RADIATIVE CORRECTIONS IN ELASTIC AND DEEP-INELASTIC ELECTRON-PROTON SCATTERING

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The electron structure function method is applied to calculate model-independent radiative corrections to an asymmetry of electron-proton scattering. The representations for both spin-independent and spin-dependent parts of the cross section are derived. Master formulae take into account the leading correction in all orders and the main contribution of of the second order next-to-leading ones and have accuracy at the level of one per mille.

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1. MASTER FORMULA

Precise polarization measurements in both inclusive and elastic scattering are crucial for understanding the structure and fundamental properties of a nucleon.

One important component of the precise data analysis is radiative effects which always accompany the processes of electron scattering. The first calculation of radiative corrections (RC) to polarized deep-inelastic scattering (DIS) was done in [1,2] where the first order correction has been found.

In present work we apply method of the electron structure function for calculation of RC. Within this approach DIS in one photon exchange approximation can be considered as the Drell-Yan process [3]. The corresponding cross section with accounting of RC can be written as an contraction of two electron structure functions and the hard part of the cross section [4,5]. Traditionally these RC include main effects caused by loop corrections as well soft and hard collinear photons and e^+e^- - pairs. But it was shown in [5] how one can improve this method by inclusion also effects due to radiation of one non-collinear photon. The corresponding procedure concludes in modification of the hard part of cross section that leads to exit beyond the leading approximation.

A straightforward calculation based on the quasi-real electron method [6] can be used to write cross section of DIS process

$$e^-(k_1) + P(p_1) \to e^-(k_2) + X(p_x) \qquad (1)$$

in the following form

$$\frac{d\sigma(k_1,k_2)}{dQ^2dy} = \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} \frac{dz_2}{z_2} D(z_1,L)D(z_2,L)\frac{d\sigma_h(\tilde{k}_1,\tilde{k}_2)}{d\tilde{Q}^2 d\tilde{y}},$$

$$L = \ln\frac{Q^2}{m^2}, \qquad (2)$$

where m is the electron mass and

$$Q^2 = -(k_1-k_2)^2, \quad y = \frac{2p_1(k_1-k_2)}{V}, \quad V = 2p_1k_1.$$

The reduced variables which define the hard cross section in the integrand are

$$\tilde{k}_1 = z_1k_1, \quad \tilde{k}_2 = \frac{k}{z_2}, \quad \tilde{Q}^2 = \frac{z_1}{z_2}Q^2, \quad \tilde{y} = 1-\frac{1-y}{z_1z_2}.$$

The electron structure function $D(z,L)$ includes contributions due to photon emission and pair production

$$D = D^\gamma + D_N^{e^+e^-} + D_S^{e^+e^-}.$$

For the photon contribution into the structure function one can use iterative form

$$D^\gamma = \delta(1-z) + \sum_{k=1}^{\infty}\frac{1}{k!}\left(\frac{\alpha L}{2\pi}\right)^k P_1(z)^{\otimes k}, \qquad (3)$$

$$P_1(z)^{\otimes k} = \underbrace{P_1(z)\otimes\cdots\otimes P_1(z)}_{k},$$

$$P_1(z)\otimes P_1(z) = \int\limits_z^1 P_1(t)P_1\left(\frac{z}{t}\right)\frac{dt}{t},$$

$$P_1(z) = \frac{1+z^2}{1-z}\theta(1-z-\Delta) + \delta(1-z)\left(2\ln\Delta + \frac{3}{2}\right), \quad \Delta\to 0.$$

The nonsinglet part of the structure function due to real and virtual pair production can be included in the iterative form of $D^\gamma(z.L)$ by replacing $\alpha L/2\pi$ on the right side of Eq. (3) with the effective electromagnetic coupling $\frac{\alpha L}{2\pi} \to \frac{\alpha_{eff}}{2\pi} = -\frac{3}{2}\ln\left(1-\frac{\alpha L}{3\pi}\right)$ that is (within the leading accuracy) the integral of the running electromagnetic constant.

The lower limits of integration with respect to z_1 and z_2 in the master equation (2) can be obtained from the condition of existence of inelastic hadronic events which reads in terms of dimentionless variables as

$$z_1z_2 + y - 1 - xyz_1 \geq z_2z_{th},$$

$$x = \frac{Q^2}{2p_1(k_1-k_2)}, \quad z_{th} = \frac{M_{th}^2 - M^2}{V},$$

which leads to

$$z_{2m} = \frac{1-y+xyz_1}{z_1-z_{th}}, \quad z_{1m} = \frac{1+z_{th}-y}{1-xy}. \qquad (4)$$

The matrix element squared of the considered process in one photon exchange approximation is proportional to contraction of leptonic and hadronic tensors. The representation (2) reflects the properties of leptonic one. Therefore, it has universal nature and can be applied to processes with different final hadronic states. In particular, we can use the electron structure function ap-

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proach to compute RC to the elastic and deep-inelastic electron-proton scattering cross sections.

On the other hand, the straightforward calculation in the first order with respect to α [1,2,6] and the recent calculation of the leptonic current tensor in the second order [7-10] for longitudinally polarized initial electron demonstrate that in the leading approximation spin-independent and spin-dependent parts of this tensor are the same for so-called nonsinglet channel contribution. The latter corresponds to photon radiation and e^+e^--pairs production through single-photon mechanism. The difference appears in the second order due to possibility of pair production by double-photon mechanism [10]. Therefore, the representation (2), being slightly modified, can be used for calculation of RC to cross sections of different processes with longitudinally polarized electron beam.

In our recent work [11] we applied this method to compute RC to the ratio of the recoil proton polarizations measured in CEBAF Jefferson Lab [12], where the proton electric form factor G_e was measured. In the present work we use the electron structure function method for calculation of model-independent part of RC to asymmetry in scattering of longitudinally polarized electrons on polarized protons at the level of per mile accuracy for elastic and deep-inelastic hadronic events.

The cross section of the scattering of the longitudinally polarized electron by the proton with given longitudinal (l) or transverse (t) polarization for both elastic and deep-inelastic events can be written as a sum of its spin-independent and spin-dependent parts

$$\frac{d\sigma(k_1,k_2,S)}{dQ^2dy} = \frac{d\sigma(k_1,k_2)}{dQ^2dy} + \eta\,\frac{d\sigma^{l,t}(k_1,k_2,S)}{dQ^2dy}, \qquad (5)$$

where S is 4-vector of the target proton polarization and η is the product of the electron and proton polarization degrees. Herein after we assume $\eta = 1$.

The master equation (2) describes the RC to spin-independent part of the cross section on the right side of Eq. (5) and the corresponding equation for the spin-dependent part reads

$$\frac{d\sigma^{l,t}(k_1,k_2,S)}{dQ^2dy} = \int\limits_{z_{1m}}^{1} dz_1 \int\limits_{z_{2m}}^{1} dz_2 D^{(p)}(z_1,L)D(z_2,L)\times \qquad (6)$$

$$\frac{d\sigma^{l,t}_{hard}(\tilde{k}_1,\tilde{k}_2,S)}{d\tilde{Q}^2d\tilde{y}}, \quad D^{(p)} = D^{\gamma} + D_N^{e^+e^-} + D_S^{e^+e^-(p)},$$

and [10]

$$D_S^{e^+e^-(p)} = \frac{\alpha^2L^2}{4\pi^2}\left[\frac{2(1-z)}{2} + (1+z)\ln z\right]\theta(1-z-2m/\varepsilon).$$

The representation (6) is valid if radiation of collinear particles does not lead to change polarization 4-vectors. In general case it is not so [13] but in this paper we will use just such polarizations which satisfy this conditions (see next Section)

The asymmetry in elastic scattering and DIS processes is defined as the ratio

$$A^{l,t} = \frac{d\sigma^{l,t}(k_1,k_2,S)}{d\sigma(k_1,k_2)},$$

therefore, RC to asymmetry requires the knowledge of RC to both spin-independent and spin-dependent parts.

2. THE LEADING APPROXIMATION

Within the leading accuracy (taking into account terms of the order $(\alpha L)^n$) the electron structure functions can be computed in all orders of the perturbation theory. In this approximation we have to take the Born cross section as hard part in integrands of master Eqs. (2), (6).

We express the Born cross section in terms of leptonic and hadronic tensors as follows

$$\frac{d\sigma}{dQ^2dy} = \frac{4\pi\alpha^2(Q^2)}{VQ^4}L^B_{\mu\nu}H_{\mu\nu}, \qquad (7)$$

where $\alpha(Q^2)$ is the runing electromagnetic constant that takes into account for the effects of vacuum polarization and

$$H_{\mu\nu} = -F_1\tilde{g}_{\mu\nu} + \frac{F_2}{p_1q}\tilde{p}_{1\mu}\tilde{p}_{1\nu} - i\varepsilon_{\mu\nu\lambda\rho}q_{\lambda}\frac{M}{p_1q}[(g_1+g_2)S - g_2\frac{Sq}{p_1q}],$$

$$L^B_{\mu\nu} = -\frac{Q^2}{2}\tilde{g}_{\mu\nu} + k_{1\mu}k_{2\nu} + k_{1\nu}k_{2\mu} + i\varepsilon_{\mu\nu\lambda\rho}q_{\lambda}p_{1\rho},$$

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}, \quad \tilde{p}_{1\mu} = p_{1\mu} - \frac{p_1q}{q^2}. \qquad (8)$$

In Eqs. (8) we assume the proton and electron polarization degrees equal to unity. The proton structure functions F_1, F_2 and g_1, g_2 depends on variables $x'=-q^2/2p_1q$, $q^2=(p_x-p)^2$. In Born approximation, $x' = x$ but they differ in general case, when radiation of photons and electron-positron pairs are allowed.

It is convenient to express the 4-vector of proton polarization in terms of 4-momenta of particles in process

$$S^t_{\mu} = \frac{up_{1\mu} + Vk_{2\mu} - [2u\tau + V(1-y)]k_{1\mu}}{\sqrt{-uV^2(1-y) - u^2M^2}}, \qquad (9)$$

$$S^l_{\mu} = \frac{2M^2k_{1\mu} - Vp_{1\mu}}{MV}, \quad u = -Q^2, \quad \tau = \frac{M^2}{V}.$$

As normalization is chosen, the elastic limit can be reached by a simple substitution in hadronic tensor

$$F_1 \to \frac{1}{2}\delta(1-x)G_m^2, F_2 \to \delta(1-x)\frac{G_e^2 + \lambda G_m^2}{1+\lambda},$$

$$g_1 \to \frac{1}{2}\delta(1-x)\left[G_mG_e + \frac{\lambda(G_m-G_e)G_m}{1+\lambda}\right],$$

$$g_1 \to -\frac{1}{2}\delta(1-x)\frac{\lambda(G_m-G_e)G_m}{1+\lambda}, \quad \lambda = -\frac{q^2}{4M^2},$$

where G_m and G_e are magnetic and electric proton form-factors which depend on q^2.

A simple calculation gives the spin-independent and spin-dependent parts of the cross section in the form

$$\frac{d\sigma^B}{dQ^2dy} = \frac{4\pi\alpha^2(Q^2)}{Q^4y}[(1-y-xy\tau)F_2 + xy^2F_1], \qquad (10)$$

$$\frac{d\sigma_l^B}{dQ^2dy}=\frac{8\pi\alpha^2(Q^2)}{V^2y}\left[\left(\tau-\frac{2-y}{2xy}\right)g_1+\frac{2\tau}{y}g_2\right], \qquad (11)$$

$$\frac{d\sigma_t^B}{dQ^2dy}=-\frac{8\pi\alpha^2(Q^2)}{V^2y}\sqrt{\frac{M^2}{Q^2}(1-y-xy\tau)}\left(g_1+\frac{2}{y}g_2\right),$$
(12)

Thus, within the leading accuracy, the radiatively corrected cross section of the process (1) is defined by Eq. (2) for its spin-independent piece with (10) as a hard cross section into integrand and by Eq. (6) with (11) or (12) as the hard part for its spin-dependent piece.

It is useful to extract the leading first order correction to the Born approximation as defined by master Eqs. (2), (6). For this purpose we can use the structure function D^γ with $L\to L-1$ and

$$\Delta\to\Delta_1=\frac{2(\Delta\varepsilon)}{\sqrt{V}(1-xy)}\sqrt{\tau+z_+},\; z_+=y(1-x),$$

for $D(z_1,L)$ and

$$\Delta\to\Delta_2=\frac{2(\Delta\varepsilon)}{\sqrt{V}(1-z_+)}\sqrt{\tau+z_+},\quad \frac{2(\Delta\varepsilon)}{\sqrt{V}}<<1$$

for $D(z_2,L)$, where $(\Delta\varepsilon)$ is the minimal energy of hard collinear photon in special system where $k_1-k_2+p_1=0$. The straightforward calculation yields the following expressions

$$\frac{d\sigma^{(1)}(k_1,k_2)}{dQ^2dy}=\frac{\alpha(L-1)}{2\pi}\Big\{\frac{d\sigma^{(B)}(k_1,k_2)}{dQ^2dy}\Big[3+2\ln\frac{4(\Delta\varepsilon)^2(z_++\tau)}{V(1-xy)(1-z_+)}\Big]+\int\limits_{z_{th}}^{z_+-\rho}dz\Big[\frac{1+z_1^2}{1-z_1}\frac{d\sigma^{(B)}(z_1k_1,k_2)}{(1-xy)dQ_t^2dy_t}+\frac{1+z_2^2}{1-z_1}\frac{d\sigma^{(B)}(k_1,k_2/z_2)}{(1-z_+)dQ_s^2dy_s}\Big]\Big\}$$

where

$$z=\frac{M_x^2-M^2}{V},\; z_1=\frac{1-y+z}{1-xy},\; z_2=\frac{1-z_+}{1-z},\; y_{t,s}=1-\frac{1-y}{z_{1,2}},$$

$$\rho=\frac{2(\Delta\varepsilon)}{\sqrt{V}}\sqrt{\tau+z_+},\; Q_t^2=-q_t^2=z_1Q^2,\; Q_s^2=-q_s^2=\frac{Q^2}{z_2}.$$

Just the same contribution can be derived also for spin-dependent part of the cross section.

3. DIS CROSS SECTION BEYOND THE LEADING ACCURACY

To go beyond the leading accuracy we have to improve the expressions for the hard part of the cross sections into master equations (2) and (6) to include effects caused by radiation of hard non-collinear photon.

To compute the improved hard cross section, one has to find the full first order correction in process (1) and subtract from it its leading contribution, which is defined by above expression, to get the double counting. Therefore, the improved hard part can be written as follows

$$\frac{d\sigma_{hard}}{dQ^2dy}=\frac{d\sigma^B}{dQ^2dy}+\frac{d\sigma^{(S+V)}}{dQ^2dy}+\frac{d\sigma^H}{dQ^2dy}-\frac{d\sigma^{(1)}}{dQ^2dy},$$

where $d\sigma^{(S+V)}$ is a correction to cross section of process (1) virtual and soft photon emission and $d\sigma^H$ is the the cross section of radiative process

$$e^-(k_1)+P(p_1)\to e^-(k_2)+\gamma(k)+X(p_x)$$

The virtual and soft corrections are factorized in the same form for both polarized and unpolarized cases and can be written as

$$\frac{d\sigma^{(S+V)}}{dQ^2dy}=\frac{d\sigma^B}{dQ^2dy}\left[1+\frac{\alpha}{2\pi}\left(\delta+(L-1)\left(3+2\ln\frac{\rho^2}{(1-xy)(1-z_+)}\right)\right)\right], \qquad (13)$$

$$\delta=-1-\frac{\pi^2}{3}-2f\left(\frac{1-y-xy\tau}{(1-xy)(1-z_+)}\right),\quad f(x)=\int\limits_o^x\frac{dt\ln(1-t)}{t}.$$

To calculate the cross section of the radiative process, we use the corresponding leptonic tesor in the form

$$L_{\mu\nu}^\gamma=\frac{\alpha}{4\pi^2}\left(L_{\mu\nu}^{H(un)}+L_{\mu\nu}^H\right)\frac{d^3k}{\omega},$$

$$L_{\mu\nu}^H=2i\varepsilon_{\mu\nu\lambda\rho}q_\lambda(k_{1\rho}R_t+k_{2\rho}R_s),$$

$$R_s=\frac{u+s}{st}-2m^2\frac{s_t}{ut^2},\quad s_t=\frac{-u(u+Vy-Vz)}{u+V},$$

$$R_t=\frac{u+t}{st}-2m^2\left(\frac{1}{t^2}+\frac{1}{s^2}\right),$$

where ω is the energy of radiated photon, $L_{\mu\nu}^{H(un)}$ is the leptonic tensor for unpolarized particles, see [14], and we use the following notation for invariants

$$s=2kk_2,\quad t=-2kk_1,\quad q^2=s+t+u.$$

The result for unpolarised case reads [5]

$$\frac{d\sigma_{hard}}{dQ^2dy}=\frac{d\sigma^B}{dQ^2dy}\left(1+\frac{\alpha}{2\pi}\delta\right)+\frac{\alpha}{VQ^2}\int\limits_{z_{th}}^{z_+}dz\Big\{\frac{1-r_1}{1-xy}\hat{P}_tN-\frac{1-r_2}{1-z_+}\hat{P}_sN+\int\limits_{r_-}^{r^+}dr\frac{2W}{\sqrt{y^2+4xy\tau}}+$$

$$\mathrm{P}\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\Big[\frac{1-\hat{P}_t}{|r-r_1|}\left(\frac{(1+r^2)N}{1-xy}+(r_1-r)T_t\right)-\frac{1-\hat{P}_s}{|r-r_2|}\left(\frac{(1+r^2)N}{1-z_+}+(r_2-r)T_s\right)\Big]\Big\}\frac{\alpha^2(rQ^2)}{r^2}, \qquad (14)$$

where $r=-q^2/Q^2$ and the limits of integration with respect to γ are

$$r_\pm(z)=\frac{2xy(\tau+z)+(z_+-z)(y\pm\sqrt{y^2+4xy\tau})}{2xy(\tau+z_+)}.$$

Here we used the following notation

$$N=2F_1+\frac{2\tilde{x}}{rxy}\left(\frac{1-y}{xy}-\tau\right)F_2,\quad W=2F_1-\frac{2\tilde{x}\tau}{rxy}F_2,$$

$$T_t=-\frac{2\tilde{x}[1-r(1-y)]}{x^2y^2r}F_2,\quad T_s=-\frac{2\tilde{x}(1-y-r)}{x^2y^2r},$$

where the hadron structure functions F_1 and F_2 depend on $\tilde{x}$ and q^2. Besides,

$$r_1=\frac{1-y+z}{1-xy},\quad r_2=\frac{1-z}{1-z_+},\quad \tilde{x}=\frac{xyr}{xyr+z}.$$

The action of the operators $\hat{P}_t$ and $\hat{P}_s$ is defined as follows

$$\hat{P}_t f(r,x\tilde{y}) = f(r_1,x_t), \quad \hat{P}_s f(r,x\tilde{y}) = f(r_2,x_s),$$

$$x_t = \frac{xyr_1}{xyr_1 + z}, \quad x_s = \frac{xyr_2}{xyr_2 + z}.$$

Note that quantity $r_1(r_2)$ coinsides with $z_1(1/z_2)$ for radiation of a single collinear photon. The hard cross section (14) has neither collinear nor infrared singularities. The different terms on the right-hand side of Eq. (14) have singularities at $r=r_1$, $r=r_2$ and $\gamma=1$. Singularities at first two points are collinear and at third one is unphysical that arise at integration. Collinear singulariries vanish due to action of operators $\hat{P}_t$ and $\hat{P}_s$ on the terms containing N. The unphysical singulariry cancels because in the limiting case $r \to 1$ we have

$$\frac{r_2 - r}{|r_2 - r|} = 1, \quad \frac{r_1 - r}{|r_1 - r|} = -1, \quad T_t + T_s = 0.$$

The corresponding result for spin-dependent hard cross section can be written by very similar form

$$\frac{d\sigma^{l,t}_{hard}}{dQ^2dy} = \frac{d\sigma^{B}_{l,t}}{dQ^2dy}\left(1 + \frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{Q^4}U^{l,t}\int\limits_{z_{th}}^{z_+} dz\left\{\frac{1-r_1}{1-xy}\hat{P}_t N_t^{l,t}\right.$$

$$- \frac{1-r_2}{1-z_+}\hat{P}_s N_s^{l,t} + \int\limits_{r_-}^{r^+} dr\frac{2W^{l,t}}{\sqrt{y^2+4xy\tau}} +$$

$$\mathrm{P}\int\limits_{r_-}^{+}\frac{dr}{1-r}\left[\frac{1-P_s}{|r-r_2|(1-z_+)}\left((1+r^2)N_s^{l,t} + \frac{2(r_2-r)}{r_2}T_s^{l,t}\right) - \right.$$

$$\left.\left.\frac{1-\hat{P}_t}{|r-r_1|}\left(\frac{(1+r^2)N_t^{l,t}}{1-xy} + 2r(r_1-r)T_t^{l,t}\right)\right]\right\}\frac{x\tilde{y}\alpha^2(rQ^2)}{r^3}, \quad (15)$$

where

$$U^l = 1, \quad U^t = \sqrt{\frac{M^2}{Q^2}(1-y-xy\tau)^{-1}}, \quad W^l = 4y\tau W,$$

$$W^t = 2y^2(1+2x\tau)W, \quad W = (1+r)xg_1 + x\tilde{y}g_2,$$

$$N_t^l = 2[2r - z - xy(r+2\tau)]g_1 - 8x\tilde{y}\tau g_2,$$

$$N_s^l = 2[2 - z - xyr(1+2\tau)]g_1 - 8x\tilde{y}\tau g_2,$$

$$N_t^t = 2[1 - y - z + r - xy(r+2\tau)](xyg_1 + 2x\tilde{y}g_2),$$

$$N_s^t = 2\left[1 - y - xy(1+2\tau) + \frac{1-z}{r}\right](xyrg_1 + 2x\tilde{y}g_2),$$

$$T_t^l = 2rg_1 - 4x'\tau g_2, \quad T_s^l = 2(z-1)(g_1 - 2x'\tau g_2),$$

$$T_t^t = 2xyrg_1 + 2x\tilde{y}(1 - y + r - 2xy\tau)g_2,$$

$$T_s^t = 2(z-1)\left[xyg_1 + x\tilde{y}\left(1 - y + \frac{1}{r} - 2xy\tau\right)g_2\right].$$

The polarized hard cross section (15) as well is free from any singularities. Note that radiation of photon at large angles by the initial and final electrons increases the region of variation for quantity r in (14) and (15), because for collinear radiation $r_1 < r < r_2$ and now we have $r_- < r_1$ and $r_+ > r_2$. It may be important if the hadron structure functions are large in these additional regions.

4. HARD CROSS SECTION FOR ELASTIC HADRONIC EVENTS

To describe the hard cross section for elastic hadronic events we use the replacement given after formulae (9) in expressions (14) and (15). For Born cross sections, which enter in these equations, see Eqs. (10-12). The function $\delta(1-x')$ is used to perform the integration with respect to inelasticity z.

The final result for unpolarized case has the following form (we do not introduce special notation for the elastic cross-section)

$$\frac{d\sigma_{hard}}{dQ^2dy} = \frac{d\sigma^{B}}{dQ^2dy}\left(1 + \frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{V^2}\left\{\frac{1-r_1}{1-xy}\hat{P}_t N - \frac{1-r_2}{1-z_+}\hat{P}_s N\right.$$

$$+ \int\limits_{r_-}^{r^+} dr\frac{2W}{\sqrt{y^2+4xy\tau}} + \mathrm{P}\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\left[\frac{1-\hat{P}_t}{|r-r_1|}\left(\frac{(1+r^2)N}{1-xy} + (r_1\right.\right.$$

$$\left.\left.\left. - r)T_t\right) - \frac{1-\hat{P}_s}{|r-r_2|}\left(\frac{(1+r^2)N}{1-z_+} + (r_2 - r)T_s\right)\right]\right\}\frac{\alpha^2(rQ^2)}{r}, \quad (16)$$

where

$$N = G_m^2 + \frac{2(1-y-xy\tau)(G_e^2 + \lambda G_m^2)}{x^2y^2r(1+\lambda)},$$

$$T_t = \frac{2[r(1-y)-1](G_e^2 + \lambda G_m^2)}{x^2y^2r(1+\lambda)},$$

$$T_s = \frac{2[r+y-1](G_e^2 + \lambda G_m^2)}{x^2y^2r(1+\lambda)}, \quad W = G_m^2 - \frac{2\tau(G_e^2 + \lambda G_m^2)}{xy\lambda(1+\lambda)}.$$

The Born cross section on the right-hand side of Eq. (16) is defined as

$$\frac{d\sigma^{B}}{dQ^2dy} = \frac{2\pi\alpha^2(Q^2)}{V^2}\Big\{$$

$$\left[G_m^2 + \frac{2[1-y(1+\tau)](G_e^2 + \lambda G_m^2)}{y^2(1+\lambda)}\right]\delta\left(y - \frac{Q^2}{V}\right).$$

When writing this last equation we take into account that $\delta(1-x) = y\delta(1-Q^2/V)$.

The spin-dependent hard cross section for elastic hadronic events can be written in the very similar to (16) form

$$\frac{d\sigma^{l,t}_{hard}}{dQ^2dy} = \frac{d\sigma^{B}_{l,t}}{dQ^2dy}\left(1 + \frac{\alpha}{2\pi}\delta\right) + \frac{\alpha}{V}U^{l,t}\left\{\frac{1-r_1}{1-xy}\hat{P}_t N_t^{l,t} + \right.$$

$$\frac{1-r_2}{1-z_+}\hat{P}_s N_s^{l,t} + \int\limits_{r_-}^{r^+} dr\frac{W^{l,t}}{\sqrt{y^2+4xy\tau}} + \mathrm{P}\int\limits_{r_-}^{r_+}\frac{dr}{1-r}\left[-\frac{1-\hat{P}_t}{|r-r_1|}\Big\{\right.$$

$$\left(\frac{(1+r^2)N_t^{l,t}}{1-xy} + 2r(r_1-r)T_t^{l,t}\right) + \frac{1-\hat{P}_s}{|r-r_2|(1-z_+)}\Big\{$$

$$\left.\left.\left((1+r^2)N_s^{l,t} + \frac{2(r_2-r)}{r_2}T_s^{l,t}\right)\right]\right\}\frac{\alpha^2(rQ^2)}{(4M^2+Q^2r)r^2},$$

$$W^l = 4y\tau W, \quad W^t = 2y^2(1+2x\tau)W,$$

$$W = r[x(1+r)-1]G_m^2 + \left[r + \frac{4\tau}{y}(1+r)\right]G_mG_e,$$

$$T_t^l = r\left[(r+2\tau)G_m^2 + 2\tau\left(\frac{2}{xy} - 1\right)G_mG_e\right],$$

$$T_s^l = -r(1+2\tau)G_m^2 - 2\tau\left(\frac{2}{xy} - r\right)G_mG_e,\ T_t^t = r\{-[r(1-xy)+$$
$$1-y-2xy\tau)]G_m^2 + (1-y-2xy\tau+r+4\tau)G_mG_e\},\quad T_s^t = r[\frac{1}{r}-$$
$$xy(1+2\tau)+1-y]G_m^2 - [2\tau(2-xyr)+1+r(1-y)]G_mG_e,$$
$$N_t^l = r(2\tau+r)(2-xy)G_m^2 + 8\tau\left[r\frac{1-xy}{xy} - \tau\right]G_mG_e,$$
$$N_s^l = r(2\tau+1)(2-xyr)G_m^2 + 8\tau\left[\frac{1}{xy}r(1+\tau)\right]G_mG_e,$$
$$N_t^t = [1-y+r-xy(r+2\tau)][2(r+2\tau)G_mG_e -$$
$$r(2-xy)G_m^2,\quad N_t^t = [1-y+\frac{1}{r}-xy(1+2\tau)]\times$$
$$[2r(1+2\tau)G_mG_e - r(2-xyr)G_m^2].$$

Note that argument of electromagnetic formfactors in Eqs. (15) and (16) is $-Q^2r$.

The Born cross sections on the right-hand side of Eq. (16) have the following form

$$\frac{d\sigma_l^B}{dQ^2dy} = \frac{4\pi\alpha^2(Q^2)}{V(4M^2+Q^2)}\Big[4\tau\left(1+\tau-\frac{1}{y}\right)G_mG_e - (1+2\tau)$$
$$\times\left(1-\frac{y}{2}\right)G_m^2\Big]\delta\left(y-\frac{Q^2}{V}\right)$$

for longitudinally polarized of the target proton and

$$\frac{d\sigma_t^B}{dQ^2dy} = \frac{8\pi\alpha^2(Q^2)}{V(4M^2+Q^2)}\sqrt{\frac{M^2}{Q^2}[1-y(1+\tau)]}\times$$
$$\Big[\left(1-\frac{y}{2}\right)G_m^2 - (1+2\tau)G_mG_e\Big]\delta\left(y-\frac{Q^2}{V}\right),$$

for transverse one. The argument of form factors in the last two formulae is $-Q^2$.

In this paper we consider model-independent QED radiative correction to the polarized DIS and elastic electron-proton scattering. Our calculations are based on the electron structure function method which allows to write both the spin-independent and spin-dependent parts of the cross section with accounting RC to the leptonic part of interaction in the form of well-known Drell-Yan representation. The corresponding RC includes explicitly the first order correction as well as the leading-log contribution in all orders of perturbation theory and the main part of the second order next to leading-log one. Moreover, any model-dependent RC to the hadronic part of interaction can be included in our analytical result by insertion it as an additive part of the hard cross section in integrand in master equations.

To derive RC, we take into account radiation of photons and e^+e^--pairs in collinear kinematics which produces a large logarithm L in the radiation probability (in D-functions) and radiation of one non-collinear photon that enlarges the limits of variation of the hadron structure function arguments. It may be important that these functions are sharp enough. In this case the loss in radiation probability (the loss of L) can be compensated by the increase in the value of the hard cross section.

On the basis of our analytical result we constructed Fortran code ESFRAD (http://www.jlab.org/~aku/RC) and perform some numerical estimations [15] for kinematical conditions of current and future experiments. We found two regions where the higher order radiative corrections can be important. These are the traditional region of high y and the region around the pion threshold.

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