APPROXIMATIONS OF OPERATING CHARACTERISTICS OF THE ELLIPTIC CROSS-SECTION BEAM POSITION MONITORS

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In the specialized storage rings it is necessary to measure a position of a beam with an absolute accuracy up to 10 μm that corresponds to a relative accuracy about 10^{-3} for the central region of a monitor. The position of a beam is usually measured with the help of electrostatic button monitors. Operating characteristics of a beam position monitor are individual for each copy, are measured on precision benches and approximated by analytical functions. In the paper the various methods of approximation of operating characteristics are considered and the variant allowing one to reduce an error of approximation in tens times in comparison with a traditional method is found.

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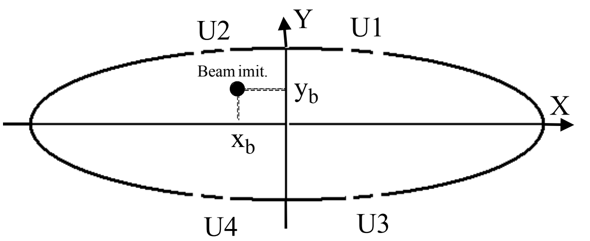
The operating characteristics (OC) of a four-electrode beam position monitor (BPM) (Figure) measured on the bench look like:

$$x_{bk} = f_1(u_{1k}, u_{2k}, u_{3k}, u_{4k}), \qquad (1)$$

$$y_{bk} = f_2(u_{1k}, u_{2k}, u_{3k}, u_{4k}), \qquad (2)$$

where x_{bk}, y_{bk} are the coordinates of the beam imitator; $u_{1k}, u_{2k}, u_{3k}, u_{4k}$ are the signals measured on the appropriate electrodes; k is the number of the measured point in the selected region of the BPM aperture. The set data obtained allows to approximate OC of BPM power polynomials or empirically picked out elementary functions.

The position of a beam defined with the help of approximating functions will have an error containing an error of work of a mechanical drive of the bench, an error of measurement of signals on electrodes, an error of a method of approximation of operating characteristics.



The layout of the BPM cross-section

In this work the possibility of discovering of functions that would have minimum errors of approximation of OC of BPM is studied. The study is carried out by simulation of bench measurement of OC of BPM. The model of a monitor for the storage ring N-100M [1] in the form of an ellipse was considered. Its geometrical parameters: major axis - 100 *mm*, minor axis - 30 *mm*, coordinates of the center of electrodes x_e= ±14.4 *mm*, y_e=±14.36 *mm*, size of electrodes - 5 *mm*, electric capacity of an electrode - 4 *pF*. The values of monitor signals were calculated with the help of the formulas defined in [2].

The signals on BPM electrodes in the elliptical coordinates are:

$$u_d = i_b \frac{2l}{\pi cC}\left\{\psi + 2\sum_{n=1}^{200}\frac{\sin n\psi}{n}\times\left[\frac{\cosh n\mu_b \cos n\nu_b}{\cosh n\mu_d}\cos n\nu_d + \frac{\sinh n\mu_b \sin n\nu_b}{\sinh n\mu_d}\sin n\nu_d\right]\right\}; \qquad (3)$$

where d=1,2,3,4 - number of an electrode, i_b – current of a beam imitator, $2l$ - longitudinal size of an electrode, c – velocity of light, C – capacity of an electrode, μ_b -radial coordinate of beam imitator, ν_b - angular coordinate of beam imitator, 2ψ - angular size of electrodes, μ_d – radial coordinate of an electrode, ν_d - angular coordinate of an electrode.

Elliptic coordinates are expressed through the Cartesian coordinates as follows:

$$\mu = \operatorname{arccos}h\sqrt{\frac{a^2 + x^2 + y^2 + \sqrt{(a^2 + x^2 + y^2)^2 - 4x^2a^2}}{2a^2}} \qquad (4)$$

$$\nu = \arccos\sqrt{\frac{a^2 + x^2 + y^2 - \sqrt{(a^2 + x^2 + y^2)^2 - 4x^2a^2}}{2a^2}} \qquad (5)$$

where $2a$ is the distance between focuses. Formulas are true for $|x|<a$.

To optimize the functions approximating OC, they are found as:

$$x_b = \Phi_1(H_g, V_g), \qquad (6)$$

$$y_b = \Phi_2(H_g, V_g). \qquad (7)$$

Arguments *Hg* and *Vg* represent the normalized linear combinations of the measured signals of BPM. Research was carried out for four variants of definition of arguments *Hg* and *Vg* (g=1,2,3,4):

$$H_1 = \frac{u_1 + u_3 - u_2 - u_4}{u_1 + u_2 + u_3 + u_4}, \quad V_1 = \frac{u_1 + u_2 - u_3 - u_4}{u_1 + u_2 + u_3 + u_4} \qquad (8)$$

$$H_2 = \frac{1}{2}\left[\frac{u_1 - u_4}{u_1 + u_4} + \frac{u_3 - u_2}{u_3 + u_2}\right], V_2 = \frac{1}{2}\left[\frac{u_1 - u_4}{u_1 + u_4} + \frac{u_2 - u_3}{u_2 + u_3}\right] \qquad (9)$$

$$H_3 = \frac{1}{2}\left[\frac{u_1 - u_2}{u_1 + u_2} + \frac{u_3 - u_4}{u_3 + u_4}\right], V_3 = \frac{1}{2}\left[\frac{u_1 - u_3}{u_1 + u_3} + \frac{u_2 - u_4}{u_2 + u_4}\right] \qquad (10)$$

$$H_4 = \frac{1}{2}\left[\frac{u_1 - u_4}{u_1 + u_4} + \frac{u_3 - u_2}{u_3 + u_2}\right], V_4 = \frac{1}{2}\left[\frac{u_1 - u_3}{u_1 + u_3} + \frac{u_2 - u_4}{u_2 + u_4}\right] \quad (11)$$

The first variant (traditional) has obtained rather wide application in practice [3,4], the second one was proposed in [5], but the propagation was not found, the third and the fourth ones are considered for the first time.

Research was carried out in the assumption of an ideally symmetric BPM. It has allowed obtaining the simplified form of approximating functions. Approximation of operating characteristics (6), (7) was carried out by polynomials up to 7-th order

$$x_b = \sum_{m=0}^{M}\sum_{n=0}^{m} A_{2m-2n+1,2n} H_g^{2m-2n+1} V_g^{2n}, \quad (12)$$

$$y_b = \sum_{m=0}^{M}\sum_{n=0}^{m} B_{2m-2n+1,2n} V_g^{2m-2n+1} H_g^{2n}, \quad (13)$$

and by the empirically picked out expression

$$x_b = C \cdot ArcTanh[H_g] + D \cdot Sinh[\sum_{m=0}^{M}\sum_{n=0}^{m} E_{2m-2n+1,2n} H_g^{2m-2n+1} V_g^{2n}], \quad (14)$$

for which M=3. Coefficients *A, B, C, D, E* were calculated with the fitting carried out by the method of least squares.

Since the accuracy of approximation depends on the distance from the beam up to the center of BPM, the simulation was carried out for two monitor regions: **1** (central) region with dimensions $|x|\le 10\,mm$, $|y|\le 5$), and **II** (far) region with dimensions $|x|\le 40\,mm$, $|y|\le 7\,mm$.

In each region of BPM the coefficients of expressions (12), (13), (14) were calculated for 4 variants of definition of arguments (8), (9), (10), (11). The data set was calculated with 1 *mm* step for both regions.

Results of simulation were estimated by the relative root-mean-square errors of approximation averaged on fixed region as follows:

$$\sigma_x = \sqrt{\left[\sum_{k=0}^{n}\left(\frac{x_{bk} - x'_{bk}}{x'_{bk}}\right)^2 / n\right]}, \quad (15)$$

$$\sigma_y = \sqrt{\left[\sum_{k=0}^{n}\left(\frac{y_{bk} - y'_{bk}}{y'_{bk}}\right)^2 / n\right]}, \quad (16)$$

where x_{bk}, y_{bk} - true values of beam coordinates assigned in the calculation of signals by formulas (3), (4), (5); x'_{bk}, y'_{bk} - beam coordinates calculated by formulas (12),(13),(14) for the corresponding true H_{gk} and V_{gk}, k – number of the point for which the error was calculated, n – quantity of the calculated points. Calculations were carried out with a 0.9 *mm* step on both coordinates.

Results of calculations are given in Table 1. From the results obtained it is seen that for the central region of BPM the best approximation of OC (6) is provided with expressions (14), (11) and the best approximation of OC (7) is provided with expressions (13), (11). The corresponding errors of the approximation are equal to $\sigma_x=1.2\times10^{-5}$, $\sigma_y=1.9\times10^{-4}$ while the traditional approximation method (the formulas (12), (13) (8)) has greater errors $\sigma_x=1.7\times10^{-3}$, $\sigma_y=7.8\times10^{-4}$. In the far region of BPM the best results is obtained with using formulas (14), (10) - $\sigma_x=5.2\times10^{-2}$ and formulas (13), (9) - $\sigma_y=0.4$. The traditional approximation method has greater errors $\sigma_x=0.3$, $\sigma_y=0.45$.

One can see, that the error of approximation of OC (7) less $\sigma_y=0.4$ cannot be obtained for the far region of BPM. Presumably, it is caused by the form of the monitor that is very extended along the axis *X*. To be convinced of this supposition the approximation of OC of BPM for the storage ring ISI-800 [6] was carried out. The BPM of ISI-800 has the following geometrical parameters: cross-section - elliptic, the big axis - 60 mm, a small axis - 36 mm, distance between of centers of electrodes along X axis - 22.86 mm and along Y axis - 32.86 mm, diameter of electrodes - 5 mm. The results of calculations are given in Table 2 with the indication of used formulas.

Approximations of OC of the ISI-800 monitor in region **1** have errors comparable with the corresponding errors of the N-100M monitor. Approximations of OC (7) by expression (13) in region **II** have errors $\sigma_y=(5.2...6.2)\times10^{-2}$, that confirms the supposition expressed above about a negative influence of the very extended form of a monitor on the accuracy of approximation.

The method of estimation of errors of approximating functions also allows analyzing a possibility to lower an order of polynomials being used. Calculations of errors for the central region of N-100M BPM are brought together in Table 3 at approximation of OC by polynomials up to 7-th, 5-th, 3-rd and 1-st orders. Traditionally used formulas (12), (13), (8) and the formulas giving the best results (13), (14), (11) are considered for comparison. From Table 3 the obvious advantage of the method of approximation of OC (6) and (7) by expressions (13), (14), (11) is visible in comparison with the traditional method at equal orders of polynomials. Use of this method of approximation enables one to apply into practice the polynomials of lower orders (5-th or even 3-rd) with a required error about 10^{-3}.

The carried out simulation and the analysis of the obtained results have shown that the approximation of operating characteristics of elliptic cross-section BPM is fulfilled most precisely by the method using expressions (13), (14), (11) for the central region of a monitor and for optimal chosen far region of a monitor. The above-mentioned method allows fulfilling precision measurements in the center of the monitor using the approximation of OC by polynomials of 5-th order. The form of the monitor very much extended along the axis X increases the errors of approximation of OC, especially the error of characteristic of a beam displacement along the axis Y in the far region.

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Table 1. Average errors of different approximation methods of OC of N-100M BPM

Region of cross-section BPM	Number of formula	σ_x(12)	σ_y(13)	σ_x(14)
1 -10≤*x*≤10 -5≤*y*≤5	(8)	0.001696	0.000788	0.000264
	(9)	0.001057	0.001542	0.000015
	(10)	0.001173	0.000226	0.000145
	(11)	0.001044	0.000195	0.000012
II -40≤*x*≤40 -7≤*y*≤7	(8)	0.304776	0.451419	0.181716
	(9)	0.420346	0.406421	0.074058
	(10)	0.298039	0.478659	0.052067
	(11)	0.285599	0.476832	0.063378

Table 2. Average errors of different approximation methods of OC of ISI-800 BPM

Region of cross-section BPM	Number of formula	σ_x(12)	σ_y(13)	σ_x(14)
1 -10≤*x*≤10 -5≤*y*≤5	(8)	0.000646	0.000385	0.000511
	(9)	0.000431	0.000522	0.000021
	(10)	0.000469	0.000195	0.000111
	(11)	0.000390	0.000186	0.000015
II -23≤*x*≤23 -10≤*y*≤10	(8)	0.100030	0.059887	0.042760
	(9)	0.062090	0.062081	0.010127
	(10)	0.079737	0.059247	0.021223
	(11)	0.062902	0.052716	0.009328

Table 3. Average errors of approximation of OC for central region of N -100M BPM

Order of polynomials	Number of formula	σ_x(12)	σ_y(13)	σ_x(14)
7–order, M=3	(11)	-	0.000195	0.000012
5–order, M=2	(11)	-	0.000876	0.000087
3–order, M=1	(11)	-	0.004655	0.000766
1-order, M=0	(11)	-	0.075856	0.002039
7–order, M=3	(8)	0.001696	0.000788	-
5–order, M=2	(8)	0.006095	0.001869	-
3–order, M=1	(8)	0.034405	0.021693	-
1-order, M=0	(8)	0.140973	0.208716	-

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АППРОКСИМАЦИИ РАБОЧИХ ХАРАКТЕРИСТИК ДАТЧИКОВ ПОЛОЖЕНИЯ ПУЧКА ЭЛЛИПТИЧЕСКОГО ПОПЕРЕЧНОГО СЕЧЕНИЯ

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В специализированных накопителях необходимо измерять положение пучка с абсолютной точностью до 10 *мкм*, что соответствует относительной точности около 10^{-3} для центральной области датчика. Положение пучка обычно измеряется с помощью электростатических кнопочных датчиков. Рабочие характеристики датчиков положения пучка являются индивидуальными для каждого экземпляра, снимаются на прецизионных стендах и аппроксимируются аналитическими функциями. В работе рассмотрены различные методы аппроксимации рабочих характеристик и найден вариант, позволяющий уменьшить погрешность аппроксимации в десятки раз по сравнению с традиционным методом.

АППРОКСИМАЦІЯ РОБОЧИХ ХАРАКТЕРИСТИК ДАТЧИКІВ ПОЛОЖЕННЯ ПУЧКУ ЕЛІПТИЧНОГО ПОПЕРЕЧНОГО ПЕРЕРІЗУ

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В спеціалізованих нагромаджувачах необхідно виміряти положення пучку з абсолютною точністю до 10 *мкм*, що відповідає відносній точності біля 10^{-3} для центральної області датчика. Положення пучку вимірюються за допомогою електростатичних кнопкових датчиків. Робочі характеристики датчиків положення пучку є індивідуальними для кожного екземпляру, одержуються на прецизійних стендах та апроксимуються аналітичними функціями. В роботі

розглянуто різні методи апроксимації робочих характеристик та знайдено варіант, що дозволяє зменшити похибку апроксимації в десятки разів в зрівнянні с традиційним методом.

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