THE SCANNING OF HADRONIC CROSS-SECTION IN e^+e^--ANNIHILATION BY RADIATIVE RETURN METHOD

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The possibility of high precision measurement of the total hadronic cross-section in electron-positron annihilation process by analysis of the initial-state radiative events is discussed. Different experimental setups are discussed. The main attention is dedicated to measurement of hadronic cross-section at DAPhNE, where the final hadronic state $\pi^+\pi^-$ dominates due to radiative return on ρ -resonance. Radiative corrections at one percent level accuracy are calculated taking into account real experimental constraints on event selection.

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In the interpretation of the recent high precision measurement of muon anomalous magnetic moment [1] the large uncertainty arouse in determine the contribution of hadron vacuum polarization contribution in (g–2) $_\mu$ [2,3].

The problem of hadron vacuum polarization contribution is that it cannot be calculated analytically because QCD loses its predictable force at low and intermediate energies, where effect must be considerable. Therefore, the only possibility to calculate it is the utilization of the measured total hadronic cross section in electron-positron annihilation, which connected with hadron vacuum polarization by dispersive relation. The necessary condition for such calculation is the knowledge of total hadronic cross section with one per cent accuracy and even better.

In the works [4,5] the radiative return method of the scanning of the total hadronic cross section in –- annihilation discussed just in connection with considered problem. For energies below 1 GeV DAPhNE machine is ideal to use this method due to its high luminosity and effect of radiative return on ρ - resonance. In [6] the analysis initial state radiation (IRS) events, has been performed for DAPhNE condition when both the energy fraction of photon inside calorimeter and the invariant mass of $\pi^+\pi^-$ -system are measured.

Because of geometry of the used at DAPhNE KLOE detector the most of ISR events, when photon belong to the blind zone, are inaccessible for detection and cannot be recorded by KLOE calorimeters. This circumstance decreases the event statistics and, respectively, the measurement precision. To avoid this problem G. Venanzoni suggested to use inclusive event selection (IES) method, when only invariant mass of the final pions is measured. The main advantage of IES is the increase of the corresponding cross section due to $\ln(E^2/m^2)$ enhancement (here E is the beam energy and m is the electron mass). To avoid uncertainties in interpretation of IES approach, some additional rules for event selection must be established.

Now to calculate analytically the Born cross-section of ISR process

$$e^-(p_1) + e^+(p_2) \to \gamma(k) + \pi^+(p_+) + \pi(p_-)$$

and the QED radiative corrections (RC) to it for IES experimental setup, taking into account appropriate additional constraints for event selection, which can be realized at DAPhNE accelerator.

1. IES SELECTION RULES

The main condition for IES approach is the precise measurement of pion invariant mass. It can be done by means of selection events with small difference between the lost (undetected) energy and the lost 3-momentum modulus in process (1).

$$2E - E_+ - E_- - \left|\vec{P}_\Phi - \vec{p}_+ - \vec{p}_-\right| < \eta E, \ \eta << 1,$$

$E_\pm$ is the energy of $\pi_\pm$, and P_Φ is the total initial-state 3-momentum which appears because laboratory system at DAPhNE does not coincides with the center mass system, $\left|\vec{P}_\Phi\right| = 12.5$ MeV. The constraint (2) avoids undetected π^0 and keeps only undetected $n\gamma$ system. The inequality (2) can be rewritten in terms of the total energy Ω and total 3-momentum modulus $\left|\vec{K}\right|$ of all photons in reaction $e^+ + e^- \to \pi^+ + \pi^- + n\gamma$ as

$$\Omega - \left|\vec{K}\right| < \eta \ E$$

The optimal value η =0.02 decreases also FSR background. The next constraint (collinear) selects events when at n=1 undetected photon is belongs to the narrow cone along the electron beam direction.

$$\vec{K}\,\vec{p}_1 > \left|\vec{K}\right| E\, c_0 \, , c_0 = \cos\theta_0 \, ,$$

where θ_0 can be chosen for about 5^o - 6^o. Due to this constraint the collinear photon radiated by the initial electron contributes into observed IES cross section and exhibits itself by $\ln(E^2\theta_0^2/m^2)$ enhancement.

Because of existence of the blind zone, KLOE detector picks out events with pion polar angles in diapason $\theta_m < \theta_\pm < \pi - \theta_m$. As show the Monte Carlo calculations [5] the choice of θ_m influences also on the value of FRS background. The optimal value of θ_m for DAPhNe conditions is 20^o. Ordinary the reduced pion phase space can be taken into account by introduction of respective acceptance factor $A(\theta_m)$.

2. BORN APPROXIMATION

To lowest order in α, the differential cross section of the process(1) can be written in terms of contraction of leptonic $L_{\mu\nu}$ and hadronic $H_{\mu\nu}$ tensors as

$$d\sigma^B = \frac{8\pi^2\alpha^2}{sq^4} L^{\gamma}_{\mu\nu}(p_1,p_2,k) H_{\mu\nu} \times \frac{\alpha}{4\pi^2}\frac{d^3k}{\omega}\frac{d^3p_+\, d^3p_-}{16\pi^2 E_+E_-}\delta(q-p_+-p_-),$$

where ω is the energy of photon and

$$H_{\mu\nu} = -4\left|F_\pi(q^2)\right|^2 \tilde{p}_{-\mu}\tilde{p}_{-\nu},\ \tilde{p}_{-\mu} = p_{-\mu} - \frac{1}{2}q_\mu,$$

$$q = p_1 + p_2 - k = p_+ + p_-$$

The pion electromagnetic form factor defines the total cross section $\sigma(q^2)$ of the process $e^+ + e^- \to \pi^+ + \pi^-$, that is the subject of experimental investigation, by means of relation

$$\left|F_\pi(q^2)\right|^2 = \frac{3q^2\sigma(q^2)}{\pi\alpha^2\varsigma},\ \varsigma = \left(1 - \frac{4m_\pi^2}{q^2}\right)^{\frac{3}{2}}.$$

The leptonic tensor for the case of collinear ISR along the electron beam direction is well known [7]:

$$L^{\gamma}_{\mu\nu}(p_1,p_2,k) = \left[\frac{\left(q^2-t_1\right)^2 + \left(q^2-t_2\right)^2}{t_1t_2} - \frac{2m^2q^2}{t_1^2}\right]\tilde{g}_{\mu\nu} + \frac{4q^2}{t_1t_2}\tilde{p}_{1\mu}\tilde{p}_{1\nu} + \left(\frac{4q^2}{t_1t_2} - \frac{8m^2}{t_1^2}\right)\tilde{p}_{2\mu}\tilde{p}_{2\nu},$$

where $t_1 = -2kp_1$, $t_2 = -2kp_2$, $s = 2p_1p_2$.

Then, we derive the distribution over pion squared invariant mass q^2 for non- cut pion phase space

$$\frac{d\sigma^B_F}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\frac{\alpha}{2\pi}\left[\left(\frac{q^4}{8\omega_0E} + \frac{q^2}{2E^2} + \frac{\omega_0}{E}\right)\left(1 + \frac{P_\Phi^2}{4E^2}\right)L_0 - \frac{q^2}{2\omega_0E} + \frac{\theta_0^2}{6}\left(\frac{q^4}{8\omega_0E} + \frac{q^2}{2E^2} - \frac{2\omega_0}{E}\right)\right].$$

To guarantee only one percent accuracy one can remove terms proportional to $P_\Phi^2/4E^2$ and $\theta_0^2/6$:

$$\frac{d\sigma^B_F}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\frac{\alpha}{2\pi}P\left(z,L_0\right),\ L_0 = \ln\left(\frac{E^2\theta_0^2}{m^2}\right),$$

$$P\left(z,L_0\right) = \frac{1+z^2}{1-z}L_0 - \frac{2z}{1-z},\ z = \frac{q^2}{4E^2}.$$

In the case of reduced pion phase space the form of distribution is more complicated. But the result is also simplified essentially if to neglect terms of the order $P_\Phi^2/4E^2$ and use QRE approximation,

$$\frac{d\sigma^B_R}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\frac{\alpha}{2\pi}P\left(z,L_0\right)A\left(z,c_m\right),$$

where $A\left(z,c_m\right)$ - acceptance factor

$$A\left(z,c_m\right) = \frac{12}{\varsigma}\int\limits_{-c_m}^{c_{\max}} dc_-\, U\frac{z\left[\left(1+z\right)K - \left(1-z\right)c_-\right]^2}{K\left[\left(1+z\right)^2 - \left(1-z\right)^2c_-^2\right]^2}.$$

Here the notations are introduced

$$U = \frac{\chi_1}{4E^2} - \frac{\chi_1^2}{16E^4} - \frac{m_\pi^2}{4zE^2},$$

$$K = \sqrt{1 - \frac{\delta^2}{z^2}\left[\left(1+z\right)^2 - \left(1-z\right)^2c_-^2\right]},$$

$$\frac{\chi_1}{4E^2} = \frac{z\left[1+z-2Kc_- + \left(1-z\right)c_-^2\right]}{\left(1+z\right)^2 - \left(1-z\right)^2c_-},\ \delta^2 = \frac{m_\pi^2}{4E^2},$$

$$c_{\max}\left(z,c_m\right) = \frac{\left(1+z\right)g}{\sqrt{\left(z - \left(1-z\right)g\right)^2 - \left(1+z\right)^2\delta^2}},$$

$$g = \frac{zc_m\left[\left(1+z\right)K\left(c_m\right) + \left(1-z\right)c_m\right]}{\left(1+z\right)^2 - \left(1-z\right)^2c_m^2} - \frac{1-z}{2}.$$

The acceptance factor as a function of pion squared invariant mass is shown on Fig. 1 for $\theta_m = 10^o, 20^o$.

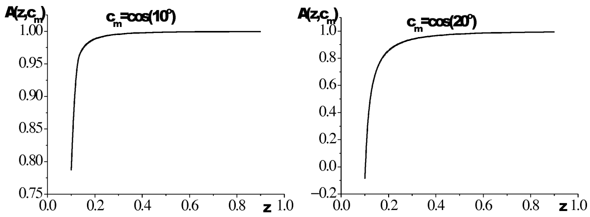

Fig. 1. *Acceptance factor defined by Eq. (8) for different* c_m

3. RADIATIVE CORRECTIONS

If events with $e^+e^-\pi^+\pi^-$ final state are rejected, only photonic RC have to be taken into account. These corrections include contributions due to virtual and real soft and hard photon emission. The soft and virtual corrections are the same for non-cut and reduced pion phase

$$\frac{d\sigma^{S+V}_F}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 C^{S+V},$$

$$\frac{d\sigma^{S+V}_R}{dq^2} = \frac{d\sigma^{S+V}_F}{dq^2}A\left(z,c_m\right),$$

$$C^{S+V} = \rho\, P\left(z,L_0\right) + D\left(L_s,L_0,z\right) + N\left(z\right).$$

All logarithmic strengthen contributions in expression for C^{S+V} are concentrated in first two terms [4]. The third term is responsible for non-logarithmic contribution [8].

As concerns contribution into RC due to additional hard photon emission, we divide it by three parts. The first one is responsible for radiation of additional photon with the energy ω_2 along the positron beam direction (provided that collinear photon with the energy ω_1 is emitted along the electron beam direction).

$$\frac{d\sigma^H_{1F}}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 P\left(z,L_0\right)2\left(L_0^{'} - 1\right)\ln\left(\frac{\eta}{2\Delta}\right),$$

$$\frac{d\sigma^H_{1R}}{dq^2} = \frac{d\sigma^H_{1F}}{dq^2} A\left(z, c_m\right). \tag{10}$$

The second part of contribution into RC caused by additional hard photon emission is responsible for radiation of two hard collinear photons (every with the energy more than ΔE) by the electron, provided that both belongs to narrow cone with the opening angle $2\theta_0$ along the electron beam direction.

$$\frac{d\sigma^H_{2F}}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 C^H, \quad \frac{d\sigma^H_{2R}}{dq^2} = \frac{d\sigma^H_{2F}}{dq^2} A\left(z, c_m\right),$$

$$C^H = B_1\left(z, \Delta\right) L_0^2 + B_2\left(z, \Delta\right) L_0 + B_3\left(z, \Delta\right). \tag{11}$$

Functions $B_1\left(z, \Delta\right)$, $B_2\left(z, \Delta\right)$ and $B_3\left(z, \Delta\right)$ were calculated in [8,9].

The third, most non-trivial for calculation, part of contribution into RC caused by two hard photon emission is connected with events when photon with the energy ω_1 is collinear and the other one (with the energy ω_2) covers angles between $\pi - \theta_0'$ and θ_0. The respective contribution into RC destroys the acceptance factor as given by Eq. (8).

To use QRE approach for description of collinear photon it is necessary to take into consideration the restrictions for event selection and the inequalities

$$-c_0' < c_2 < c_0, \; E\Delta < \omega_1 < \Omega - E\Delta_1, \; c_0' = \cos\theta_0' \tag{12}$$

for possible angles of the non-collinear photon and energies of the collinear one.

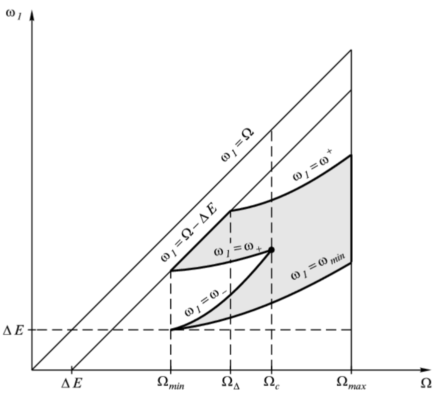

***Fig. 2.** The integration region with respect to ω_1 and Ω as given by inequalities (3,4,12)*

From these restrictions we can determine the integration region for ω_1 and Ω, see Fig. 2. Here

$$\Omega_{\min} = E\left(1 - z\right)\left(1 + \frac{\Delta\left(1 - c_0\right)}{2}\right),$$

$$\Omega_\Delta = E\left(1 - z\right)\left(1 + \Delta\right), \; \Omega_{\max} = E\left(1 - z\right)\left(1 + \frac{\eta}{2}\right),$$

$$\Omega_c = E\left(1 - z\right)\left(1 + \frac{\left(1 - c_0\right)\left(1 - z\right)}{8}\right), \tag{13}$$

$$\omega_{\min} = \frac{2E\Omega_z}{\Omega - \left|\vec{K}\right| c_0}, \; \left|\vec{K}\right| = \sqrt{\Omega^2 - 4E\Omega_z},$$

$$\omega^\pm = \frac{\Omega}{2}\left[1 \pm \sqrt{1 - \frac{4E\Omega_z}{\Omega^2}\left(1 + \frac{1 - c_0'}{2}\right)}\right],$$

$$\omega_\pm = \frac{\Omega}{2}\left[1 \pm \sqrt{1 - \frac{8E\Omega_z}{\Omega^2\left(1 - c_0\right)}}\right]. \; \Omega_z = \Omega - E\left(1 - z\right)$$

To executed the integration with respect to ω_1 and Ω over region (13) we can write

$$\frac{d\sigma^H_{3F}}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 \left[2P\left(z, L_0\right) G_1 + L_0 G_2 + G_3\right],$$

$$G_1 = \frac{\pi^2}{2} + \frac{1}{2}\ln^2\frac{1 - c_0}{2} - \ln\frac{\eta}{2\Delta}\ln\frac{\left(1 - c_0\right)\left(1 - c_0'\right)}{4} - \ln^2\xi,$$

$$\xi = \frac{\eta}{\left(1 - c_0\right)\left(1 - z\right)},$$

$$G_2 = \left(1 - z\right)\left[\ln\frac{\left(1 + \xi\right)\left(1 + \xi z\right)}{\xi^2} - 2\right] + \left(1 + z\right)\Big[\left(1 + \ln\xi\right)\ln z$$

$$+ Li_2\left(-\xi z\right) + Li_2\left(-\frac{1 - z}{z}\right) - Li_2\left(-\xi\right) - Li_2\left(1 - z\right)\Big],$$

$$G_3 = -\left(1 - z\right)\left(\frac{\pi^2}{3} - \ln^2\xi\right) + 2Li_2\left(-\xi\right) +$$

$$2z\left[Li_2\left(1 - z\right) - Li_2\left(-\xi z\right) - Li_2\left(-\frac{1 - z}{z}\right)\right]. \tag{14}$$

Unfortunately, the calculations in the case of reduced pion phase space cannot be performed analytically. Nevertheless, the dependence on unphysical auxiliary parameters Δ and θ_0', which have to vanish in final result for total RC, can be extracted.

To extract Δ-dependence it is enough investigate limit $\omega_2 \to 0$. The cross section we rewrite as the sum of its hard and soft parts

$$\frac{d\sigma^H_{3R}}{dq^2} = \frac{d\sigma^{H_h}_{3R}}{dq^2} + \frac{d\sigma^{H_s}_{3R}}{dq^2}, \quad \frac{d\sigma^{H_h}_{3R}}{dq^2} = \frac{d\sigma^H_{3R}}{dq^2} - \frac{d\sigma^{H_s}_{3R}}{dq^2},$$

The hard part of the cross section is not singular at $\omega_2 \to 0$, whereas just integration of the soft one over the region (47) extracts all Δ - dependence.

$$\frac{d\sigma^{H_s}_{3R}}{dq^2} = \frac{\sigma\left(q^2\right)}{4E^2}\left(\frac{\alpha}{2\pi}\right)^2 P\left(z, L_0\right) G_1 A\left(z, c_m\right), \tag{15}$$

This soft part absorbs also all dependence on angular auxiliary parameter θ_0'. That is why the hard part of the cross section depends on physical parameters only and can be computed numerically.

4. TOTAL RADIATIVE CORRECTIONS

The total RC to the Born cross section in the case of non-cut and reduced pion phase space is represented by

$$\frac{d\sigma^{RC}_{F,R}}{dq^2} = \frac{d\sigma^{S+V}_{F,R}}{dq^2} + \frac{d\sigma^H_{1F,R}}{dq^2} + \frac{d\sigma^H_{2F,R}}{dq^2} + \frac{d\sigma^H_{3F,R}}{dq^2}, \tag{16}$$

Since both auxiliary parameters, infrared Δ and collinear angular θ_0', are canceled in Eq. (16), the total RC depends only on physical parameters and can be written in the case of non-cut phase space as

$$\frac{d\sigma_F^{RC}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\frac{\alpha}{2\pi}P(z,L_0)\,\delta_F^{RC}, \qquad (17)$$

$$\delta_F^{RC} = \frac{\alpha}{2\pi}\frac{F_0 + L_0F_1 + F_2}{P(z,L_0)},$$

$$F_0 = \frac{1}{2}L_0^2P_{2\theta}(z) + P(z,L_0)\Big[L_s\Big(\frac{3}{2} + 2\ln\frac{\eta}{2}\Big) + \ln\frac{4}{\theta_0^2}\Big(\frac{3}{2} + \ln\frac{\eta}{z\theta_0}\Big) - 2\ln^2\xi - 2\ln\frac{\eta}{2} + 3\ln z + \frac{5\pi^2}{3} - \frac{9}{2}\Big],$$

$$F_1 = \frac{3 - 8z + z^2}{2(1-z)} - \frac{2(1+z)^2}{1-z}\ln(1-z) + \Big[\frac{4z}{1-z} + (1+z)(1+\ln\xi)\Big]\ln z + \frac{1}{2}(1+z)\ln^2 z +$$
$$\frac{1+z^2}{1-z}\Big(\frac{\pi^2}{3} - 2Li_2(z)\Big) + (1-z)\ln\frac{(1+\xi)(1+\xi z)}{\xi^2} +$$
$$(1+z)\Big[Li_2(-\xi z) + Li_2\Big(-\frac{1-z}{z}\Big) - Li_2(-\xi) - Li_2(1-z)\Big],$$

$$F_2 = (1-z)\ln^2\xi - \frac{3+4z}{3(1-z)} + \frac{4z}{1-z}\ln(1-z) +$$
$$\Big[-2z\ln\xi + \frac{3 - 18z + 7z^2}{3(1-z)^2}\Big]\ln z + 2z\ln^2(1-z) +$$
$$\frac{3 - 12z + 30z^2 - 36z^3 + 7z^4}{6(1-z)^3}\ln^2 z + \frac{\pi^2}{6}\Big(4 + \frac{14z}{3} - \frac{5}{1-z}\Big)$$
$$+ Li_2(z)\Big(4z - 6 + \frac{5}{1-z}\Big) + \Big(1 - \frac{z}{3}\Big)Li_2(1-z) +$$
$$2Li_2(-\xi) - 2z\Big[Li_2(-\xi z) + Li_2\Big(-\frac{1-z}{z}\Big)\Big] + J,$$

here $P_{2\theta}(z)$ is θ – term of second order electron structure function [10].

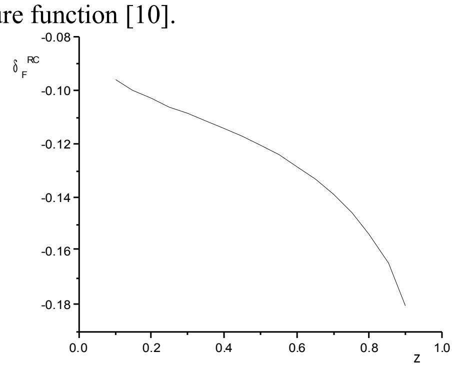

Fig. 3. *The full first order radiative correction to the Born cross section (7) for the case of non-cut pion phase space, that is defined by Eq. (17)*

So the IES Born cross section and RC to it have a factorized form: the low energy pion pair production cross-section $\sigma(q^2)$, which is the object of precise measurement, enters into right side of Eq. (17) as factorized multiplier. The other multiplier has pure electrodynamical origin and is not connected with strong interaction of pions.

The z – dependence of δ_F^{RC}, that is the total radiative correction to the Born cross section (7), is shown on Fig. 3. Note that the contribution of non-logarithmic terms in δ_F^{RC} parametrically equals to $\alpha/(2\pi L_0)$ that is of the order 10^{-4}. This order is the same as relative contribution of terms proportional to $|P_\Phi|^2/4E^2$ and $\theta_0^2/6$ in the Born cross section (see Eq. (6)). That is why the exact account of the Born IES cross section requires with necessity the calculation of RC with inclusion of non-logarithmic contribution.

By analogy with (17) we can write the total RC in the case of reduced pion phase space in the following form

$$\frac{d\sigma_R^{RC}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\frac{\alpha}{2\pi}P(z,L_0)A(z,c_m)\,\delta_R^{RC} + \frac{d\sigma_{3R}^{H_h}}{dq^2}, \qquad (18)$$

where

$$\delta_R^{RC} = \frac{\alpha}{2\pi}\frac{F_0 + L_0F_{1R} + F_{2R}}{P(z,L_0)},$$

$$F_{1R} = F_{1F} - G_2, \quad F_{2R} = F_{2F} - G_3.$$

Because the multiplier $\sigma(q^2)/4E^2$ enters into last term on the right side of Eq. (18) too, the total RC in this case has as well factorized form.

5. PAIR PRODUCTION CONTRIBUTION INTO IES CROSS SECTION

If $e^+e^-\pi^+\pi^-$ final state is not rejected from analysis, there is an additional contribution caused by initial-state hard e^+e^- pair production [4]. The main part of this contribution arises due to collinear kinematics. In framework of NLO approximation keeping only logarithmically strengthen terms, the corresponding cross section can be written as

$$\frac{d\sigma_F^{e^+e^-(c)}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\Big(\frac{\alpha}{2\pi}\Big)^2\Big[P_1(z)L_0^2 + P_2(z)L_0\Big],$$

$$\frac{d\sigma_R^{e^+e^-(c)}}{dq^2} = \frac{d\sigma_F^{e^+e^-(c)}}{dq^2}A(z,c_m), \qquad (19)$$

where functions $P_{1,2}(z)$ given in [11].

Within NLO accuracy one has to compute also the contribution caused by semicollinear kinematics of pair production when created electron belong to narrow cone along the electron beam direction and created positron does not. In the case of non-cut pion phase space the respective differential cross section [12] we can integrate over region on Fig. 2 with substitution $\omega/E \to x$, here x is the energy fraction of created collinear electron.

$$\frac{d\sigma_F^{e^+e^-(s)}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\Big(\frac{\alpha}{2\pi}\Big)^2 L_0S(z,\xi), \qquad (20)$$

$$S(z,\xi) = (1-z)$$

$$\left[\ln\frac{\xi}{1+\xi} - \frac{2}{3}\left(1 + \frac{(1-z)^2\xi}{(1+z\xi)^2} + \frac{2(1+z+z^2)}{z}\ln\frac{1+z\xi}{\xi}\right)\right] +$$
$$2(1+z)\left[\left(\ln\xi - \frac{2}{3z}(1-z+z^2)\right)\ln z +\right.$$
$$\left. Li_2(-z\xi) + Li_2\left(-\frac{1-z}{z}\right) - Li_2(1-z) - Li_2(-\xi)\right].$$

So, the contribution of the pair production into IES cross section for the case of non-cut pion phase space is

$$\frac{d\sigma_F^{e^+e^-}}{dq^2} = \frac{\sigma(q^2)}{4E^2}\left(\frac{\alpha}{2\pi}\right)P(z,L_0)\,\delta_F^{e^+e^-},$$

$$\delta_F^{e^+e^-} = \frac{\alpha}{2\pi}\frac{P_1(z)L_0^2 + \left(P_2(z) + S(z,\xi)\right)L_0}{P(z,L_0)}. \qquad (21)$$

Function $\delta_F^{e^+e^-}$ is shown in Fig. 4.

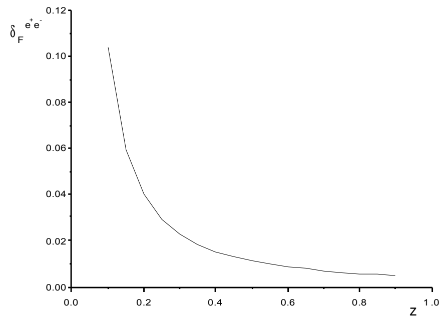

Fig. 4. *The radiative correction to the IES Born cross section caused by e^+e^- -pair production*

The total contribution of e^+e^- - pair production into IES cross section for the case of reduced pion phase spacehas not any singularity and can be calculated numerically.

As DAPhNE conditions allow select and detect also the same events when collinear particles are emitted by the initial positron, all derived cross sections have to be doubled.

CONCLUSION

We compute the corresponding ISR Born cross section and radiative corrections to it in the framework of QRE approximation. The cases of non-cut and reduced pion phase space are considered. In the first one the photonic contribution into RC is calculated analytically with NNLO accuracy and the contribution caused by e^+e^- -pair production with NLO one.

The photonic RC is large and negative in wide diapason of the pion invariant mass. The RC caused by pair production is positive and small as compared with absolute value of photonic one. Only in the region near the threshold (small z), where cross section is very small, it has approximately the same value. So, we conclude that RC due to pair production must be taken into account to guarantee one percent accuracy.

In the case of reduced pion phase space we derived the analytical form of acceptance factor for the Born cross section and the part of RC which includes contributions due to virtual and real collinear photons and e^+e^- - pair. As concerns the contribution into IES cross section caused by semicollinear kinematics for double photon emission and pair production, the respective acceptance factor cannot be calculated analytically. In this case formulae suitable for numerical calculation are derived.

REFERENCES

1. By Muon (g-2) Collaboration (H.N. Brown et al.) Improved measurement of the positive muon anomalous magnetic moment // *Phys. Rev. Lett.* 2001, v. 86, p. 2237-2231.
2. M. Davier and H. Hocker. New results of the hadronic contribution to $\alpha\,(M_Z^2)$ and to the $(G-2)_\mu$ // *Phys. Lett.* 1998, v. B435, p. 427-440.
3. S. Eidelman, F. Jegerlehner. Hadronic contributions to G-2 of the leptons and to the effective fine structure constant $\alpha\,(M_Z^2)$ // *Z. Phys*. 1995, v. C67, p. 585-602.
4. A.B. Arbuzov, E.A. Kuraev, N.P. Merenkov, L. Trentadue. Hadronic cross-sections in electron-positron annigilation with tagged photon // *JHEP*. 1998, v. 9812, p. 009.
5. S. Binner, J.H. Kuhn, and K. Melnikov. Measuring $\sigma\,(e^+e^- \to hadrons)$ using tagged foton // *Phys. Lett.* 1999, v. B459, p. 279-287. S. Spagnollo. The had- ronic contribution to the muon G-2 from hadron production in initial state radiation events at e^+e^- collider DAPhNE // *Eur. Phys. J.* 1999, v. C6, p. 637-645. M. Konchatnij, N.P. Merenkov. Scanning of hadron cross sections at DAPhNE by analysis of the initial-state radiative events // *JETP Lett.* 1999, v. 69, p. 811-818.
6. V.A. Hose, M.I. Konchatnij, N.P. Merenkov et al. Radiative corrections to the hadronic cross-section measurement at DAPhNE // *Eur. Phys. J.* 2001, v. C18, p. 481-490.
7. E.A. Kuraev, N.P. Merenkov, V.S. Fadin. The compton effect tensor with heavy foton // *Yad. Phys*. 1987, v. 45, p. 782-789.
8. G.I. Gakh, M.I. Konchatnij, N.P. Merenkov, O.N. Shekhovtzova. QED correction to electron -positron annihilation cross-section with tagged collinear photons and -e^+e^- -pairs // *Ukr. Fiz. Journ*. 2001, v. 46, №7, p. 652-660.
9. N.P. Merenkov. Study of the process of emission of two hard photons in ep collisions at high-energy // *Sov. J. Nucl. Phys*. 1988, v. 48, p. 1073-1078.
10. S. Jadach, M. Skrzypek, B.F.L. Ward. Exact result on $e^+e^- \to e^+e^-2\gamma$ at SLC/LEP energies // *Phys. Rev.* 1993, v. D47, p. 2682-2689.
11. A.B. Arbuzov, E.A. Kuraev, N.P. Merenkov, L. Trentadue. Pair production in small angle Bhabha scattering // *Zh. Eksp. Teor. Fiz.* 1995, v. 108, p. 1164-1178.
12. M.I. Konchatnij, N.P. Merenkov, O.N. Shekhovtzova. Current tensor with heavy photon for pair production by a longitudionally

polarized electron. // *JETP*. 2000, v. 1, №.1, p. 1-15.