NUMERICAL MODELLING OF MULTIBEAM ACCELERATING STRUCTURES

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The three-dimensional multibeam numerical simulation has been carried out on the basis of integral equations. Dispersion equations and expressions for shunt impedance and Q-factor have been obtained for E and H oscillations in cylindrical cavities. The tensor Green function and filament-like representation of the beam were used.

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With the purpose of creation of industrial linear accelerators of ions with a current of a bunch more than 100 mA, works on studying multibeam accelerating structures working on E and H kinds of fluctuations, and also resonators with the electrodes of spiral type overlapping a range of lengths of waves 2...15 m recently are actively conducted.

Three-dimensional modelling of similar systems with the help of modern software packages of applied electrodynamics such as MAFIA, ISFEEL 3D, HFSS 7.0 or ANSIS 7.0 is or expensive and designed on the use of high-efficiency computers, or basically unpromising, as in the case with spiral systems and systems on basis H-resonators in a range of meter wavelengths. In this work the numerical algorithm of calculation of similar systems is offered.

1. MODELLING MULTIBEAM SYSTEMS ON BASIS E AND H RESONATORS

As is known [1] system of Maxwell equations which describes electromagnetic fields in the given area, can be shown to two wave equations concerning vector and scalar sizes which name potentials of an electromagnetic field. Electric and magnetic fields can be determined from the following equations:

$$\vec{E}(\vec{r},t) = -grad\phi - \frac{d\vec{A}(\vec{r},t)}{dt}, \quad \vec{B}(\vec{r},t) = rot\vec{A}(\vec{r},t) \qquad (1)$$

where φ, A are defined from the d'Alambert equation:

$$\Delta \vec{A} - \frac{1}{c^2}\cdot\frac{\partial^2 A}{\partial t^2} = -\mu_0 \cdot \vec{\delta}, \quad \Delta\phi - \frac{1}{c^2}\cdot\frac{\partial^2\phi}{\partial t^2} = -\frac{\rho}{\varepsilon_0} \qquad (2)$$

In the right parts of the equations the vector of density of a current and the volumetric density of a charge stand respectively. In that specific case the time dependence can be considered harmonious, and for the solutions of equations (2) to search as late potentials:

$$\vec{A}_m = \int_V \frac{\mu_0 \cdot \delta(\vec{r}_0)}{4\pi R(\vec{r}_0, r)} dV, \quad \phi_m = \int_V \frac{\varepsilon_0 \cdot \rho(\vec{r}_0)}{4\pi R(\vec{r}_0, r)} dV, \qquad (3)$$

where A_m, ϕ_m - complex amplitudes, $\vec{r}_0$ - vector radius of a point source, r - vector radius of an observation point, R - distance from a source up to the observer. For an external problem of electrodynamic a unique condition, set in (2) is the regularity on infinity or the condition of radiation which apparently is carried out.

We shall consider now an internal problem of electrodynamics in the following statement: it is required to define an electromagnetic field in cylindrical area with the limited conducting surface. Then the system (2) remains constant, but the solution will be searched in the following form:

$$A = \int_V \mu_0 \vec{G}(r,\phi,z,r_0,\phi_0,z_0)\,\delta(r_0,\phi_0,z_0)\,dV, \qquad (4)$$

where $\vec{G}(r,\phi,z,r_0,\phi_0,z_0)$ is so-called tensor or a Green dyad [2] taking into account influence of external borders on electromagnetic process in volume. As shown in [3] for the cylindrical system of coordinates the dyad has an explicit expression in index designations. As the table this tensor can be presented as follows:

$$\vec{G} = \begin{pmatrix} G_{11} & G_{12} & 0 \\ G_{21} & G_{22} & 0 \\ 0 & 0 & G \end{pmatrix}, \qquad (5)$$

where the factors of a matrix are the following:

$$G_{11} = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{1}{\tilde{k}_{mn}\cdot r\cdot r_0}\cdot\frac{\partial\tilde{\psi}_{mn}(r,\phi)}{\partial\phi}\cdot\frac{\partial\tilde{\psi}_{mn}(r_0,\phi_0)}{\partial\phi_0}\cdot\tilde{f}_{mn}(z,z_0) + \frac{1}{k_{mn}^2 r r_0}\frac{\partial\psi_{mn}(r,\phi)}{\partial r}\cdot\frac{\partial\psi_{mn}(r_0,\phi_0)}{\partial r_0}\cdot f_{mn}(z,z_0),$$

$$G_{12} = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{1}{\tilde{k}_{mn}^2\cdot r}\cdot\frac{\partial\tilde{\psi}_{mn}(r,\phi)}{\partial\phi}\cdot\frac{\partial\tilde{\psi}_{mn}(r_0,\phi_0)}{\partial r_0}\cdot\tilde{f}_{mn}(z,z_0) + \frac{1}{k_{mn}^2 r_0}\frac{\partial\psi_{mn}(r,\phi)}{\partial r}\cdot\frac{\partial\psi_{mn}(r_0,\phi_0)}{\partial\phi_0}\cdot f_{mn}(z,z_0),$$

$$G_{21} = \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \frac{1}{\tilde{k}_{mn}^2\cdot r_0}\cdot\frac{\partial\tilde{\psi}_{mn}(r,\phi)}{\partial r}\cdot\frac{\partial\tilde{\psi}_{mn}(r_0,\phi_0)}{\partial\phi_0}\cdot\tilde{f}_{mn}(z,z_0) + \frac{1}{\tilde{k}_{mn}^2 r}\frac{\partial\psi_{mn}(r,\phi)}{\partial\phi}\cdot\frac{\partial\psi_{mn}(r_0,\phi_0)}{\partial r_0}\cdot f_{mn}(z,z_0),$$

$$G_{22}=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\frac{1}{\tilde{k}_{mn}^{2}}\cdot\frac{\partial\tilde{\psi}_{mn}(r,\phi)}{\partial r}\cdot\tilde{f}_{mn}(z,z_0)+$$

$$+\frac{1}{\tilde{k}_{mn}^{2}rr_0}\frac{\partial\psi_{mn}(r,\phi)}{\partial r}\cdot\frac{\partial\psi_{mn}(r_0,\phi_0)}{\partial r_0}\cdot f_{mn}(z,z_0),$$

$$G_{33}=\sum_{m=1}^{\infty}\sum_{n=0}^{\infty}\psi_{mn}(r,\phi)\cdot\psi_{mn}(r_0,\phi_0)\cdot g_{mn}(z,z_0),$$

where ψ_{mn}, f_{mn}, g_{mn} are the own functions of the Helmholtz equations with boundary conditions such as an electric wall, and the functions $\tilde{\psi}_{mn}, \tilde{f}_{mn}$ define solutions with boundary conditions such as magnetic walls for the given area. Definition of the given functions and calculation of the component Green tensor allows to obtain the explicit solution.

Representing accelerating systems as a closed area with the currents distributed inside (current-carrying elements are: a tube of drift for the E-resonator, pin holders for the H-resonator) it is possible to obtain the integrated equations describing the electromagnetic process in the resonator. So for E - the resonator relative to the z-component the current density on a drift tube is presented as Fourier series:

$$\int_z g_{mn}(z,z_0)\cdot\delta_z(z_0)dz_0=$$

$$\frac{c_p}{k_z^2+\left(\frac{p\pi}{l}\right)^2}\cdot\left\{\cos\frac{p\pi}{l}\cdot z-\frac{(-1)^p shk_z\cdot(L-l)}{shk_z\cdot L}\cdot chk_z\cdot z\right\} \qquad (6)$$

For H - the resonator the integrated equation can be written down as:

$$I(r)+\int_{r_1}^{r_2}K\,I(r_0)\,r_0dr_0=0, \qquad (7)$$

where $K=0.5a\left\{\frac{1}{r}\frac{\partial}{\partial r}(r\cdot G_{21})-\frac{1}{r}\frac{\partial}{\partial\varphi}G_{11}\right\}$.

The decision of these equations can be carried by Galerkin method, and for (1) it is more preferable to choose Fourier series as basis, and for (2) to use sewing together on points. Results of calculation are shown on fig.1, fig.2, fig.3.

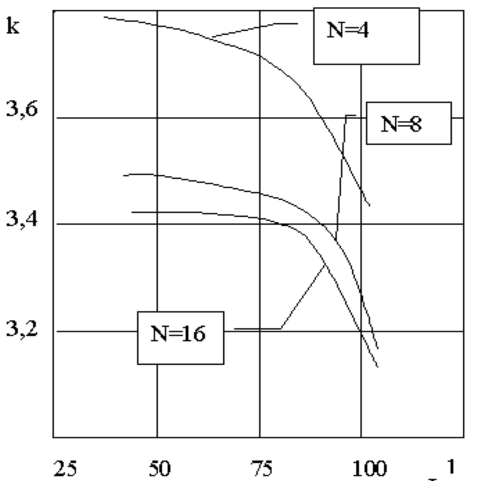

Fig.1. Dependence of wave number of period length (E - mode)

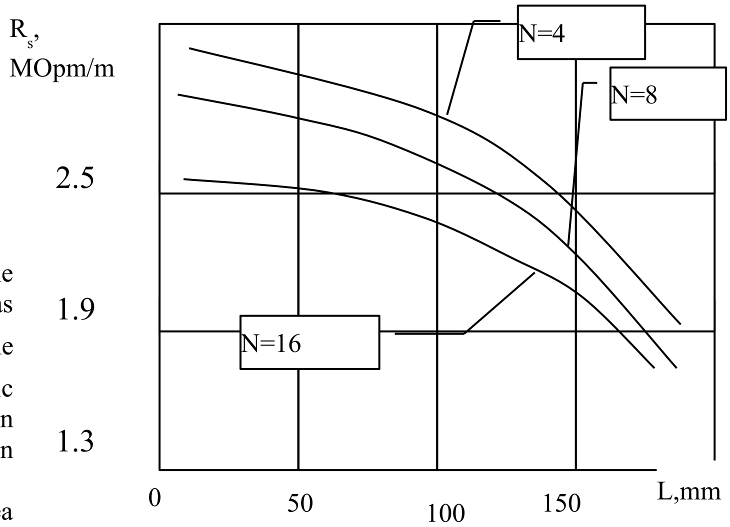

Fig.2. Dependence of shunt resistance from length period (E - mode)

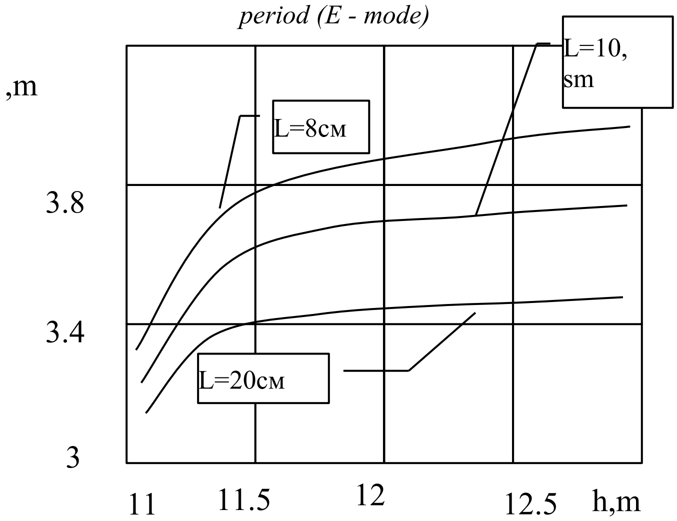

Fig.3. Dependence of wavelength from length of the resonator (H - mode)

Testing of the program was carried out by comparison of results of calculation by an offered method and calculation with the help of package ISFEEL 3D. Comparisons (for E-modes) have shown, that the error of definition of frequency is lesser than 3% at a time of processing 10...20 seconds while computer modelling for four drift tubes on an azimuth has taken 75 minutes for one point. By the shunt impedance the error lays within the limits of 10%.

2. MODELLING MULTIBEAM SYSTEMS ON BASIS OF SPIRAL ELECTRODES

Of greatest interest for acceleration of heavy ions are structures with electrodes of a spiral type. In [4] one of possible variants of the similar accelerator is presented. Actually the resonator works as follows: the spiral electrode being a resonant element is activated on its own lowest frequency. The currents reaching the spiral form the distribution of superficial charges at surfaces of conducting ring (CR) with drift tubes (TD) and then between TD the accelerating potential difference is formed. Thus, from the experiment it is known, that the size of the external resonator does not influence the frequency of a spiral, as well as accommodation of an electrode. At spatial arrangement of spirals with quadrupoles the characteristic frequency of a working mode of fluctuations ceases to depend on the number of electrodes, that also is the experimental fact.

All above-mentioned listed features allow us to formulate the following mathematical consequences: in the solution of the Helmholtz equation, there is no necessity to use the Green tensor function as the influence of the screen on the oscillatory process is insignificant. The taking into account of several electrodes is also unessential. The problem of the analysis as a matter of fact is reduced to a problem about a field created by a spiral, fixed on the conducting plane. Thus, for the solution we shall use a method of secondary sources: conducting surface we shall replace by the mirror image of a spiral, and for a current we shall write down, using threadlike model:

$$\delta\, dV_0 = I(s)\,\tau(s)\,ds, \qquad (8)$$

where ds is the element of spiral length $\tau(s)$ is the vector of a tangent to a spiral.

The vector potential will look like:

$$\vec{A} = \frac{\mu_0}{4\pi}\int_0^s \left\{\frac{e^{-jkR_+}}{R_+} - \frac{e^{-jkR_-}}{R_-}\right\} I(s)\,\vec{\tau}(s)\,ds, \qquad (9)$$

where R_+, R_- the distances between an observation point of a source and a point on the mirror image respectively.

The equation of a spiral in this case will be written down as follows:

$$\begin{cases} r_0 = a \\ \varphi_0 = \dfrac{2\pi}{b}, \quad z_0 = ctg\psi\, \dfrac{z_0}{a}, \\ z_0 \in [0, l] \end{cases} \qquad (10)$$

where r_0, φ_0, z_0 are the coordinates on a spiral, a is the radius of the coil, b is the step of a spiral, ψ_0 is the angle of winding. Distance up to observation points :

$$R_\pm = (z \mp z_0)^2 + r^2 - a^2 - 2ar\cos(\varphi - \varphi_0) \qquad (11)$$

The length element is correct in the form

$$ds = \frac{dz_0}{\sin\psi}, \qquad (12)$$

where $\sin\psi = \dfrac{b}{\sqrt{b^2 + (2\pi a)^2}}$.

Note the importance of Equation (12) as parametrical forms of an length element, since in this case the current acts as an explicit function of the coordinate z that allows essentially to facilitate the model.

Substituting (10) in expression for vector potential, we shall receive the integrated equation being mathematical model of a physical problem:

$$I(z) - \frac{\delta}{2\sin\psi}\int_0^l \left\{\vec{N} \cdot rot\,\vec{G}\right\}\cdot\vec{\tau}\cdot I(z_0)\,dz_0. \qquad (13)$$

Making consistently all the vector operations and entering explicit expressions for the Frene's trihedron, it is possible to write the kernel of the integrated equation in the following final form:

$$K = \left\{\cos\psi\left(\frac{\delta}{b}\sin\alpha\,\sin^2\psi + \cos\psi\right) + \sin^2\psi\right\} \times$$
$$\times \left\{\left(\left[\frac{1}{r}\frac{\partial}{\partial\varphi}G\right]\sin\psi - \left[\frac{\partial}{\partial z}G\right]\cos\psi\right)\sin\alpha + \cos\alpha\left[\frac{\partial}{\partial r}G\right]\right\} \qquad (14)$$

and to reduce the equation to the initial form:

$$I(z) - \lambda\int_0^l K\, I(z_0)\,dz_0 = 0, \qquad (15)$$

with $\lambda = \dfrac{\delta}{2\sin\psi}$.

The solution can be obtained also by the method of sewing together on points. All given algorithms are realized as program modules, the time of the account by which is some tens seconds. Results are submitted on

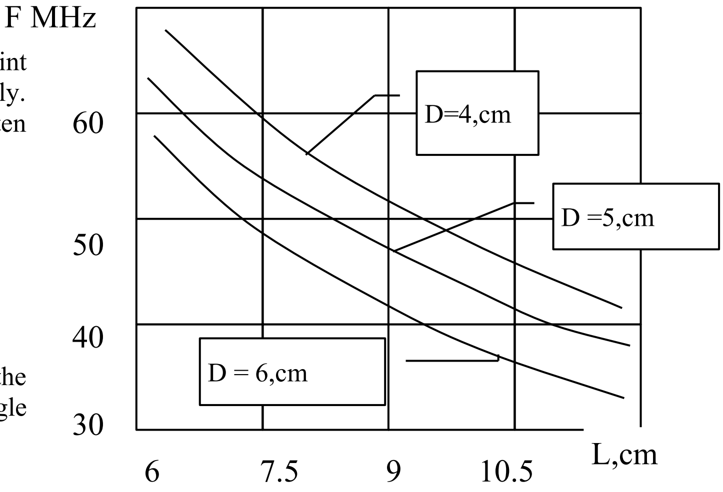

Fig.4. Dependence of resonant frequency of a spiral on its length

Results of calculations compared with experimental data showed that the error of definition of frequency lays within the limits of 5%.

3. CONCLUSIONS

In this paper were presented the electrodynamic models of multibeam systems based on E and H resonators and structures with the spiral electrodes, admitting numerical realization in MATCAD environment.

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ МНОГОЛУЧЕВЫХ УСКОРЯЮЩИХ СТРУКТУР

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На основе метода интегральных уравнений проведено трехмерное численное моделирование многолучевых ускоряющих структур. Для Е- и Н- резонаторов при использовании нитевидного представления тока источника и тензорной формы функции Грина для цилиндрического резонатора получены дисперсионные уравнения, выражения для шунтового импеданса и добротности.

ЧИСЕЛЬНЕ МОДЕЛЮВАННЯ БАГАТОПРОМЕНЕВИХ ПРИСКОРЮЮЧИХ СТРУКТУР

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На основі методу інтегральних рівнянь проведене тривимірне чисельне моделювання багатопроменевих прискорюючи структур. Для Е- и Н- резонаторів при використанні нитковидного зображення струму джерела і тензорної форми функції Гріна для циліндричного резонатора отримані дисперсійні рівняння, вирази для шунтового імпедансу і добротності.