$ form left triple $\vec{P}\vec{E}\vec{H} < 0$ and two of them have equal lengths

$$|\vec{E}| = |\vec{H}| \neq P^0.$$

At the boundary of this region if mutually orthogonal vectors form left triple and all of them have equal modulus

$$|\vec{E}| = |\vec{H}| = P^0,$$

then three SUSYs are preserved. All the SUSYs cannot be preserved because of $\Pi_3 = 4P_0 \neq 0$.

In the table all possible preservations of fractions of target-space supersymmetry for massless ($m = 0$) and massive ($m \neq 0$) D=4 N=1 superparticle with tensorial central charges are given. Except the case of massless superparticle with ¼ unbroken SUSY, in Table 1 it is mention the full set of necessary conditions. For massless superparticle ($m = 0$) with ¼ unbroken SUSY these conditions are complicated. We give required conditions at orthogonal vectors $\vec{E}$, $\vec{H}$ to $\vec{P}$,

$$\vec{P}\vec{E} = \vec{P}\vec{H} = 0.$$

Here $\varepsilon = +1$ at $\vec{P}\vec{E}\vec{H} > 0$ or $\varepsilon = -1$ at $\vec{P}\vec{E}\vec{H} < 0$.

In the following we present the twistorial formulation of the superparticle with tensorial central charges in which all allowed cases of supersymmetry violation can be realized both for massive superparticle and massless one.

The conditions on ‘electric’ and ‘magnetic’ vectors of tensorial central charges and energy-momentum vector for states of N=1 D=4 massless ($m = 0$) and massive ($m \neq 0$) superparticles preserving fractions ¼, ½ and ¾ of target-space supersymmetry

	¼	½	¾
$m=0$	$\vec{E}\times\vec{H} = 0$, $\vec{E}^2 + \vec{H}^2 = 4P_0^2$ ($\vec{P}\vec{E} = \vec{P}\vec{H} = 0$)	$\vec{E}\vec{H} = 0$, $\vec{E}^2 - \vec{H}^2 = 0$, $\vec{P}\vec{E} = \vec{P}\vec{H} = 0$,	
$m\neq 0$	$\vec{E}\vec{H} = 0$, $\lvert\vec{E}\rvert - \varepsilon\lvert\vec{H}\rvert = \pm 2P^0$ ($\vec{P}\vec{E} = \vec{P}\vec{H} = 0$)	$\frac{\vec{E}^2 + \vec{H}^2}{2} \neq P_0^2$	$\frac{\vec{E}^2 + \vec{H}^2}{2} = P_0^2$

$m \neq 0$	$(\vec{E}^2 + \vec{H}^2 - m^2)^2 = 4\lvert \vec{E}\times\vec{H}\rvert^2 \neq 0$	$\vec{E}\times\vec{H} = 0$, $\vec{E}^2 + \vec{H}^2 = m^2$	no

3. LAGRANGIAN OF TWISTORIAL SUPERPARTICLE WITH TENSORIAL CENTRAL CHARGES

A trajectory of superparticle is parameterized by the usual superspace coordinates x^μ, θ^α, $\bar\theta^{\dot\alpha}$ and tensorial central charge coordinates $y^{\alpha\beta} = y^{\beta\alpha}$, $\bar y^{\dot\alpha\dot\beta} = (\overline{y^{\alpha\beta}})$. For description of superparticle energy-momentum we use the pair of bosonic spinors (bitwistor) $v_\alpha{}^a$, $\bar v_{\dot\alpha a} = (\overline{v_\alpha{}^a})$, $a = 1,2$. The tensor central charges are written in terms of even spinor variables $u_\alpha{}^a$, $\bar u_{\dot\alpha a} = (\overline{u_\alpha{}^a})$, $a = 1,2$. We use D=4 Weyl spinor and σ-matrices conventions of [9] where the metric tensor $\eta_{\mu\nu}$ has mostly plus and $\sigma^{(\mu}_{\alpha\dot\beta}\tilde\sigma^{\nu)\dot\beta\gamma} = -\delta_\alpha^\gamma$. The indices a, b, c, … which are carried by the spinors $v_\alpha{}^a$, $\bar v_{\dot\alpha a}$ as well as $u_\alpha{}^a$, $\bar u_{\dot\alpha a}$, are raised and lowered as $SU(2)$ ones.

For description of the superparticle, both massless and massive, with tensorial central charges we take the action $S = \int d\tau\, L$ with Lagrangian [8] in twistor-like form

$$L = P_\mu \Pi_\tau^\mu + Z_{\alpha\beta}\Pi_\tau^{\alpha\beta} + \bar Z_{\dot\alpha\dot\beta}\bar\Pi_\tau^{\dot\alpha\dot\beta} - \\ - \lambda_v h_v - \bar\lambda_v \bar h_v - \lambda_u h_u - \bar\lambda_u \bar h_u. \qquad (1)$$

Here the one-forms

$$\Pi^\mu \equiv d\tau\, \Pi_\tau^\mu = dx^\mu - id\theta\sigma^\mu\bar\theta + i\theta\sigma^\mu d\bar\theta\ ,$$

$$\Pi^{\alpha\beta} \equiv d\tau\, \Pi_\tau^{\alpha\beta} = dy^{\alpha\beta} + i\theta^{(\alpha}d\theta^{\beta)},$$

$$\bar\Pi^{\dot\alpha\dot\beta} \equiv d\tau\, \bar\Pi_\tau^{\dot\alpha\dot\beta} = d\bar y^{\dot\alpha\dot\beta} + i\bar\theta^{(\dot\alpha}d\bar\theta^{\dot\beta)}$$

are invariant under global supersymmetry transformations

$$\delta\theta^\alpha = \varepsilon^\alpha,\ \delta\bar\theta^{\dot\alpha} = \bar\varepsilon^{\dot\alpha},$$

$$\delta x^\mu = i\theta\sigma^\mu\delta\bar\theta - i\delta\theta\sigma^\mu\bar\theta\ ,$$

$$\delta y^{\alpha\beta} = i\theta^{(\alpha}\delta\theta^{\beta)},\ \delta\bar y^{\dot\alpha\dot\beta} = i\bar\theta^{(\dot\alpha}\delta\bar\theta^{\dot\beta)}$$

acting in the extended superspace parameterised by the usual superspace coordinates x^μ, θ^α, $\bar\theta^{\dot\alpha}$ and by the tensorial central charge coordinates $y^{\alpha\beta}$, $\bar y^{\dot\alpha\dot\beta}$.

In the action the quantities P_μ, $Z_{\alpha\beta} = Z_{\beta\alpha}$, $\bar Z_{\dot\alpha\dot\beta} = (\overline{Z_{\alpha\beta}})$ which play the role of the momenta for x^μ, $y^{\alpha\beta}$, $\bar y^{\dot\alpha\dot\beta}$ are taken in the form

$$P_{\alpha\dot\beta} = P_\mu\sigma^\mu_{\alpha\dot\beta} = v_\alpha{}^a\bar v_{\dot\alpha a}, \qquad (2)$$

$$Z_{\alpha\beta} = u_\alpha{}^a u_\beta{}^b C_{ab},\ \bar Z_{\dot\alpha\dot\beta} = \bar u_{\dot\alpha a}\bar u_{\dot\beta b}\bar C^{ab} \qquad (3)$$

where C_{ab}, $\bar C^{ab} = (\overline{C_{ab}})$ are symmetric constants. Thus they are resolved in terms of bosonic Weyl spinors $v_\alpha{}^a$, $\bar v_{\dot\alpha a}$ and $u_\alpha{}^a$, $\bar u_{\dot\alpha a}$.

The last terms in Lagrangian (1) are the sum of the kinematic constraints for the even spinors

$$h_v \equiv v^{\alpha a}v_{\alpha a} - 2m \approx 0,\ \bar h_v \equiv \bar v_{\dot\alpha a}\bar v^{\dot\alpha a} - 2m \approx 0 \qquad (4)$$

$$h_u \equiv u^{\alpha a}u_{\alpha a} - 2 \approx 0,\ \bar h_u \equiv \bar u_{\dot\alpha a}\bar u^{\dot\alpha a} - 2 \approx 0 \qquad (5)$$

with Lagrange multipliers.

Due to the constraints (4) which are equivalent to

$$v^{\alpha a}v_{\alpha b} = m\delta_b^a,\ \bar v_{\dot\alpha a}\bar v^{\dot\alpha b} = m\delta_a^b$$

we have $\det(v_\alpha{}^a) = m$ and

$$P^2 \equiv P^\mu P_\mu = -m^2.$$

Thus the constant $|m|$ plays the role of the mass.

In both cases, massless and massive, we have twistorial resolution [10] of energy-momentum vector in term of bosonic spinors.

In the massless case ($m = 0$) the spinors $v_\alpha{}^1$ and $v_\alpha{}^2$ are proportional to each other $v_\alpha{}^1 \propto v_\alpha{}^2$ as the consequence of the constraint (4) $v^{\alpha 1}v_\alpha{}^2 = 0$.

The constraints (5) are equivalent to

$$u^{\alpha a}u_{\alpha b} = \delta_b^a,\ \bar u_{\dot\alpha a}\bar u^{\dot\alpha b} = \delta_a^b$$

and they give $\det(u_\alpha{}^a) = 1$. Thus the spinors $u_\alpha{}^a$, $\bar u_{\dot\alpha a}$ play the role of harmonic variables [11-13] parametrizing an appropriate homogeneous subspace of the Lorentz group.

In addition to kinematical constraints (4), (5) the bosonic spinor variables of the model are satisfied to constraints on spinor momenta $p_v \approx 0$, $p_u \approx 0$ and c.c. A part of these constraints conjugate to constraints (2-5) and are second class constraints. The rest of the constraints on spinor momenta, which conserve the constraints (2), (3), are first class constraints and correspond to stability subgroup of Lorentz group acting on indices a, b,… The choices of gauge allow to connect harmonic spinors and twistor ones for certain cases of the central charges and particle mass.

4. SUPERPARTICLE STATES PRESERVING ARBITRARY FRACTIONS OF TARGET-SPACE SUPERSYMMETRY

The expressions (3) of central charges containing three complex (six real) constants are collected in C_{ab}, $\bar C^{ab}$. The symmetric matrices are expanded in symmetric Pauli matrices

$$(\sigma_i)_{ab} = \varepsilon_{ac}(\sigma_i)_a{}^c,\ (\sigma_i)^{ab} = \varepsilon^{ac}(\sigma_i)_c{}^b,$$

where $(\sigma_i)_a{}^b$, $i = 1,2,3$ are usual Pauli matrices. Thus

$$C_{ab} = C_i(\sigma_i)_{ab},\ \bar C^{ab} = -\bar C_i(\sigma_i)^{ab}$$

where $\bar C_i = (\overline{C_i})$. We obtain directly

$$C_{ab}C^{bc} = -C_iC_i\delta_a^{\ c} = (\vec{E}^2 - \vec{H}^2 + 2i\vec{E}\vec{H})\delta_a^{\ c},$$

$$C_{ab}\overline{C}^{ab} = 2C_i\overline{C}_i = 2(\vec{E}^2 + \vec{H}^2),$$

where real 3-vectors $\vec{E}$ and $\vec{H}$ are defined by the equality

$$\vec{C} = i(\vec{E} + i\vec{H}).$$

The vectors $\vec{E}$ and $\vec{H}$ are 'electric' and 'magnetic' vectors of tensorial central charges, which was used in Section 2. But now they are determined with respect to the basis of Lorentz harmonics $u_\alpha^{\ a}$, $\overline{u}_{\dot\alpha\, a}$. In this basis the components of energy-momentum are

$$P^{(0)} = \frac{1}{2}P_a^{\ a},\ P^{(i)} = \frac{1}{2}P_a^{\ b}(\sigma_i)_b^{\ a}$$

where matrices $P_a^{\ b}$ and $P_{\alpha\dot\beta}$ are connected by the Lorentz transformation generated by harmonic matrix $u_\alpha^{\ a}$, i.e.

$$P_{\alpha\dot\beta} = u_\alpha^{\ a}\overline{u}_{\dot\beta\, b}P_a^{\ b}.$$

Thus the condition

$$u_\alpha^{\ a}\overline{u}_{\dot\beta\, b}P_a^{\ b} = v_\alpha^{\ a}\overline{v}_{\dot\beta\, a} \qquad (6)$$

connects bitwistor representation of the energy-momentum and its representation in harmonic basis.

For given energy-momentum vector P the fraction of preserving supersymmetry is determined by the choice of 'electric' $\vec{E}$ and 'magnetic' $\vec{H}$ vectors. Since central charges are written in terms of harmonics, any choice of $\vec{E}$ and $\vec{H}$ does not lead to violation of Lorentz invariance. In the following it is convenient to make the analysis in the standard momentum frame.

Massive ($m \neq 0$) case

In the frame of standard momentum $P^{(0)} = m$, $P^{(i)} = 0$ the expression (6) gives

$$m\, u_\alpha^{\ a}\overline{u}_{\dot\beta\, a} = v_\alpha^{\ a}\overline{v}_{\dot\beta\, a}.$$

Hence the harmonic spinors $u_\alpha^{\ a}$ and twistor ones $v_\alpha^{\ a}$ are identical up to unitary transformations acting on index a. Without loss of generality we can take

$$m^{1/2}u_\alpha^{\ a} = v_\alpha^{\ a}.$$

This identification can be obtained by gauge fixing of the harmonic degrees of freedom, which are pure gauge ones in the initial action (1). As result, we derive the model of superparticle with only twistor spinors (or only with Lorentz harmonics). Such model was considered in [8] and gives all possible cases of target-space supersymmetry preserving for massive superparticle.

The case with ½ unbroken SUSY is reached if central charge coefficients satisfy the conditions

$$C^{ab}\overline{C}_{ab} = 2m^2,\ C^{ab}C_{ab}\overline{C}^{cd}\overline{C}_{cd} = 4m^4 \qquad (7)$$

which are equivalent to

$$\vec{E}^2 + \vec{H}^2 = m^2,\ \vec{E}\times\vec{H} = 0.$$

As it been obtained in [8], the conditions (7) give the unitary condition on the coefficient matrix of central charges

$$C^{ab}\overline{C}_{bc} = m^2\delta^a_{\ c}.$$

Without loss of generality we can take diagonal matrix C_{ab} with m multiplied on phase multipliers on diagonal of them. This corresponds to vectors $\vec{E}$ and $\vec{H}$ which both set in plane XY.

The case with ¼ unbroken SUSY is obtained for $\vec{E}\times\vec{H} \neq 0$ when

$$|\vec{E}^2 + \vec{H}^2 - m^2| = 2|\vec{E}\times\vec{H}|$$

is satisfied. Here only one from two elements of diagonal matrix C_{ab} has the modulus equal to m.

Massless ($m = 0$) case

In the standard energy-momentum frame for massless particle

$$P^\mu = (P^{(0)}, 0, 0, P^{(0)})$$

and

$$P_a^{\ b} = 2P^{(0)}(\sigma_+)_a^{\ b}$$

where $\sigma_+ = (1_2 + \sigma_3)/2$. The twistor spinors, which are proportional each other in massless case can be taken equal

$$v_\alpha \equiv v_\alpha^{\ 1} = v_\alpha^{\ 2}.$$

Then the expression (6) give

$$2P^{(0)}u_\alpha^{\ 1}\overline{u}_{\dot\beta\, 1} = 2v_\alpha\overline{v}_{\dot\beta}.$$

Therefore one harmonic spinor $u_\alpha^{\ 1}$ is expressed in form twistor one v_α up to phase transformation. Thus we can remain with twistor spinor v_α and one harmonic spinor $u_\alpha^{\ 2}$. The second harmonic spinor is obtained directly from twistor one (for example, $u_\alpha^{\ 1} = (P^{(0)})^{-1/2}v_\alpha$) if it is necessary.

In terms of vectors $\vec{E}$ and $\vec{H}$ the matrix C_{ab} has the following expression

$$C_{ab} = \begin{pmatrix} H_1 - E_2 - i(H_2 + E_1) & -H_3 + iE_3 \\ -H_3 + iE_3 & -H_1 - E_2 - i(H_2 - E_1) \end{pmatrix}.$$

In case of ½ and ¾ unbroken SUSY vectors $\vec{E}$ and $\vec{H}$ are orthogonal to each other and to vector $\vec{P}$ (see Table 1) which along third axis. Therefore $E_3 = H_3 = 0$ and matrix C_{ab} is diagonal. In these cases (½ and ¾ unbroken SUSY) vectors $\vec{E}$ and $\vec{H}$ have equal lengths $|\vec{E}| = |\vec{H}| \equiv V$ and the nonzero elements of the matrix C_{ab} are $C_{11} = 2Ve^{-i\phi}$ in case $\vec{P}\vec{E}\vec{H} > 0$ or $C_{22} = -2Ve^{i\phi}$ in case $\vec{P}\vec{E}\vec{H} < 0$ where ϕ is angle between first axis and vector $\vec{H}$. In case of ½ unbroken SUSY it is just $V \neq P^{(0)}$ whereas in case of ¾ unbroken SUSY it is fulfilled $V = P^{(0)}$. The case with $\vec{P}\vec{E}\vec{H} > 0$

and nonzero only $C_{11} \neq 0$ correspond to twistor model of the superparticle with tensorial central charges considered in [2].

For massless particle the states which preserving ¼ SUSY are realised at $|\vec{E}| \neq |\vec{H}|$ when $\vec{E}\vec{H} = 0$. At these conditions the matrix C_{ab} has two nonzero diagonal elements. Namely

$$C_{11} = (|\vec{E}| + |\vec{H}|)e^{-i\phi}, \quad C_{22} = (|\vec{E}| - |\vec{H}|)e^{i\phi}$$

in case $\vec{P}\vec{E}\vec{H} > 0$ or

$$C_{11} = -(|\vec{E}| - |\vec{H}|)e^{-i\phi}, \quad C_{22} = -(|\vec{E}| + |\vec{H}|)e^{i\phi}$$

in case $\vec{P}\vec{E}\vec{H} < 0$. Thus the case with only one κ -symmetry is realised for massless superparticle if two spinors are presented in model. The using of one twistor spinor as in [2] is insufficient for realization of ¼ unbroken SUSY. The more strong violation of target-space supersymmetry requires using more numbers of the spinors represented central charges of supersymmetry algebra [14].

5. CONCLUSION

In present paper we construct the model of N=1 D=4 superparticle with tensorial central charges. By corresponding choice of tensorial central charges we obtain all possibilities of violated target-space supersymmetry. It should be noted that these choices are fulfilled in Lorentz-covariant way due to using Lorentz harmonics in model.

The twistor formulation of the massive superparticle is required to use of pair of twistors. For massless superparticle the cases with ¾ and ½ unbroken SUSY are realized by using only one twistor spinor, the energy-momentum and the tensorial central charges are resolved in form of which. But for realization of only ¼ unbroken SUSY it is necessary to use two spinors in model, one from which is twistor whereas second used spinor can be harmonic one.

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