REFLECTION OF ELECTROMAGNETIC ULTRA-SHORT PULSE ON NON-UNIFORM DIELECTRIC MEDIUM

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In this paper the reflection of ultra-short pulse electromagnetic waves on inhomogeneous dielectric matter determined by dielectric susceptibility $\varepsilon(z)$ is discussed. Coefficient of reflection is obtained. Frequency dispersion is neglected. Ultra-short pulse waves are considered as a row of Lugger polynoms.

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INTRODUCTION

The problem of carrying out researches related to the propagation of monochromatic and, particularly, ultra-short pulse (few-cycle) electromagnetic waves in non-uniform medium is very topical because we have to consider this problem in geophysics, optics of semiconductors and polymers, physics of laboratory and cosmic plasma in the first case and in electrodynamics of ultra-short (transient) pulse electromagnetic waves in the latter case. Exact solutions of Maxwell equations for the profile of dielectric permeability

$$\varepsilon(z) = n_0^2\left(1 + \frac{s_1}{L_1}z + \frac{s_2}{L_2}z^2\right)^{-2}$$

and reflection coefficients of ultra-short pulse waves are obtained in this paper.

1. THE MODEL AND BASIC EQUATIONS

Paper [1] deals with the common (universal) approach used while studying propagation of planar monochromatic waves in non-uniform dielectric media. Paper [2] concerns the propagation of ultra-short pulse waves in non-uniform dielectric media. By using the approach stated in [1], let us consider propagation of a planar electromagnetic wave in non-uniform, non-magnetic dielectrics determined by the dielectric permeability ε depending on the coordinate z. Suppose that dependence $\varepsilon(z)$ is as follows:

$$\varepsilon(z) = n_0^2 U^2(z), \tag{1.1}$$

where n_0 - refraction index in the neighborhood of the dielectric medium boundary $z = 0$; $U(z)$ - dimensionless function determining spatial distribution of the dielectric permeability.

$$U(z)\big|_{z=0} = 1. \tag{1.2}$$

Let us write Maxwell equations for the continuous medium:

$$rot\vec{E} = -\frac{1}{c}\frac{\partial B}{\partial t}, \tag{1.3}$$

$$rot\vec{H} = \frac{1}{c}\frac{\partial D}{\partial t}. \tag{1.4}$$

Suppose that $\vec{D} = \varepsilon(z)E_x e_x$, $B = H_y e_y$ and, by using equations (1.3), (1.4) we can obtain:

$$\frac{\partial E_x}{\partial z} = -\frac{1}{c}\frac{\partial H_y}{\partial t}, \tag{1.5}$$

$$-\frac{\partial H_y}{\partial z} = \frac{n_0^2 U^2(z)}{c}\frac{\partial E_x}{\partial t}. \tag{1.6}$$

Taking into account the fact that the vector-potential A determines the electric and magnetic components of the electromagnetic field by equations (1.7) and (1.8)

$$\vec{E} = -\frac{1}{c}\frac{\partial A}{\partial t}, \tag{1.7}$$

$$\vec{H} = rot\vec{A}, \tag{1.8}$$

where $A = A_x e_x = A_0\psi(z,t)e_x$ in our case, we obtain

$$\frac{\partial^2\psi}{\partial z^2} - \frac{n_0^2U^2(z)}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0. \tag{1.9}$$

Let us use new functions F and Q, and variable η in order to simplify equation (1.9).

$$\psi = F(\eta,t)U^{-\frac{1}{2}}(z), \tag{1.10}$$

$$Q(z) = U^{-1}(z), \tag{1.11}$$

$$\eta = \int_0^z U(z_1)dz_1. \tag{1.12}$$

Equation (1.9) gives in this case

$$\frac{\partial^2F}{\partial\eta^2} - \frac{n_0^2}{c^2}\frac{\partial^2F}{\partial t^2} = p^2F, \tag{1.13}$$

$$\text{where } p^2 = \frac{1}{4}\left(\frac{\partial Q}{\partial z}\right)^2 - \frac{1}{2}Q\frac{\partial^2Q}{\partial z^2}. \tag{1.14}$$

Let us make an assumption that $p = const$ in order to determine the profile U. By substituting infinite series $\sum_{i=-\infty}^{\infty} d_i z^i$ in (1.14), we can deduce that

$$Q = 1 + s_1z/(L_1) + s_2z^2/L_2^2, \tag{1.15}$$

$$p^2 = s_1^2/(4L_1^2) - s_2/L_2^2, \tag{1.16}$$

$$\text{where } s_1 = 0,\pm1;\ s_2 = 0,\pm1. \tag{1.17}$$

2. PROPAGATION OF ULTRA-SHORT PULSE

Using new variables (2.1)

$$\tau = ctn_0^{-1},\ \beta = \eta p,\ \upsilon = \tau p, \tag{2.1}$$

equation (1.13) can be written as

$$\frac{\partial^2F}{\partial\beta^2} - \frac{\partial^2F}{\partial\upsilon^2} = F. \tag{2.2}$$

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This is the Klein-Gordon equation. Equation (2.2), exact non-periodic solutions describing non-stationary electromagnetic waves propagating in non-uniform dielectrics from boundary $\beta = 0$ ($\eta = 0$, $z = 0$) into its depth are determined by the formulae as follows:

$$F = \sum_q a_q f_q(\beta, \upsilon), \quad (2.3)$$

$$f_q(\beta, \upsilon) = \left[\zeta_{q-1}(\beta, \upsilon) - \zeta_{q+1}(\beta, \upsilon)\right], \quad (2.4)$$

$$\zeta_q(\beta, \upsilon) = \left(\frac{\nu - \beta}{\upsilon + \beta}\right)^{\frac{q}{2}} J_q\left(\sqrt{\upsilon^2 - \beta^2}\right), \quad (2.5)$$

where J_q - Bessel function. It is evidently that

$$\frac{\partial \zeta_q}{\partial \upsilon} = \frac{1}{2}\left(\zeta_{q-1} - \zeta_{q+1}\right), \quad (2.6)$$

$$\frac{\partial \zeta_q}{\partial \beta} = -\frac{1}{2}\left(\zeta_{q-1} + \zeta_{q+1}\right). \quad (2.7)$$

Thus, the potential of the electromagnetic field is equal to

$$A_x = A_0\psi = A_0 F U^{\frac{1}{2}}(z) = A_0 U^{\frac{1}{2}}(z)\sum_q f_q(\beta, \upsilon) =$$

$$= \frac{1}{2}A_0 U^{\frac{1}{2}}(z)\sum_q a_q\left[\zeta_{q-1}(\eta p, \tau p) - \zeta_{q+1}\right]. \quad (2.8)$$

Electrical and magnetic components are as follows:

$$E_x = -\frac{A_0 p}{n_0}U^{\frac{1}{2}}(z)\sum_q a_q e_q, \quad (2.9)$$

$$H_y = -\frac{A_0 p}{n_0}U^{\frac{1}{2}}(z)\sum_q a_q h_q, \quad (2.10)$$

where $e_q = \frac{1}{4}\left[\zeta_{q-2}(\eta p, \tau p) - 2\zeta_q + \zeta_{q+2}\right]$, (2.11)

$$h_q = \frac{1}{4}n_0\left\{U(z)\left[\zeta_{q-2}(\eta p, \tau p) - \zeta_{q+2}\right] - \right.$$

$$\left. - \frac{1}{pU(z)}\frac{\partial U(z)}{\partial z}\left[\zeta_{q-1} - \zeta_{q+1}\right]\right\}. \quad (2.12)$$

Variable η can be deduced by using formula (1.11). It is equal, for instance, to

$$\eta\big|_{p^2>0, s_2<0, s_1>0} = \frac{L_2}{\sqrt{1+y^2}}\, arth\frac{zL_2^{-1}\sqrt{1+y^2}}{1+s_1 z/(2L_1)}, \quad (2.13)$$

$$\eta\big|_{\substack{p^2>0, s_2<0 \\ s_1>0, y^2>1}} = \frac{L_2}{\sqrt{y^2-1}}\, arth\frac{zL_2^{-1}\sqrt{y^2-1}}{1+s_1 z/(2L_1)}, \quad (2.14)$$

where y denotes the expression $y = L_2/(2L_1)$. Electrical and magnetic components calculated in the boundary under consideration ($\eta|_{z=0} = 0$, $U(z)|_{z=0} = 1$) are equal to

$$E_x^{(0)} = -\frac{A_0 p}{n_0}\sum_q a_q e_q^{(0)},$$

$$H_y^{(0)} = -\frac{A_0 p}{n_0}\sum_q a_q h_q^{(0)}, \quad (2.15)$$

$$e_q^{(0)} = \frac{1}{4}\left[J_{q-2}(\tau p) - 2J_q(\tau p) + J_{q+2}(\tau p)\right], \quad (2.16)$$

$$h_q^{(0)} = \frac{1}{4}n_0\left\{\left[J_{q-2}(\tau p) - J_{q+2}(\tau p)\right] - \right.$$

$$\left. - s_1 p^{-1}L_1^{-1}\left[J_{q-1}(\tau p) - J_{q+1}\right]\right\} \quad (2.17)$$

Let us consider the ultra-short pulse electrical component to be determined by the series expansion on Lugger functions $L_m(x)$.

$$E(x) = \sum_{m=0}^{\infty} b_m L_m(x), \quad (2.18)$$

where $L_m(x) = \frac{exp(x/2)}{m!}\frac{d^m}{dx^m}\left[exp(-x)x^m\right]$,

$x = \left(t - zc^{-1}\right)/t_0$, t_0 - parameter. It is known that

$\int_0^{\infty} L_m(x)L_n(x)dx = \delta_{mn}$, $L_0 = exp(-x/2)$, $L_1 = (1-x)L_0$,

$L_2 = \left(1 - 2x + x^2/2\right)L_0$. Let $E(t) = E_{in}F_m(x)$, where

$$F_m(x) = B\left[L_m(x) - L_{m+2}\right] = 2Bx\left(1 + \sum_{k=1}^{\infty} C_k x^k\right).$$

Let us consider wave $E(t) = E_{in}F_0(x)$, where $F_0(x) = B\left[L_0(x) - L_2(x)\right]$, falling from the vacuum onto the dielectric medium. It is necessary $E_x^{(0)}$ and $H_y^{(0)}$ to be written by using a series expansion on Lugger functions in order to obtain reflection coefficients because we are going to equalize wave harmonics by using continuity conditions.

$$E_x^{(0)} = -\frac{A_0 p}{n_0}\sum_{m=0}^{\infty} T_{1m}L_m(x'),$$

$$H_y^{(0)} = -\frac{A_0 p}{n_0}\sum_{m=0}^{\infty} T_{2m}L_m(x'), \quad (2.19)$$

$$T_{1m} = \sum_{q=3}^{\infty} a_q P_{mq}(\alpha), \; T_{2m} = \sum_{q=3}^{\infty} a_q Q_{mq}(\alpha), \quad (2.20)$$

$$P_{mq}(\alpha) = \int_0^{\infty} L_m(x')e_q^{(0)}(x'\alpha)dx', \quad (2.21)$$

$$Q_{mq}(\alpha) = \int_0^{\infty} L_m(x')h_q^{(0)}(x'\alpha)dx', \quad (2.22)$$

where $x' = t/t_0$, $\alpha = t_0 c n_0^{-1} p$. By using the continuity conditions we get the infinite set of equations. It is as follows for the electrical component:

$E_{in}B(1 + R_0) = -A_0 p n_0^{-1}T_{10}$, $E_{in}BR_1 = -A_0 p n_0^{-1}T_{11}$,

$E_{in}B(-1 + R_2) = -A_0 p n_0^{-1}T_{12}$, and so on. (2.23)

In general, the set is written as:

$$E_{in}B(l_m + R_m) = -A_0 p n_0^{-1}T_{1m},$$

$$E_{in}B(l_m - R_m) = -A_0 p n_0^{-1}T_{2m}, \quad (2.24)$$

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where $l_m = 0, \pm 1$. From here, the desired reflection coefficients are determined by (2.25).

$$R_m(\alpha) = l_m(1 - T_{2m}/T_{1m})/(1 + T_{2m}/T_{1m}) . \qquad (2.25)$$

Coefficients a_q can be calculated by using the infinite set of algebraic equations (2.24).

CONCLUSIONS

Coefficient of reflection of electromagnetic pulse waves determined by (2.18) on the boundary between vacuum and non-uniform dielectric medium described by spatial distribution of dielectric permeability (1.1), (1.11), (1.15), (1.17) is obtained. The approach used in [1] in order to solve exactly Maxwell equations is generalized while deducing reflection coefficients (2.25).

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ОТРАЖЕНИЕ ЭЛЕКТРОМАГНИТНОГО ИМПУЛЬСА ОТ НЕОДНОРОДНОЙ ДИЭЛЕКТРИЧЕСКОЙ СРЕДЫ

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В работе рассмотрено отражение видеоимпульсов от неоднородной диэлектрической среды с диэлектрической проницаемостью $\varepsilon(z)$. Частотная дисперсия не учитывается. Для видеоимпульса используется разложение в ряд по функциям Лаггера. Найдены коэффициенты отражения.

ВІДБИТТЯ ЕЛЕКТРОМАГНІТНОГО ІМПУЛЬСУ ВІД НЕОДНОРІДНОГО ДІЕЛЕКТРИЧНОГО СЕРЕДОВИЩА

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У роботі розглянуто відбиття відеоімпульсів від неоднорідного діелектричного середовища із діелектричною проникністю $\varepsilon(z)$.Частотна дисперсія не враховується. Для відеоімпульсу використовується розклад в ряд за функціями Лаггера. Знайдено коефіцієнти відбиття.