EFFECT OF MAGNETIC ISLANDS ON THE IMPURITY FLOWS IN NCSX GEOMETRY

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A new physics idea to protect a plasma from impurity ions with the use of magnetic islands can be examined on the new National Compact Stellarator Experiment device (NCSX), which is under construction at Princeton Plasma Physics Laboratory, USA. On the basis of the MHD approach the impurity ions flows are studied in the configuration with the parameters of NCSX with the magnetic islands m/n=5/3 and m/n=6/3, which can be excited with the trim coils in NCSX. It is shown that solving the flow trajectory equations $d\mathbf{r}\times\mathbf{u}_\alpha = 0$ can lead us to the conclusion that ion flow trajectories are concentrated in the region of the magnetic islands.

PACS: 52.55.-s

1. MOTIVATION OF STUDY

The impurity ion flux into the core of plasma can be partly stopped with the magnetic island chains at the periphery of the magnetic confinement volume. On the new National Compact Stellararor Experiment device (NCSX), which is under construction at Princeton Plasma Physics Laboratory, USA [1,2], there exists the possibility to examine physics ideas about the control of the impurity ion flows with the use of the magnetic islands. These islands with "wave" numbers $m=5$, $n=3$ and $m=6$, $n=3$ can be produced with the trim coils.

2. PHYSICS MODEL

2.1. MAIN EQUATIONS

For our analysis the starting equations are chosen in the following form

$$-\nabla p_\alpha + e_\alpha n_\alpha \mathbf{E} + e_\alpha n_\alpha (\mathbf{u}_\alpha \times \mathbf{B}) - \mathbf{F}_{\alpha 1} = 0, \quad (1)$$

$$-\frac{5}{2} n_\alpha \nabla T_\alpha + e_\alpha n_\alpha \frac{1}{p_\alpha} (\mathbf{q}_\alpha \times \mathbf{B}) - \mathbf{F}_{\alpha 2} = 0, \quad (2)$$

where

$$\mathbf{F}_{\alpha 1} = \sum_\beta l_{11}^{\alpha\beta} \mathbf{u}_\beta - \frac{2}{5} l_{12}^{\alpha\beta} \frac{\mathbf{q}_\beta}{p_\beta}, \quad (3)$$

$$\mathbf{F}_{\alpha 2} = \sum_\beta l_{21}^{\alpha\beta} \mathbf{u}_\beta - \frac{2}{5} l_{22}^{\alpha\beta} \frac{\mathbf{q}_\beta}{p_\beta}.$$

Here $l_{ij}^{\alpha\beta}$ are the matrix elements, which depend on the plasma parameters [2,3]. As we can see the friction forces are taken into account in equations (1) – (3). From these equations a linear analysis can be carried out. The variables $\mathbf{u}_\alpha$ and $\mathbf{q}_\alpha$ can be expressed via the main magnetic field and profiles of the density n_α, the temperature T_α and the electric field $\mathbf{E}$ properties (characteristics). The effects of inertia and viscosity terms should be taken into account separately.

2.2. ANALYTICAL EXPRESSIONS FOR FLOWS OF MASS OF SPECIES α

From the equations (1)-(3) the analytical expressions for variables $\mathbf{u}_\alpha$ and $\mathbf{q}_\alpha$ can be obtained and they have the following form [3]:

$$\mathbf{u}_\alpha = \frac{1}{D_{u_\alpha}} \left\{ \frac{5}{2} n_\alpha (\mathbf{B}\times\nabla T_\alpha) + \frac{B_{q_\alpha}}{A_{q_\alpha}} \left[(\mathbf{B}\times\nabla p_\alpha) - e_\alpha n_\alpha (\mathbf{B}\times\mathbf{E}) \right] + e_\alpha n_\alpha \mathbf{B} \begin{bmatrix} L_1(\mathbf{B}\nabla p_\alpha) + L_2 e_\alpha n_\alpha (\mathbf{BE}) + L_3 n_\alpha (\mathbf{B}\nabla T_\alpha) + \\ + \sum_{\beta\neq\alpha} \left(L_{4\beta}(\mathbf{Bu}_\beta) + L_{5\beta}(\mathbf{B}\frac{\mathbf{q}_\beta}{p_\beta}) \right) \end{bmatrix} + \right.$$
$$\left. + L_6\left(\nabla p_\alpha - e_\alpha n_\alpha \mathbf{E}\right) + L_7 \frac{5}{2} n_\alpha \nabla T_\alpha + \sum_{\beta\neq\alpha} \begin{bmatrix} (L_6 l_{11}^{\alpha\beta} - L_7 l_{21}^{\alpha\beta})\mathbf{u}_\beta + (-\frac{2}{5} L_6 l_{12}^{\alpha\beta} + \frac{2}{5} L_7 l_{22}^{\alpha\beta}) \frac{\mathbf{q}_\beta}{p_\beta} + \\ + L_{8\beta}(\mathbf{B}\times\mathbf{u}_\beta) + L_{9\beta}(\mathbf{B}\times\frac{\mathbf{q}_\beta}{p_\beta}) \end{bmatrix} \right\}, \quad (4)$$

$$\frac{\mathbf{q}_\alpha}{p_\alpha} = \frac{1}{D_{q_\alpha}} \left\{ (\mathbf{B}\times\nabla p_\alpha) - e_\alpha n_\alpha (\mathbf{B}\times\mathbf{E}) - \frac{A_{u_\alpha}}{B_{u_\alpha}} \frac{5}{2} n_\alpha (\mathbf{B}\times\nabla T_\alpha) + e_\alpha n_\alpha \mathbf{B} \begin{bmatrix} M_1(\mathbf{B}\nabla p_\alpha) + M_2 e_\alpha n_\alpha (\mathbf{BE}) + M_3 n_\alpha (\mathbf{B}\nabla T_\alpha) \\ + \sum_{\beta\neq\alpha} \left(M_{4\beta}(\mathbf{Bu}_\beta) + M_{5\beta}(\mathbf{B}\frac{\mathbf{q}_\beta}{p_\beta}) \right) \end{bmatrix} + \right.$$
$$\left. + M_6\left(\nabla p_\alpha - e_\alpha n_\alpha \mathbf{E}\right) + M_7 \frac{5}{2} n_\alpha \nabla T_\alpha + \sum_{\beta\neq\alpha} \begin{pmatrix} (M_6 l_{11}^{\alpha\beta} - M_7 l_{21}^{\alpha\beta})\mathbf{u}_\beta + (-\frac{2}{5} M_6 l_{12}^{\alpha\beta} + \frac{2}{5} M_7 l_{22}^{\alpha\beta}) \frac{\mathbf{q}_\beta}{p_\beta} \\ + M_{8\beta}(\mathbf{B}\times\mathbf{u}_\beta) + M_{9\beta}(\mathbf{B}\times\frac{\mathbf{q}_\beta}{p_\beta}) \end{pmatrix} \right\}. \quad (5)$$

These are the general expressions, which can be useful for linear analysis, particularly to compare the MHD velocity of the background ions $\mathbf{u}_i$ and the MHD velocity of the impurity ions $\mathbf{u}_I$. In order to carry out such an analysis it is necessary to substitute the expression for $\mathbf{q}_\alpha$ into the expression for $\mathbf{u}_i$ and $\mathbf{u}_I$. The matrix elements mentioned above are expressed through the coefficients, which can be found in [4], they are obtained as moments of the collision operators from the kinetic description [4].

2.3. THE DESCRIPTION OF THE ISLAND STRUCTURE

A function which describes the structure of two island chains with the "wave" numbers: m,n and $m^{'},n^{'}$, has the following form:

$$\Psi_{m,n,m^{'},n^{'}} = \frac{1}{r_{m,n}^{6}}\int\left(r^2 - r_{m,n}^2\right)\left(r^2 - r_{m^{'},n^{'}}^2\right)dr + \frac{1}{2}\left(\frac{\Delta r_{m,n}}{r_{m,n}}\right)^2\left(\frac{r}{r_{m,n}}\right)^m \frac{r^2 - r_{m^{'},n^{'}}^2}{r_{m,n}^2}\cos\left(m\vartheta - n\varphi + \delta_{m,n}\right) +$$
$$+\frac{1}{2}\left(\frac{\Delta r_{m^{'},n^{'}}}{r_{m^{'},n^{'}}}\right)^2\left(\frac{r}{r_{m^{'},n}}\right)^{m^{'}}\left(\frac{r_{m^{'},n^{'}}}{r_{m,n}}\right)^4\frac{r^2 - r_{m,n}^2}{r_{m,n}^2}\cos\left(m^{'}\vartheta - n^{'}\varphi + \delta_{m^{'},n^{'}}\right) \qquad (6)$$

Here $r_{m,n}$ and $r_{m^{'},n^{'}}$ are the radii of the rational magnetic surfaces with the rotational transform values $\iota(r_{m,n}) = n/m$ and $\iota(r_{m^{'},n^{'}}) = n^{'}/m^{'}$; $\Delta r_{m,n}$ and $\Delta r_{m^{'},n^{'}}$ are the half widths of the magnetic islands which occur as the splitting of the appropriate rational magnetic surfaces. Such a form (6) of the magnetic surface function Ψ results after the so-called renormalization procedure [3].

2.4. FLOW TRAJECTORY EQUATIONS

The mass flow trajectories can be obtained as the solution of the equations

$$d\mathbf{r} \times \mathbf{u}_\alpha = 0 \cdot \qquad (7)$$

For the analytical solution of the equation (7) it is possible to represent $\mathbf{u}_\alpha$ in the form

$$\left(\mathbf{u}_\alpha\right)_r = u_{\nabla\Psi}\frac{\partial\Psi}{\partial r} + u_{\mathbf{B}\times\nabla\Psi}\left(B_\vartheta\frac{1}{R}\frac{\partial\Psi}{\partial\varphi} - B_\varphi\frac{1}{r}\frac{\partial\Psi}{\partial\vartheta}\right)$$
$$\left(\mathbf{u}_\alpha\right)_\varphi = u_{\nabla\Psi}\frac{1}{R}\frac{\partial\Psi}{\partial\varphi} + u_{\mathbf{B}\times\nabla\Psi}\left(B_r\frac{1}{r}\frac{\partial\Psi}{\partial\vartheta} - B_\vartheta\frac{\partial\Psi}{\partial r}\right) \qquad (8)$$
$$\left(\mathbf{u}_\alpha\right)_\vartheta = u_{\nabla\Psi}\frac{1}{r}\frac{\partial\Psi}{\partial\vartheta} + u_{\mathbf{B}\times\nabla\Psi}\left(B_\varphi\frac{\partial\Psi}{\partial r} - B_r\frac{1}{R}\frac{\partial\Psi}{\partial\varphi}\right)$$

Some simplified assumptions are made for beginning this study. The derivatives of the functions $p_\alpha = p_\alpha(\Psi)$, $T_\alpha = T_\alpha(\Psi)$, $\Phi = \Phi(\Psi)$ enter $u_{\nabla\Psi}$ and $u_{\mathbf{B}\times\nabla\Psi}$. The parabolic functions of the equilibrium quantities the functions $p_\alpha = p_\alpha(\Psi)$, $T_\alpha = T_\alpha(\Psi)$, $\Phi = \Phi(\Psi)$ are taken here. In that case the magnitudes $u_{\nabla\Psi}$ and $u_{\mathbf{B}\times\nabla\Psi}$ are independent on r,ϑ,φ. In this way we can underscore the effect of the island structure. The general case will be considered further.

3. PHYSICS RESULTS

Flow trajectories in the configurations *without* and *with* islands are shown in figures 1 and 2 respectively. One can see that the flow trajectories change their direction when the islands are present. The initial positions for the fluid elements are the same on both figures. When the islands appear the fluid elements near the islands change their direction and come to the island region. One can imagine two sets of experiments: the first one when only one chain of island is excited, for example with the "wave" numbers $m = 5, n = 3$ and the second one when two chins of islands are excited: with the "wave" numbers $m = 5, n = 3$

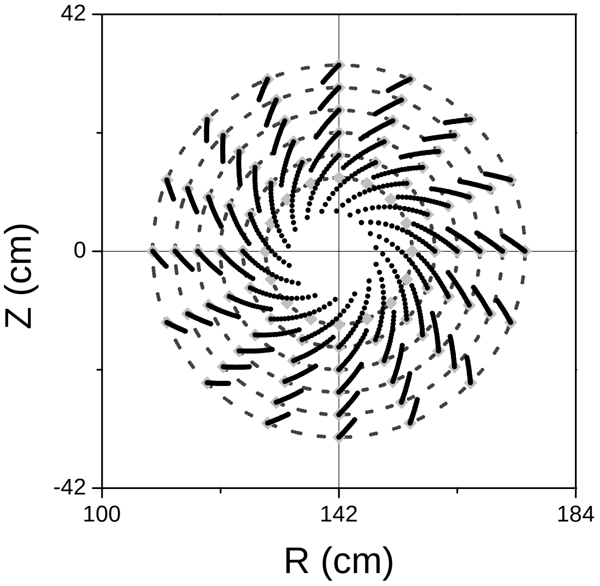

Fig 1. Flow trajectories in NCSX configuration with unperturbed closed nested magnetic surfaces

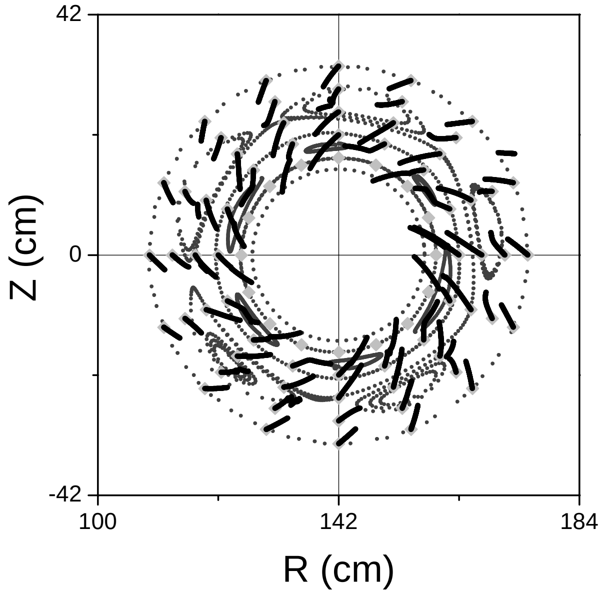

Fig 2. Flow trajectories in NCSX configuration with n/m=5/3 and n/m=6/3 islands

and $m=6, n=3$. The accumulation of the impurity ions inside the islands can be expected.

4. CONCLUSIONS

4.1 The mass flow trajectories as the solution of the equation $d\mathbf{r}\times\mathbf{u}_{\alpha}=0$ have been obtained for the configurations *with* and *without* islands. One can see the strong effect of the islands on the mass flows. On NCSX it is possible to examine this dependence of the impurity fluxes on the magnetic islands.
4.2 We conclude that magnetic islands can be transport barriers to impurity ions moving into the plasma core. We currently believe that similar behavior can be expected for the main plasma ions also.
4.3 The change of the island geometry: the excitation of the islands with "wave" numbers $m=5$, $n=3$ and $m=6$, $n=3$ separately (in different discharges), which can be produced with the trim coils, and then the excitation of both adjacent resonances can help to control impurity flows in the helical plasmas. NCSX may be the most appropriate device for such a study.

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ВЛИЯНИЕ МАГНИТНЫХ ОСТРОВОВ НА ПОТОКИ ПРИМЕСЕЙ В ГЕОМЕТРИИ NCSX

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Новая физическая идея - защитить плазму от проникновения примесных ионов с использованием магнитных островов - может быть проверена в экспериментах на новой установке National Compact Stellarator Experiment (NCSX), строящейся в Princeton Plasma Physics Laboratory, USA. На основе МГД приближения потоки ионов примеси изучаются для конфигурации с параметрами NCSX с магнитными островами с «волновыми» числами m/n=5/3 и m/n=6/3, которые могут быть возбуждены специальными токовыми катушками (trim coils) в NCSX. В результате решения уравнений траекторий потоков $d\mathbf{r}\times\mathbf{u}_{\alpha}=0$ показано, что потоки ионов могут концентрироваться в области магнитных островов.

ВПЛИВ МАГНІТНИХ ОСТРОВІВ НА ПОТОКИ ІОНІВ ДОМІШКИ В ГЕОМЕТРІЇ NCSX

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Нову фізичну ідею захисту плазми від проникнення іонів домішки за допомогою магнітних островів можна перевірити в експериментах на новому пристрої National Compact Stellarator Experiment (NCSX), який споруджується в Princeton Plasma Physics Laboratory, USA. На основі МГД наближення потоки іонів домішки вивчаються для конфігурації з параметрами NCSX з магнітними островами з «хвильовими» числами m/n=5/3 и m/n=6/3, які можна збудити спеціальними токовыми катушками (trim coils) у NCSX. Рішення рівнянь траєкторій потоків $d\mathbf{r}\times\mathbf{u}_{\alpha}=0$ доводить, що потоки іонів можуть концентруватися в межах магнітних островів.