NON-LINEAR EVOLUTION OF VORTICES IN HIGH-CURRENT ELECTROSTATIC PLASMA LENS

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The spatial structure and nonlinear dynamics of vortices in plasma lens for high-current ion-beam focusing have been investigated theoretically.

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1. INTRODUCTION

It is known from numerical simulations and experiments that vortices are long-lived structures in vacuum. However, the acceleration of evolution of vortices in electron plasma was observed in laboratory experiments. Same dynamics of vortices should take place in near wall turbulence of nuclear fusion installations, where the crossed configuration of electrical and magnetic fields also is realized.

The charged plasma lens, intended for focusing of high-current ion beams, has the same crossed configuration of fields [1]. It is important to know the properties of vortices at the nonlinear stage of their evolution. It has been shown theoretically in this paper, that after reaching the quasi-stationary state the electrons in a field of a vortex rotate around its axis with the higher velocity in comparison with the velocity of azimuthal drift of electrons in the fields of the lens. Slow and quick vortices are contacting combinations of two vortices rotated in the opposite directions.

The instability development in the initially homogeneous plasma causes that the vortices are born pairs. Namely, if the vortex-bunch of electrons is generated, the vortex-hole of electrons occurs near it. It has been shown, that at small inhomogeneous electron density in the real experimental lens the preference is realized in the behaviour of vortices. Namely, the vortex - bunch goes to the region of a higher electron density n_e , and vortex – hole goes to the region of lower n_e .

2. JOINT DEVELOPMENT OF TWO INSTABILITIES

In [2] the dispersion law of oscillations, possible in the plasma lens is presented. The obtained dispersion law describes the joint development of two instabilities. Namely, in a limiting case $l_\theta\omega_{\theta o}<<k_zV_{bi}$, basically, the instability of the ion stream relative to electrons develops. Here V_{bi} is the ion beam velocity, k_z is the longitudinal wave vector, l_θ is the azimuthal angular number, $\omega_{\theta o}$ is the angular velocity of the electron drift in the crossed fields. Thus, the growth rate of the instability development in the case $l_\theta\omega_{\theta o}<k_zV_{bi}$ increases with the growth of k_z. In the limiting case $l_\theta\omega_{\theta o}>>k_zV_{bi}$ the instability of electrons, drifting relative to ions in the crossed fields in the cylindrical system with a radial gradient of the magnetic field develops. Slow vortices have the highest growth rate. Joint development of two instabilities under conditions typical for experiments $l_\theta \omega_{\theta o}>k_zV_{bi}$, results to that the growth rate of the slow vortical perturbation is more for the most homogeneous perturbation in the longitudinal direction, as the finite dimensions of the lens allow.

At instability development the vortices are born as follows. The non-uniform electric field $\mathbf{E}(\mathbf{r}, t)$ ($\equiv-\nabla \varphi(\mathbf{r}, t)$), arising as a result of the instability development, leads to the nonuniform electron dynamics with a velocity perturbation $\delta\mathbf{V}(\mathbf{r}, t)\approx(e/m_e\omega_{ce})[\mathbf{e}_z ,\nabla\varphi]$. As a result of a nonuniform $\delta\mathbf{V}(\mathbf{r}, t)$ the electron bunching is performed which results in the nonuniform distribution of the electron density perturbation $\delta n_e\approx n_o(\mathbf{k}\delta\mathbf{V})/(\omega-l_\theta\omega_{\theta o})$. Last automatically results in the vortical movement of the electrons with a vorticity $\alpha\equiv\mathbf{e}_z\mathrm{rot}\mathbf{V}\approx(\omega^2_{pe}/\omega_{ce})\delta n_e/n_{eo}$.

3. SPATIAL STRUCTURE OF VORTICES

Let us describe the structure of a quick vortex in the rest frame, rotating with the angular velocity $\omega_{ph}\equiv V_{ph}/r_q$. Let us consider a chain on θ of vortices - bunches and vortexes - holes of electrons. Neglecting nonstationary and nonlineary- on $\boldsymbol{\phi}$- terms, we derive the following equation

$$\mathbf{V}_\perp=-(e/m_e\omega_{He})[\mathbf{e}_z,\mathbf{E}_{ro}]+(e/m\omega_{He})[\mathbf{e}_z,\nabla\boldsymbol{\phi}], \qquad (1)$$

describing the quasi-stationary dynamics of electrons in the fields of the lens and the vortical perturbation. From (1) we obtain the expression for radial and azimuthal electron velocities

$$V_r=-(e/m_e\omega_{He})\nabla_\theta\phi , \quad V_\theta=V_{\theta o}+(e/m_e\omega_{He})\nabla_r\phi ,$$

$$V_{\theta o}=-(e/m_e\omega_{He})E_{ro}=(\omega^2_{pe}/2\omega_{He})(\Delta n/n_{oe})r \qquad (2)$$

V_θ can be presented as a sum of the phase velocity of perturbation, V_{ph}, and velocity of azimuth electron oscillations, δV_θ , in the field of perturbation, $V_\theta=V_{ph}+\delta V_\theta$. Because $V_\theta=rd\theta/dt$, we present $d\theta/dt$ as

$$d\theta/dt=d\theta_1/dt+\omega_{ph} ,$$

where $\omega_{ph}=(\Delta n/n_{oe})(\omega^2_{pe}/2\omega_{He})|_{r=rv}$, r_v is the radius of the vortical perturbation location. Then from (2) we obtain

$$d\theta_1/dt=(\omega^2_{pe}/2)(\Delta n/n_{oe})[1/\omega_{He}(r)-$$
$$-1/\omega_{He}(r_v)]+(e/rm_e\omega_{He})\partial_r\phi , \quad dr/dt=-(e/m_e\omega_{He}r)\partial_\theta\phi \qquad (3)$$

At small diversions of r from r_v, decomposing $\omega_{He}(r)$ on $\delta r\equiv r-r_v$ and integrating (3), we derive

$$(\delta r)^2-2\omega_{He}(r_v)\phi/\pi e\Delta nr_v(\partial_r\omega_{He})|_{r=rv}=\mathrm{const} \qquad (4)$$

The vortex boundary separates the trapped electrons, forming the vortex and moving on closed trajectories and untrapped electrons, moving outside the boundary of the vortex and oscillating in its field. For vortex boundary we derive the following expression from the condition $\delta r|_{\phi=-\phi_o}=\delta r_{cl}$

$$\delta r=\pm[2(\phi+\phi_o)\omega_{He}(r_v)/\pi e\Delta n r_v(\partial_r\omega_{He})|_{r=rv}+(\delta r_{cl})^2]^{1/2} \quad (5)$$

Here δr_{cl} is the radial width of the vortex - bunch of electrons. From (5) the radial size of the vortex - hole of electrons follows

$$\delta r_h\approx 2[\phi_o\omega_{He}(r_v)/\pi e\Delta n r_v(\partial_r\omega_{He})|_{r=rv}]^{1/2} \quad (6)$$

From the equation of electron motion and Poisson equation it is possible to derive approximately the expression for the vorticity $\alpha\equiv\mathbf{e}_z\mathrm{rot}\mathbf{V}$, which is characteristic of the vortical motion of electrons

$$\alpha\approx-2eE_{ro}/rm\omega_{He}+(\omega^2_{pe}/\omega_{He})\delta n_e/n_{eo}$$

From here it follows that up to certain amplitude of vortices the structure of electron trajectories in the field of the chain on θ of quick vortices in the system of rest, rotated with $\omega_{ph}\equiv V_{ph}/r_q$, is similar to the structure, presented in [2].

For large amplitudes of quick vortices in the region of electron bunches the contraflows are formed. The vortex - hole rotates in the rest frame, rotating with a frequency $\omega_{ph}\equiv V_{ph}/r_q$, in the same direction as unperturbed plasma. The vortex - bunch rotates in the opposite direction of rotation of unperturbed plasma at $\delta n_e>\Delta n\equiv n_{oe}-n_{oi}$. It is seen the size of the vortex is inversely proportional to $[(\Delta n/n_{oe})(\omega_{pe}/\omega_{eH})\partial_r\omega_{He}]^{1/2}$ and is proportional to $\phi^{1/2}_o$. That is the size of the vortex essentially depends on the gradient of the magnetic field. At low $\Delta n/2n_{oe}$ and ω_{pe}/ω_{He} already at small perturbations of electron density the sizes of the vortex, δr_h, can reach $\delta r_h\approx R/2$, R is the plasma lens radius

(3) can be integrated without decomposition $\omega_{He}(r)$ on $\delta r\equiv r-r_v$. For this purpose we approximate $\omega_{He}(r)=\omega_{Ho}(1+\mu r^2/R^2)$. Then, integrating (3), we derive

$$2\phi+\pi e\Delta n r^2[1-\omega_{Ho}/2\omega_{He}(r_v)-\omega_{He}(r)/2\omega_{He}(r_v)]=\mathrm{const} \quad (7)$$

From the condition $r|_{\phi=-\phi_o}=r_v+\delta r_{cl}$ and (7) we obtain the expression, determining the boundary of the vortex - hole of electrons,

$$[r^2-(r_v+\delta r_{cl})^2][1-\omega_{Ho}/\omega_{He}(r_v)]-[r^4-$$
$$-(r_v+\delta r_{cl})^4]\omega_{Ho}\mu/2R^2\omega_{He}(r_v)+2(\phi+\phi_o)/\pi e\Delta n=\mathrm{const} \quad (8)$$

From (8) and $r|_{\phi=\phi_o}=r_v+\delta r_h$ we derive the expression, determining the radial width of the vortex - hole of electrons,

$$\phi_o 4R^2\omega_{He}(r_v)/\pi e\Delta n]\omega_{Ho}\mu=$$
$$=(\delta r_h-\delta r_{cl})(2r_v+\delta r_h+\delta r_{cl})[r_v(\delta r_h+\delta r_{cl})+(\delta r_h^2+\delta r_{cl}^2)/2] \quad (9)$$

Let us consider the vortex with the small phase velocity V_{ph} in comparison with the drift electron velocity, $V_{ph}<<V_{\theta o}$. The spatial structure of the electron trajectories in its field for small amplitudes of the vortex looks like that shown in Fig.1. It is determined by that in all lens α has an identical sign, $\alpha>0$. In other words, the radial electric field, created by the vortex is less, than the electric field of the lens, $E_{rv}<E_{ro}$. Then in all lens the azimuthal electron velocities have an identical sign and there are not contraflows of electrons. The slow vortex of a small amplitude does not have a separatrix. For the description of the electron trajectories we use (2). Using in them $V_\theta=rd\theta/dt$ and excluding θ, we obtain for boundary of the vortex $r(\theta)$

$$r=[r^2_s+(\phi_o-\phi)2/\pi e\Delta n]^{1/2} \quad (10)$$

In the case of small amplitudes (10) becomes

$$\delta r\equiv r-r_s=(\phi_o-\phi)/\pi e\Delta n r_s \quad (11)$$

From (10) we derive the radial size of the slow vortex

$$\delta r_s\equiv r|_{\phi=-\phi_o}-r_s=[r^2_s+4\phi_o/\pi e\Delta n]^{1/2}-r_s \quad (12)$$

In the case of small amplitudes (12) becomes

$$\delta r_s\approx 2\phi_o/\pi e\Delta n r_s \quad (13)$$

For the description of the slow vortex structure one can also use the equation

$$d_t\omega_{He}/n_e\approx 0,\ d_t=\partial_t+(\mathbf{V}_\perp\nabla_\perp)-V_{ph}\nabla_\theta \quad (14)$$

We obtain approximately from (14) the equation, describing the slow vortex of the small amplitude

$$dr/dt\approx-[n_o\omega_{He}/\partial_r\omega_{He}(r)][\partial_\tau-V_{ph}\nabla_\theta+V_{\theta o}\nabla_\theta](1/(n_o+\delta n)),$$
$$d\theta/dt\approx V_{\theta o}$$

or

$$\delta r\equiv r-r_v\approx\omega_{He}(r_v)\,\delta n/n_o r_v\partial_r\omega_{He}(r_v)$$

Because on $r=r_v$, $\delta n_o(r=r_v)=0$, on it the electron moves with $V_{\theta o}$ without radial perturbations. At $r>r_v$ there is a positive radial displacement, and at $r<r_v$ - negative radial displacement of the electrons. The radial size of the slow vortex is inversely proportional to the radial gradient of the magnetic field.

In the case of large amplitudes, $\delta n_e>\Delta n$ (or $E_{rv}>E_{ro}$), in the region where the electron holes are placed, the characteristic of the vortical motion α accepts an opposite sign, $\alpha<0$. In other words, on the axis, connecting the vortex - hole and vortex - bunch, the inequality $E_{rv}>E_{ro}$ is fulfilled, and there is an azimuthal contraflow of electrons. Then in some regions the electrons rotate in the direction, opposite to their rotation in crossed fields of the lens. The slow vortex is a dipole perturbation of the electron density, disjointed on radius. At $\delta n_e>\Delta n$ the structure of the slow vortex is similar to the structure of the Rossby vortex.

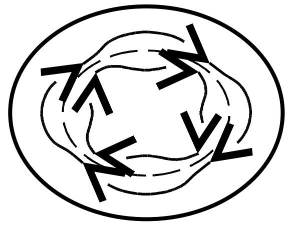

Fig.1.

4. SATURATION OF EXCITED HOMOGENEOUS SLOW VORTICAL TURBULENCE

For quick vortices the cause of the instability is the gradient of the velocity $\partial_r V_{\theta o}$, therefore for development of instability the nonadiabatic dynamics of electrons is necessary. For slow vortices the reason of the instability is the interaction of the drifting electron stream with ions, therefore amplitude of the saturation of the slow vortex is determined from the condition of the ion trapping

$$V_{tri}\approx V_{phs}. \quad (15)$$

or from the condition of the electron trapping

$$V_{tre}\approx(V_{\theta o}-V_{phs}) \quad (16)$$

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and is determined by smaller of them. For the plasma lens, close to the optimum plasma lens, the saturation is determined by electron trapping. For the plasma lens, far from the optimum plasma lens, the saturation is determined by ion trapping. The slow homogeneous turbulence is not separated into single vortices.

5. NONLINEAR DYNAMICS OF VORTICES

The development of instability in initially homogeneous plasma lens causes that the vortices are born pairs: if the vortex - bunch of electrons is generated, the vortex - hole of electrons occurs near it.

Let us consider how the nonhomogeneity of electron density effects on the behaviour of vortices. Finiteness of time of the vortices symmetrization and also the reflection of resonant electrons from vortices - bunches result that the vortices are asymmetrical. Namely, on opposite on θ parties of vortices the small bunches and holes are formed. It results in formation of polarization azimuth electric fields E_θ, directed along **e** $_\theta$. The formation of fields E_θ causes the radial drift and spatial separation of vortices (see fig.2). In other words, the property of preference of motion of the vortex - hole on the peripherals of the plasma column and the vortex - bunch to its axis is realized. The polarization electric fields in the vortex - hole and the vortex - hole have opposite signs. Then the velocities of radial drift of the vortex - hole and vortex - bunch have opposite signs. Namely, the vortex - hole goes to the region of a lower electron density, and the vortex - bunch goes to the region of higher electron density).

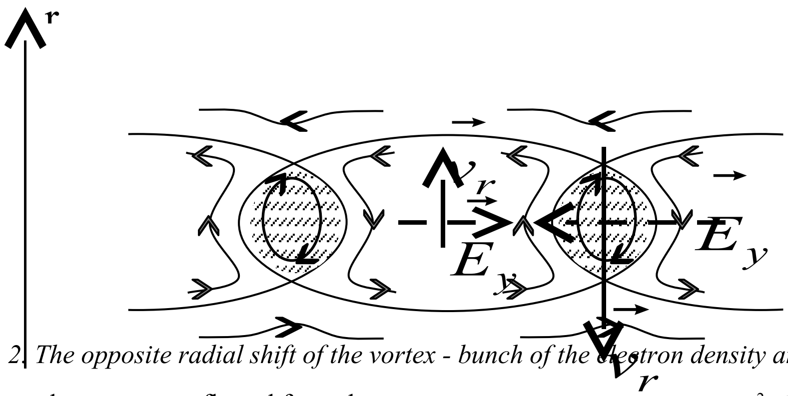

Fig. 2. The opposite radial shift of the vortex - bunch of the electron density and the vortex - hole

The resonant electrons are reflected from the vortex-bunch. Thus the distribution of the electron density being asymmetrical on azimuth is formed. It results in the radial motion of the vortex - bunch of electrons and leads to simultaneous formation of spiral distribution of the electron density. In the case of the azimuthally symmetrical vortex its velocity of radial drift is equal to

$$V_{rv} \approx (\omega^2_{pe}/2\omega_{He})(R^2_v/n_{oe})(dn_{oe}/dr)\big|_{r=rv} \quad (17)$$

R_v is the radius of the vortex. The width of the spiral is equal to the radial width of the vortex in the case of its high radial velocity. In the case of a low radial velocity of the vortex the width of the spiral is less, than the radial width of the vortex.

When two vortices - bunches of electrons begin to concern each other, the electrons of each vortex, taking place near to its boundary, are reflected from the next vortex. Thus the asymmetry is formed on the azimuth distribution of the electron density in the neighbourhood of each vortex. It leads to occurrence of a relative velocity of vortices.

$$V_{mer} \approx [\omega^2_{pe}(\delta n_e)/2\omega_{He}]R_v \quad (18)$$

The similar behaviour of electrons was observed in experiments in the only electron plasma, in the charged plasma of the lens [1,3] and in the plasma, placed in crossed radial electrical and longitudinal magnetic fields.

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НЕЛИНЕЙНАЯ ЭВОЛЮЦИЯ ВИХРЕЙ В СИЛЬНОТОЧНОЙ ЭЛЕКТРОСТАТИЧЕСКОЙ ПЛАЗМЕННОЙ ЛИНЗЕ

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Проведено теоретическое исследование пространственной структуры и нелинейной динамики вихрей в плазменных линзах для фокусировки больших ионных пучков.

НЕЛІНІЙНА ЕВОЛЮЦІЯ ВИХРІВ У ПОТУЖНОСТРУМОВІЙ ЕЛЕКТРОСТАТИЧНІЙ ПЛАЗМОВІЙ ЛІНЗІ

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Проведено теоретичне дослідження просторової структури і нелінійної динаміки вихрів у плазмових лінзах для фокусування великих іонних пучків.