CALCULATIONS OF THE MAGNETIC WELL DETERMINED BY DIFFERENT PLASMA PRESSURE PROFILES AND AN EXTERNAL TRANSVERSE MAGNETIC FIELD IN THE TORSATRON

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The present paper deals with the calculations of a relative magnetic-well depth in a torsatron using the formulas of averaging over magnetic surfaces. The calculations were made for different functions of vacuum angles of rotational transform in the radius and different plasma pressure profiles with due regard for an external uniform transverse magnetic field. The distribution of the vacuum angle of field lines rotation was calculated by the expression $t(r) = t(r_0)[\alpha+(1-\alpha)\cdot r^2/r_0^2]$, where $\alpha=t(0)/t(r_0)$ is the ratio of the angle of rotational transform on the magnetic axis to its value at the plasma boundary of radius r_0. The authors have considered three laws of plasma pressure distribution over magnetic surfaces: $P_1=P_0$; $P_2=P_0(1-\psi(r)/\psi(r_0))$; $P_3= P_0(1-\psi(r)/\psi(r_0))^2$; where P_0 is the plasma pressure on the axis of the system, $\psi(r)$ is the averaged function of vacuum magnetic surfaces. Analytical expressions have been derived in the paper to calculate the relative magnetic well depth determined by different plasma pressure profiles and by the external transverse magnetic field. The relative depth of the magnetic well was calculated by the formula $\delta U/U = B_{01}/\langle B(r_1)\rangle - 1$, where $\langle B(r_1)\rangle$ is the longitudinal magnetic field on the radius r_1 averaged over the magnetic surfaces.
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The magnetic field minimum is an important factor exerting an effect on plasma stability [1]. It has been demonstrated elsewhere that a rise in the plasma pressure gives rise to a magnetic well, that removes the restriction on the plasma-stability limiting pressure for a number of most dangerous MHD instabilities [2, 3]. The present paper describes analytical calculations of an average magnetic well in a torsatron for different functions of vacuum angles of rotational transform in the radius and for different plasma pressure profiles with due regard for an external transverse magnetic field. The calculations were performed using the formulas of averaging over magnetic surfaces [4]. The distribution of the vacuum angle of field lines rotation was calculated by the expression $t(r) = t(r_0)[\alpha+(1-\alpha)\cdot r^2/r_0^2]$, where $\alpha=t(0)/t(r_0)$ is the ratio of the angle of rotational transform on the magnetic axis to its value at the plasma boundary of radius r_0. The authors have considered three laws of plasma pressure distribution over vacuum magnetic surfaces: $P_1=P_0$ (uniform distribution); $P_2=P_0(1-\psi(r)/\psi(r_0))$; $P_3= P_0(1-\psi(r)/\psi(r_0))^2$, where P_0 is the plasma pressure on the axis of the system, $\psi(r)=\int t(r)rdr$ is the averaged function of vacuum surfaces [4]. The equations of averaged magnetic surfaces in the cylindrical coordinate system with due regard for the external transverse magnetic field have the following forms [5]:

for the pressure distribution $P_1=P_0$

$$\Psi(r)=\frac{\alpha}{2}\frac{r^2}{r_0^2}+\frac{1-\alpha}{4}\frac{r^4}{r_0^4}+M_\perp\frac{r}{r_0}\cos\theta-N\frac{r}{r_0}\cos\theta \qquad (1)$$

for the pressure distribution $P_2=P_0(1-\psi(r)/\psi(r_0))$

$$\Psi(r)=\frac{\alpha}{2}\frac{r^2}{r_0^2}+\frac{1-\alpha}{4}\frac{r^4}{r_0^4}+M_\perp\frac{r}{r_0}\cos\theta-\frac{N}{1+\alpha}\left(1-\frac{r^2}{2r_0^2}\right)\frac{r}{r_0}\cos\theta \qquad (2)$$

for $P_3=P_0(1-\psi(r)/\psi(r_0))^2$

$$\Psi(r)=\frac{\alpha}{2}\frac{r^2}{r_0^2}+\frac{1-\alpha}{4}\frac{r^4}{r_0^4}+M_\perp\frac{r}{r_0}\cos\theta- \frac{N}{1+\alpha}\left(\frac{2}{3}\frac{2+\alpha}{1+\alpha}-\frac{r^2}{r_0^2}\frac{2}{3(1+\alpha)}\left(\alpha\frac{r^4}{r_0^4}+\frac{1-\alpha}{4}\frac{r^6}{r_0^6}\right)\right)\frac{r}{r_0}\cos\theta \qquad (3)$$

where $M_\perp=\frac{H_\perp A}{H_0 t(r_0)}$; $N=\frac{\beta A}{t^2(r_0)}$; $A = R/r_0$ is the aspect ratio, R is the major radius of torus, $H_\perp/H_0$ is the ratio of the external uniform transverse magnetic field $H_\perp$ to the longitudinal field H_0. The average magnetic well determined by the pressure profiles $P_1 = P_0$ and $P_2 = P_0(1-\psi(r)/\psi(r_0))$ was calculated by the following formula [6]

$$\frac{\delta\overline{U}}{U}=\frac{1+\sqrt{1+c+b}/\sqrt{1+c-b}}{\sqrt{2}\sqrt{1+c+b}\sqrt{\sqrt{(1+c)^2-b_2}+1-3c}}\Big/ \frac{1+d_1}{(1-d_1)^2+c(1+6d_1+d_1^2)\ -b(1-d_1^2)}x\left[\frac{(1-d_1)^2}{2\sqrt{d_1}}+ \frac{4cd_1-b(1-d_1)+[4c-b(1-d_1)]\sqrt{1+c+b}/\sqrt{1+c-b}}{\sqrt{2}\sqrt{1+c+b}\sqrt{\sqrt{(1+c)^2-b^2}+1-3c}}\right]-1, \qquad (4)$$

where $d_1=\frac{1-r_1/R}{1+r_1/R}$. The coefficients a, b, c for the $P_1=P_0$ distribution are written as:

$$a=\alpha+(1-\alpha)r_1^2/r_o^2+2(1-\alpha)r_c^2/r_o^2,$$
$$b=3(1-\alpha)r_c r_1/ar_o^2,\quad c=(1-\alpha)/a\,r_c^2/r_o^2, \qquad (5)$$

and for the plasma pressure distribution $P_2 = P_0(1-\psi(r)/\psi(r_0))$ the coefficients a, b, c have the form:

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$$a=\alpha+(1-\alpha)\frac{r_1^2}{r_o^2}+2(1-\alpha)\frac{r_c^2}{r_o^2}-2\frac{r_c}{r_o}\frac{(1-\alpha)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)+\alpha\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)}{1-3r_c^2/2r_o^2};$$

$$b=\frac{1}{a}\left(-3(1-\alpha)\frac{r_c}{r_o}\frac{r_1}{r_o}+\frac{3}{2}\frac{r_1}{r_o}\frac{(1-\alpha)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)+\alpha\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)}{1-3r_c^2/2r_o^2}\right) \quad (6)$$

$$c=\frac{1}{a}\left((1-\alpha)\frac{r_c^2}{r_o^2}-\frac{r_c}{r_o}\frac{(1-\alpha)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)+\alpha\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)}{1-3r_c^2/2r_o^2}\right)$$

where $r_{c\perp}$ is the initial displacement due to the external transverse magnetic field.

For the plasma pressure distribution $P_3 = P_0(1-\psi(r)/\psi(r_0))^2$, the $\delta U/U$ ratio was calculated as [6]:

$$\frac{\delta U}{U}=\frac{\frac{A+B/\sqrt{q}}{\sqrt{p+2\sqrt{q}}}+\frac{C+D/\sqrt{q_1}}{\sqrt{p_1+2\sqrt{q_1}}}}{\frac{1+d_2}{2}\left[\frac{A_1+B_1/\sqrt{q}}{\sqrt{p+2\sqrt{q}}}+\frac{C_1+D_1/\sqrt{q_1}}{\sqrt{p_1+2\sqrt{q_1}}}+\frac{E_1}{\sqrt{d_2}}\right]}-1,$$

where $A=\frac{(q-q_1)(q-3)+(p-3)(p_1q-pq_1)+p-p_1}{(q-q_1)^2+(p-p_1)(q_1p-qp_1)}$,

$$B=\frac{(p-p_1)[p-q(3-q_1)]+(q-q_1)[-1+q(3-p_1)]}{(q-q_1)^2+(p-p_1)(q_1p-qp_1)}, \quad (7)$$

$$C=\frac{(q-q_1)[3-q_1-p_1(3-p_1)]+(p-p_1)[-1+q_1(3-p_1)]}{(q-q_1)^2+(p-p_1)(q_1p-qp_1)},$$

$$D=\frac{(q-q_1)(1-3q_1)-(p-p_1)(p_1-3q_1)+q_1(qp_1-pq_1)}{(q-q_1)^2+(p-p_1)(q_1p-qp_1)},$$

the A_1, B_1, C_1, D_1, E_1 values are calculated from the following equations

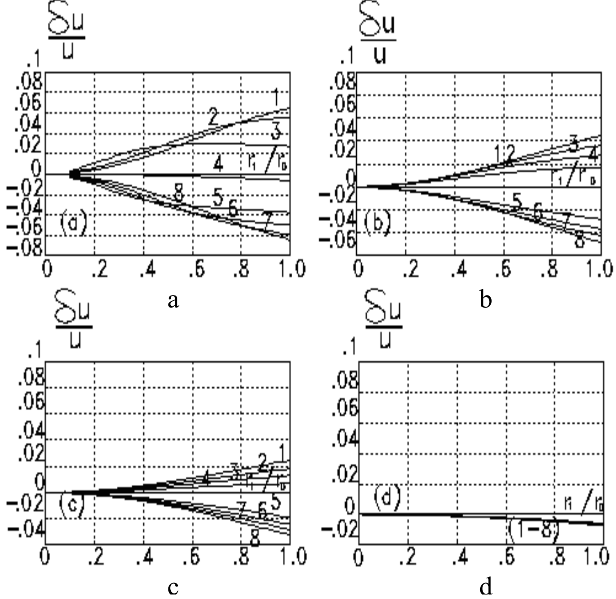

Fig. 1. The $\delta U/U$ ratio versus average radius r_1/r_0 at a uniform plasma pressure distribution $P_1 = P_0$. $A = 3.5$; $r_{c\perp}/r_0 = 0.5$; a– $\alpha=0$; b– $\alpha=0.3$; c– $\alpha=1.0$ (1– $r_c/r_0=0.5$; 2– $r_c/r_0=0.4$; 3– $r_c/r_0=0.3$; 4– $r_c/r_0=0.2$; 5– $r_c/r_0=-0.2$; 6– $r_c/r_0=-0.3$; 7– $r_c/r_0=-0.4$; 8– $r_c/r_0=-0.5$). The sign "minus" at the magnetic axis displacement r_c/r_0 means that the magnetic axis displacement is found on the ray $\vartheta=0$ directed to the outside of the torus

$$(p+p_1)E_1+d_2B_1+(1+d_2p_1)A_1+d_2D_1+$$
$$+(1+d_2p)C_1=4, \qquad E_1+d_2(A_1+C_1)=1,$$
$$(q+q_1+pp_1)E_1+(1+d_2p_1)B_1+(p_1+d_2q_1)A_1+$$
$$+(1+d_2p)D_1+(p+d_2q)C_1=6,$$
$$(p_1q+pq_1)E_1+(p_1+d_2q_1)B_1+q_1A_1+$$
$$+(p+d_2q)D_1+qC_1=4, \qquad qq_1E_1+q_1B_1+qD_1=1,$$

where

$$p=\frac{b_1+\sqrt{8y+b_1^2-4c_1}}{2},\ p_1=\frac{b_1-\sqrt{8y+b_1^2-4c_1}}{2},$$
$$q=y+\frac{b_1y-d_1}{\sqrt{8y+b_1^2-4c_1}},\ q_1=y-\frac{b_1y-d_1}{\sqrt{8y+b_1^2-4c_1}},$$
$$y=\sqrt[3]{-q_o+\sqrt{E_o}}-\sqrt[3]{-q_o-\sqrt{E_o}}+\frac{c_1}{6},\ E=q_o^2+p_o^3,$$
$$p_0=\frac{b_1d_1-4\ell}{12}-\frac{c_1^2}{36},\ q_o=-\frac{c_1^3}{216}+\frac{c_1b_1d_1}{48}-\frac{b_1^2\ell_1}{16}+$$
$$+\frac{\ell_1\ell}{6}-\frac{d_1^2}{16},\ a_1=1-b+c-d+\ell,\ b_1=(4-2b-4c+$$
$$+14d-28\ell)/a_1,\ c_1=(6-10c+70\ell)/a_1,$$
$$d_1=(4+2b-4c-14d-28\ell)a_1,$$
$$\ell_1=(1+b+c+d+\ell)/a_1,\quad d_2=1/(1+r_1/R),$$
$$a=\alpha+(1-\alpha)r_1^2/r_o^2+2((1-\alpha)r_c^2/r_o^2-2Nr_c^2/r_o^2\{-2+$$
$$+6M\alpha r_c^2/r_o^2+3M(1-\alpha)r_c^4/r_o^4+3M[2\alpha+$$
$$+3(1-\alpha)r_c^2/r_o^2]r_1^2/r_o^2+3M(1-\alpha)r_1^4/r_o^4\},$$
$$b=r_1/a\,r_o\{3(1-\alpha)r_c/r_o-N[3(-1+9M\alpha r_c^2/r_o^2+$$
$$+15/2M(1-\alpha)r_c^4/r_o^4)+5r_1^2/r_o^2M[\alpha+9/2(1-\alpha)r_c^2/r_o^2+$$
$$+7/4M(1-\alpha)r_1^4/r_o^4]\},$$
$$c=r_c/ar_o\{(1-\alpha)r_c/r_o-N[-2+8M\alpha r_c^2/r_o^2$$
$$+9/2M(1-\alpha)r_c^4/r_o^4+4r_1^2/r_o^2M[2\alpha+4(1-\alpha)r_c^2/r_o^2]+$$
$$+9/2M(1-\alpha)r_1^4/r_o^4]\},\ d=-3MNr_c^2r_1/ar_o^3[\alpha+ \quad (8)$$
$$+5/4(1-\alpha)r_c^2/r_o^2+5/4(1-\alpha)r_1^2/r_o^2],\ \ell=-MN(1-\alpha)r_c^3r_1^2/ar_o^5,$$
$$N=\frac{\alpha(1+\alpha)\left(\frac{r_{c\perp}}{r_o}-\frac{r_c}{r_o}\right)+(1+\alpha^2)\left(\frac{r_{c\perp}^3}{r_o^3}-\frac{r_c^3}{r_o^3}\right)}{K-3\frac{r_c^2}{r_o^2}+M[5\alpha\frac{r_c^4}{r_o^4}+\frac{7}{4}(1-\alpha)\frac{r_c^6}{r_o^6}]}=\frac{\beta A_o}{t^2(r_o)(1+\alpha)},$$
$$K=2/3\frac{2+\alpha}{1+\alpha},\quad M=\frac{2}{3(1+\alpha)}.$$

The $\delta U/U$ ratio was determined in terms of the longitudinal magnetic field $\langle B(r_1)\rangle$ averaged over the magnetic surfaces [4]:

$$\delta U/U = B_{01}/\langle B(r_1)\rangle - 1,$$

where $B_{01} = B_0/(1+r_c/r_0)$ is the longitudinal magnetic field value on the magnetic axis. Using the expressions derived, we have calculated the $\delta U/U$ ratio caused by the pressure profile $P_1 = P_0$ (uniform distribution) for different α values that describe the distribution of vacuum angles of field lines rotation in the radius. Figure 1 shows the behavior of $\delta U/U$ as a function of the displacement r_c/r_0 specified by the plasma pressure and the initial magnetic axis displacement $r_{c\perp}/r_0 = 0.5$ due to the external transverse magnetic field. It is evident from the figure that for the magnetic axis displacements r_c/r_0 being on the $\vartheta=\pi$ ray directed inwards the torus, we have $\delta U/U>0$, i.e., there is a magnetic hill (antiwell). With a magnetic axis displacement to the outside of the torus due to a plasma pressure rise (Fig. 1a), the δU/U ratio becomes negative, i.e., the magnetic well (δU/U<0) appears. At $\alpha \approx 1$ (Fig. 1d), with an increasing plasma pressure the magnetic well value tends to δU/U = – 0.005. As it was indicated in ref. [7], at $r_c/r_0 = 0$ in the central region of the magnetic configuration the rise in the plasma pressure due to the pressure profiles $P_2 = P_0(1-\psi(r)/\psi(r_0))$ or $P_3 = P_0(1-\psi(r)/\psi(r_0))^2$ and the opposing external transverse field, leads to magnetic axis splitting and gives rise to two families of independent magnetic surfaces. Figure 2 shows these two families of magnetic surfaces.

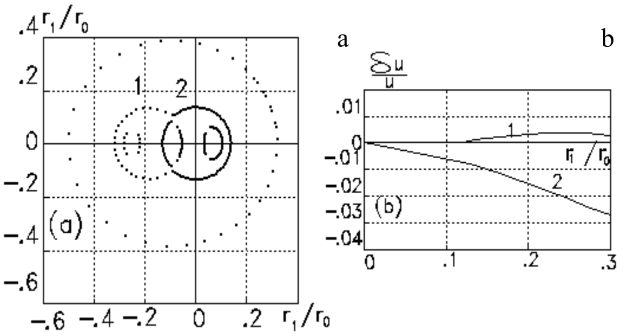

Fig. 2. a) shapes of magnetic surfaces arising in the central region of the magnetic configuration, due to the plasma pressure profile $P_2 = P_0(1-\psi(r)/\psi(r_0))$ and the opposing transverse magnetic field at the magnetic axis displacement $r_c/r_0 = 0$; $\alpha = 0$; $R/r_0 = 8.9$; $r_{c\perp}/r_0 = 0.5$; b) distribution of δU/U versus the average radius r_1/r_0 of the magnetic surfaces formed

The first family will have the magnetic hill (δU/U>0, Fig. 2b, 1), and the second family will have the magnetic well (δU/U<0, Fig. 2b, 2).

So, we have derived analytical expressions to calculate the average magnetic well determined by different plasma pressure profiles and the external transverse magnetic field. It has been shown that with a magnetic axis displacement to the inside of the torus due to the external transverse magnetic field and $\alpha = 0$, the resulting magnetic configuration will have a magnetic hill (δU/U>0). If the magnetic axis is displaced to the outside of the torus, due to the plasma pressure, the magnetic configuration will have the magnetic well (δU/U<0), and may attain δU/U = 11 % at the displacement $r_c/r_0 = 0.3$ and the pressure distribution $P_3 = P_0(1-\psi(r)/\psi(r_0))^2$.

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РАСЧЕТЫ МАГНИТНОЙ ЯМЫ, ОБУСЛОВЛЕННОЙ РАЗЛИЧНЫМИ ПРОФИЛЯМИ ДАВЛЕНИЯ ПЛАЗМЫ И ВНЕШНИМ ПОПЕРЕЧНЫМ МАГНИТНЫМ ПОЛЕМ В ТОРСАТРОНЕ

В.И. Тюпа, И.Б. Пинос

Авторами выполнены аналитические расчеты магнитной ямы, обусловленной различными профилями давления плазмы в зависимости от параметра α, характеризующего профили вакуумных углов поворота силовых линий, а также от смещения магнитной оси, вызванного внешним поперечным магнитным полем. Рассмотрены три закона распределения давления плазмы по вакуумным магнитным поверхностям: $P_1=P_0$ (пологое распределение); $P_2=P_0(1-\psi(r)/\psi(r_0))$; $P_3=P_0(1-\psi(r)/\psi(r_0))^2$. Расчеты показали, что при смещении магнитной оси внутрь тора магнитная конфигурация будет обладать магнитным бугром (δU/U>0). При смещении магнитной оси наружу тора, вызванного давлением плазмы, магнитная конфигурация будет обладать магнитной ямой (δU/U<0).

РОЗРАХУНКИ МАГНІТНОЇ ЯМИ, ЗУМОВЛЕНОЇ РІЗНИМИ РОЗПОДІЛАМИ ТИСКУ ПЛАЗМИ ТА ЗОВНІШНІМ ПОПЕРЕЧНИМ МАГНІТНИМ ПОЛЕМ У ТОРСАТРОНІ

В.І. Тюпа, І.Б. Пінос

Авторами роботи виконані аналітичні розрахунки магнітної ями, зумовленої різними розподілами тиску плазми в залежності від параметра α, який характеризує розподіли вакуумних кутів повороту силових ліній, а також від зміщення магнітної осі, викликаного зовнішнім поперечним магнітним полем. Авторами роботи розглянуто три закони розподілу тиску плазми по вакуумним магнітним поверхням: $P_1=P_0$ (пологий розподіл); $P_2=P_0(1-\psi(r)/\psi(r_0))$; $P_3=P_0(1-\psi(r)/\psi(r_0))^2$. Розрахунки показали, що при зміщенні магнітної осі усередину тора магнітна конфігурація буде володіти магнітним горбом (δU/U>0). При зміщенні магнітної осі на зовнішню сторону тора, викликаного тиском плазми, магнітна конфігурація буде мати магнітну яму (δU/U<0).