ACCELERATION OF CHARGED PARTICLES BY ELLIPTIC POLARIZED WAVES OF LARGE AMPLITUDE

A.V. Buts, V.A. Buts, V.V. Kuzmin
National Scientific Center "Kharkov Institute of Physics & Technology"
61108 Akademicheskaya st.1, Kharkov, Ukraine
E-mail: vbuts@kipt.kharkov.ua

In this paper the results of investigations on the charged particle dynamics in the homogeneous constant magnetic field and in the field of a flat wave of arbitrary intensity are presented. When the magnetic field is absent the particle trajectory is defined. It is shown, that a particle can be effectively drawn along the wave vector of the wave. Average speed of this drawing is proportional to the square of the wave force parameter. The trajectories of particles are significantly dependent on initial conditions. In the presence of constant external magnetic field the particles can be effectively accelerated by the wave field. The particle dynamics is always chaotic if the wave force parameter is enough large. This dynamic is characterized by the alternation.

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1. INTRODUCTION

Interaction of charged particles (electrons) with fields of electromagnetic waves underlies in the theory of accelerators, amplifiers and generators. At present the time dynamics of particles at such interaction in details is investigated, when intensity of electromagnetic wave fields is small enough. Under the value smallness we understand a smallness of $\mathcal{E}=eE/mc\omega$, which sometimes is named as a parameter of a wave force or parameter of nonlinearity. For 10 *cm* radiation this parameter is of about unit when the intensity of an electric field of a wave reaches 10^5 *V/cm*. For laser radiation the ($\lambda \approx 10^{-4}$ *cm*) intensity of a field should by higher than 10^{10} *V/cm*. In majority of experiments this parameter is small. In this case the significant exchange of energy between particles and electromagnetic waves occurs for times considerably large then the period, i.e. interaction of particles with a field should be long. Therefore the special role conditions of synchronism of the charged particle with phases of electromagnetic waves play. These conditions look like resonant conditions. As the field intensity is sufficiently high that a particle gets a velocity close to the velocity of light during the period of an electromagnetic wave, i.e. the exchange of energy may be very fast, as compared with resonant conditions in this case lose the exclusiveness. We shall note the important feature of interaction of a flat transverse electromagnetic wave with particles in vacuum. If parameter $\mathcal{E}$ is small, the particle in main makes transverse (concerning a direction of wave distribution) oscillation. If the parameter of nonlinearity is great ($\mathcal{E}$ >>1), the particle makes a basic movement along the direction of wave distribution. In the same direction it gets also the maximum velocity, therefore, the period of a wave, which is perceived by a particle, is essentially increased.

Some results of research on particle dynamics in intense ($\mathcal{E} \geq 1$) electromagnetic waves are represented. The basic models of interaction of the charged particles with electromagnetic waves are formulated in section 2, the equations describing these models, and also some integrals are written out. Some features of the charged particle dynamics in the field of a transverse electromagnetic wave with linear polarization in vacuum are analyzed in section 3. The basic result of this section consists in that the particles quickly scatter in the cross direction. In section 4 the particle dynamics in the transverse electromagnetic wave with elliptic polarization is considered. In Section 5 the dynamics of particles in a field of an electromagnetic wave with linear polarization at presence of a constant homogeneous magnetic field are considered. In this case the dynamics of particles has alternated character. The average energy of the particles grows. However this growth is not monotonous, i.e. the particle in the casual moments of time can gain or loss energy.

2. THE BASIC EQUATIONS AND INTEGRALS

Let us consider the movement of the charged particle in the field of a flat electromagnetic wave with any polarization. Components of electric and magnetic fields of such a wave can be presented as

$$\mathbf{E} = \mathrm{Re}(\mathbf{E}_0 e^{i\psi}), \mathbf{H} = \mathrm{Re}\left(\frac{1}{k_0}[\mathbf{kE}]\right), \qquad (1)$$

where $\psi \equiv \omega t - \mathrm{kr}$, $\mathbf{E}_0 = \alpha E_0$; $\alpha = \{\alpha_x, i\alpha_y, \alpha_z\}$ – a vector of polarization of a wave; $k_0 = \omega / c$; ω , k – frequency and a wave vector of a wave. We shall enter the following dimensionless variables:

$$\mathbf{p}_1 = \mathbf{p}/mc, \quad \mathbf{k}_1 = \mathbf{k}/k_0, \ \tau = \omega t, \ \mathbf{r}_1 = k_0\mathbf{r},$$

$$\mathcal{E} = e\mathbf{E}_0/mc\omega, \ \mathbf{v}_1 = \mathbf{v}/c, \ \upsilon_{ph1} = \upsilon_{ph}/c = \omega/kc.$$

In these variables the equation of movement gets a form (the index "1" is omitted)

$$\dot{\mathbf{p}} \equiv \frac{d\mathbf{p}}{d\tau} = \mathrm{Re}\left\{\left[(1-\mathbf{kv})\mathcal{E}+\mathbf{k}(\mathbf{v}\mathcal{E})\right]e^{i\psi}\right\}. \qquad (2)$$

It is convenient to add to equations (2) the equation which defines the particle energy and may be obtained from system (2):

$$\dot{\gamma} = \mathrm{Re}\left(\mathbf{v}\mathcal{E}e^{i\psi}\right), \qquad (3)$$

where $\gamma = \sqrt{1 + p^2}$ is the dimensionless energy of a particle (measured in terms of mc^2).

Equations (2) and (3) have the integrals:

$$\mathbf{p} - \mathbf{k}\gamma + \mathrm{Re}\left(i\mathbf{E} e^{i\psi} \right) = \mathbf{p}_0 - \mathbf{k}\gamma_0 + \\ + \mathrm{Re}\left(i\mathbf{E} e^{i\psi_0} \right) = \mathrm{const} = \mathbf{C} \quad (4)$$

Here the index "0" designates initial variables. Further, without restriction of a generality, we shall consider, that the wave travels along the axis z, i.e. $\mathbf{k} = \{0, 0, k\}$.

3. DYNAMICS OF PARTICLES AT INTERACTION WITH THE WAVE OF LINEAR POLARIZATION

If the particle moves in the field of only one wave then the dynamics of its movement may be expressed by analytical expressions. So, for values of the pulses and values of its energy, one can obtain such expressions (the solving of system of equations (2)):

$$p_x = p_{x0} + \mathbf{E}_x \left(\sin\psi - \sin\psi_0 \right), \\ p_z = p_{z0} \pm \frac{\left(p_x^2 \right) - \left(p_{x0}^2 \right)}{2\gamma\dot{\psi}}, \quad (5) \\ \gamma = \gamma_0 \pm \left(p_z - p_{z0} \right),$$

where $\mathbf{E}_x \equiv eE_0 / mc\omega$; The upper sign (+) in the expressions for γ and p_z corresponds to the case $k = 1$. The inferior sign (–) corresponds to the case $k = -1$. Let us look at the particles which originally were in rest:

From formulas (5) it follows that the value of a transverse pulse essentially depends on the initial position of a particle concerning a wave, i.e. from ψ_0. Really, even if originally the particle has no transverse velocity ($p_x = 0$), depending on of an initial phase ψ_0, the maximal values of the module of a transverse pulse vary from $\mathbf{E}_x$ up to $2\mathbf{E}_x$. Besides, in an initial phase, the size of an average transverse pulse varies too. The average pulse is equal to 0, for particles, which are located in phases πn. Such particles are not displaced in a transverse direction concerning the initial position. The average value of a longitudinal pulse, which the particles get during interaction with a wave, reaches the value $\mathbf{E}_x^2 / 4$. For particles which are located in phases $\psi_0 = \pi(n + 1/2)$, the maximal values of a transverse pulse on the module reach the value $2\mathbf{E}_x$. And the average value of the transverse pulse for such particles is not equal to 0 and also reaches value $\mathbf{E}_x$.

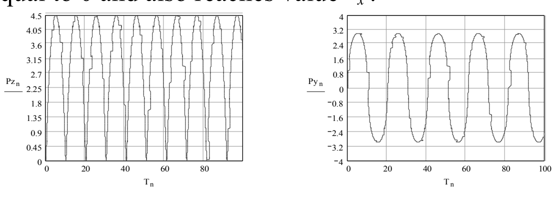

Fig 1. Pulses of particles taking place in an initial phase πn, at $\mathbf{E}_x = 3$. A) a longitudinal pulse, B) a transverse pulse

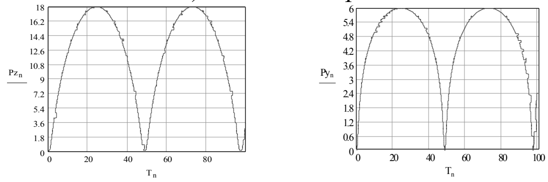

Fig 2. Pulses of particles taking place in an initial phase $\psi_0 = \pi(n + 1/2)$, at $\mathbf{E}_x = 3$. A) a longitudinal pulse, B) a transverse pulse

The average size of a longitudinal pulse of such particles reaches size $\mathbf{E}_x^2$. Thus, all particles, except for those which are located in initial phases πn, will run up quickly in the transverse direction. Clearly, that such a scheme of laser acceleration of the charged particles will be not effective.

4. INTERACTION OF PARTICLES WITH THE WAVE OF CIRCULAR POLARIZATION

As is shown above, if a charged particle is accelerated by the wave with linear polarization, the particles run up in the transverse direction, and for the purpose of acceleration it is necessary to find conditions at which their run up will be limited. One of the simplest way of restriction of run up, apparently, may be acceleration of particles by a wave with circular polarization. Under this condition we can expect, that the trajectory of running up particles will be a spiral trajectory. Below we shall show that indeed it takes place. Really the dynamics of particles in the elliptic polarized wave may be expressed by analytical formulas which can be obtained from equation (2):

$$p_x = p_{x0} + \mathbf{E}_x \left(\sin\psi - \sin\psi_0 \right), \\ p_y = p_{y0} + \mathbf{E}_y \left(\cos\psi - \cos\psi_0 \right), \\ p_z = p_{z0} \pm \frac{\left(p_x^2 + p_y^2 \right) - \left(p_{x0}^2 + p_{y0}^2 \right)}{2\gamma\dot{\psi}}, \quad (6) \\ \gamma = \gamma_0 \pm \left(p_z - p_{z0} \right),$$

Despite of simplicity of these formulas, they describe dynamics in a rather complex manner. It is caused by that decisions (6) are submitted in an implicit form since a phase ψ itself is a function of a pulse.

For a more detailed investigation of particle dynamics in a wave with elliptic polarization the system of equations (2) was investigated by numerical methods. In figures the most typical examples of this dynamics are given.

As follows from the analytical research and the numerical analysis the main feature of movement of a particle in a wave with elliptic polarization consists in that its trajectory represents a spiral with an wave axis directed along the wave propagation and with the radius $\mathbf{E}$ in space of pulses (see figure 3).

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Thus, acceleration of particles by a field elliptic polarization may remove a part of the difficulties connected with the run up of electrons in the transverse direction that is characteristic for the scheme of interaction with a linearly polarized wave.

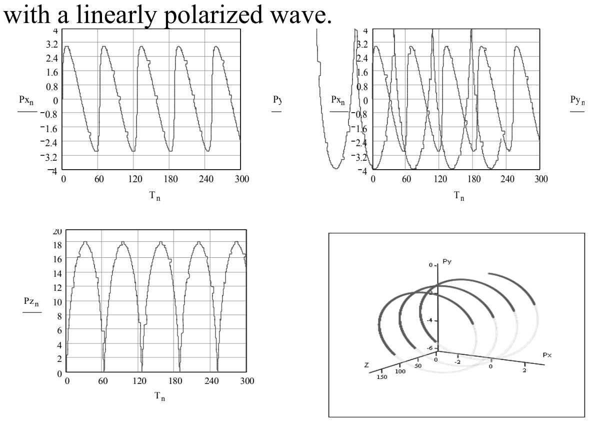

Fig.3. Dynamics of a particle at interaction with a wave of circular polarization, at $\mathbf{E} = 3$

Really, in the space of transverse pulses the trajectory of a particle describes by circle. The equation of this circle can be presented as:

$$(P_x - A_x)^2 + (P_y - A_y)^2 = \mathbf{E}^2 \qquad (7)$$

where $A_x = P_{x0} + \mathbf{E}_x^2 \cos\psi_0$, $A_y = P_{y0} + \mathbf{E}_y^2 \sin\psi_0$

It is visible, that if initial particle pulses (pulses of coming particles in area of interactions with a wave) will be large than $\mathbf{E}^2$ ($P_{x0} >> \mathbf{E}_x^2$, $P_{y0} >> \mathbf{E}_y^2$), all particles will get an average transverse pulse which differs insignificantly from the initial particle pulse. In this case accelerated particles practically will not be run up since their average pulses will not depend on the arrangement of particles with respect to the wave phase ψ_0.

5. DYNAMICS OF PARTICLES IN FIELD OF THE WAVE IN PRESENCE OF THE CONSTANT HOMOGENEOUS MAGNETIC FIELD

In the presence of a constant magnetic field $\mathbf{H}_0$, having components $\mathbf{H_0} = \{0, H_{0y}, 0\}$, the equation of movement (2) will become:

$$\frac{d\mathbf{p}}{dt} = \mathrm{Re}\{[(1-\mathbf{k}\cdot\mathbf{v})\mathbf{E}+\mathbf{k}(\mathbf{v}\cdot\mathbf{E})]e^{i\psi}\} + [\mathbf{vB_0}], \quad (8)$$

where $\mathbf{B}_0 = e\mathbf{H}_0 / mc\omega \sqrt{}$

The system of equations (8) was analyzed by numerical methods. Characteristic results of calculations are given in figure 4.

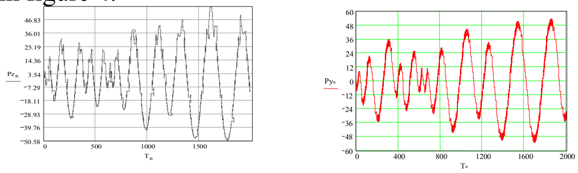

Fig.4. Dynamics of particles in a wave field with linear polarization in the presence of a constant homogeneous magnetic field. $\mathbf{E} = 3$, $\mathbf{B_0} = 1$

A feature of movement of a particle in a wave, following from the numerical analysis in the presence of a constant homogeneous magnetic field, consists in that the dynamics has alternated character. Under these conditions the particle receives the average energy which is lower than it can receive as a result of diffusion in space of energy. Really, all conditions for stochastic acceleration of charged particles in this case are satisfied. As it is known, under this condition, the particle should get the energy under the law $\Delta\gamma \sim \mathbf{E}\cdot\sqrt{\tau}$. Numerical calculations show, that there are intervals of time at which the particle gets the energy considerably more quickly, however, on a whole, for rather large times its energy is lower than it follows from the law of diffusion. Discussion of this fact is contained in other our paper submitted at this conference.

УСКОРЕНИЕ ЗАРЯЖЕННЫХ ЧАСТИЦ ЭЛЛИПТИЧЕСКИ ПОЛЯРИЗОВАННЫМИ ВОЛНАМИ БОЛЬШОЙ АМПЛИТУДЫ

А.В. Буц, В.А. Буц, В.В. Кузьмин

Изложены результаты исследования динамики заряженных частиц в однородном постоянном магнитном поле и в поле плоской волны произвольной напряженности. В отсутствии магнитного поля определен вид траектории частиц. Показано, что частицы могут эффективно увлекаться полем внешней волны. Средняя скорость увлечения пропорциональна квадрату параметра силы волны. Вид траектории существенно зависит от начальных условий. При наличии постоянного магнитного поля частицы могут эффективно ускоряться полем волны. При больших напряженностях поля динамика частиц всегда хаотична. Динамика характеризуется перемежаемостью.

ПРИСКОРЕННЯ ЗАРЯДЖЕНИХ ЧАСТОК ЄЛЕПТИЧНО ПОЛЯРІЗОВАНИМИ ХВИЛЯМИ ВЕЛИКОЇ АМПЛІТУДИ

О.В.Буц, В.О. Буц, В.В. Кузьмін

Викладені результаті дослідження динаміки заряджених часток у однорідному постійному магнітному полі та в полі плоскої хвилі довільної напруженості. За відсутності постійного магнітного поля вигляд траєкторії частки був визначений. Показано, що частка може ефективно захоплюватися полем зовнішньої хвилі. Середня швидкість захоплення пропорційна квадрату параметру сили хвилі. При наявності постійного магнітного поля. частки можуть ефективно прискорюватись полем хвилі. При великих напруженостях поля динаміка часток завжди хаотична. Динаміка характеризується перемежністю.