DYNAMICS OF CHARGED PARTICLES IN A FIELD OF INTENSIVE ELECTROMAGNETIC WAVES

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The results of analytical and numerical investigation of charged particles dynamics in a field of intense electromagnetic waves and electromagnetic pulses are reported. Conditions for effective acceleration of charged particles by the field of high-frequency or laser radiation in vacuum are found. It is shown, that the presence of frictional force can promote power transmission from a high-frequency field to particles. There are regimes at which all moments grow in time. Moreover, the higher moments grow faster then lowest. It means existence of the superdiffusion. It is shown, that there are conditions, when the leading centers of particles with different masses move in different directions

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1. INTRODUCTION

Dynamics of charged particles in fields of moderate intensity electromagnetic waves is well investigated. We call a field –"field of a moderate intensity"

f the wave force parameter ε is much less then one. (here $\varepsilon = eE/mc\omega$; E – electric strength of the wave; ω - its frequency)

A considerable success has been achieved recently in generating electromagnetic fields of extremely large intensity. The wave force parameter of such fields is already close to unity and can even be substantially large. For, as an example, a ten-centimeter wave band it means that the electric strength of the wave exceeds 10^5 V/cm. For a laser radiation ($\lambda \sim 10^{-4}$ cm) this intensity should be higher than 10^{10} V/cm.

In the 2-5 parts of this report we present the results of investigation of particles dynamics under influence of a field of one electromagnetic wave only or in a field of an electromagnetic impulse of high intensity. Let's mention that there are many similar results published (see ref. [1-8]). In the 6-8 parts we analyze the dynamics of particles under influence of electromagnetic wave, external magnetic field and a friction force.

2. BASIC EQUATIONS AND INTEGRALS

Let's consider a charged particle, which moves in an external permanent magnetic field of magnitude H_0 in a direction along Z-axis and in a field of an electromagnetic wave of arbitrary polarization. The wave has following components:

$$\vec{\varepsilon} = \mathrm{Re}\,\vec{E}\exp\left(i\vec{k}\vec{r} - i\omega t\right);$$
$$\vec{E} \equiv \left\{E_0\left(\alpha_x, i\alpha_y, \alpha_z\right)\right\}, \qquad (1)$$
$$\vec{H} = \mathrm{Re}\frac{c}{\omega}\left[\vec{k}\vec{E}\right]\exp\left(i\vec{k}\vec{r} - i\omega t\right),$$

where $\alpha \equiv \left\{\alpha_x, i\alpha_y, \alpha_z\right\}$ – vector of polarization of the wave.

Without loss of generality it is possible to assume, that the vector k has only two non-zero components: k_x and k_z. If one measures the time in the units of ω^{-1}, velocity in c, wave vector k in $\frac{\omega}{c}$, impulse in mc, and introduces dimensionless amplitude of field as $\varepsilon_0 = eE_0/mc\omega$, the equations of a particle motion can be rewritten in a form:

$$\dot{\vec{P}} = \left(1 - \frac{\vec{k}\vec{p}}{\gamma}\right)\mathrm{Re}\left(\vec{\varepsilon}\, e^{i\psi}\right) + \frac{\omega_H}{\gamma}\left[\vec{p}\vec{e}\right] + \frac{\vec{k}}{\gamma}\mathrm{Re}\left(\vec{p}\vec{\varepsilon}\right)e^{i\psi};$$
$$\dot{\vec{r}} = \vec{p}/\gamma;\ \dot{\psi} = \vec{k}\vec{p}/\gamma - 1, \qquad (2)$$

where:

$$\tau \equiv \omega t,\ \vec{e} \equiv \vec{H}/H_0;\ \omega_H \equiv eH_0/mc\omega;$$
$$\psi = \vec{k}\vec{r} - \tau.$$

It is convenient to add to these equations the equation for energy:

$$\dot{\gamma} = \mathrm{Re}\left(\vec{v}\vec{\varepsilon}\right)e^{i\psi} \qquad (3)$$

From (2) and (3) one can found an integral of motion:

$$\vec{p} - \mathrm{Re}\left(i\vec{\varepsilon}\, e^{i\psi}\right) + \omega_H\left[\vec{r}\vec{e}\right] - \vec{k}\gamma \equiv \vec{I} = const \qquad (4)$$

3. INTERACTION WITH A PLAIN POLARIZED WAVE

The most important features of a particle dynamics in a field of a plain polarized wave are summarized in [1-2]. Here we only mention that charged particles are dragged by a field of plain polarized wave in a direction of propagation of this wave. The velocity of the entrainment is sufficiently high. Such dynamics can be potentially used for acceleration of charged particles. However particles placed in different phases of an electromagnetic wave, move differently. Therefore, a bunch of accelerated particles scatters. Let's indicate also that different applications for this dynamics have been described in [2].

4. INTERACTION OF PARTICLES WITH A CIRCULARLY POLARIZED WAVE

If a wave has elliptic, particularly circular polarization then the dynamics of particles has some important features. Let's discuss some of them. Major feature of dynamics of particles in a field of a wave with circular polarization is the independence of a longitudinal momentum of a particle from its initial phase. This is illustrated by Fig.1. From Fig.1 one can see that the magnitudes of longitudinal impulses of all particles completely coincide. From the equations (2) it is possible to obtain an expression, which describes this important feature:

$$P_z = 2E\left[1-\cos(\psi-\psi_0)\right]. \qquad (5)$$

From this expression follows, that the magnitude of a longitudinal impulse is identical for all particles. It does not depend on an initial distribution of particles over initial phases.

Spatial dynamics of particles in a field of a wave with circular polarization unfortunately is not so correlated: in a longitudinal direction all particles trajectories are completely similar, but in a transverse direction all particles disperse.

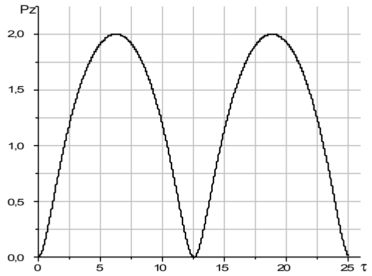

Fig.1. Dependence of longitudinal momentum of particle from its initial phase: 0, π/4, π/2, 3π/4, π at E =1

More detailed analytical and numerical examinations reveal (see [10] for details), that the trajectory of a particle in a circularly polarized wave is a spiral with an axis directed along the wave-vector of the wave. The radius of the spiral is E in the impulse space. Thus, the acceleration of particles by a field of circularly polarized wave can remove some problems with scattering of electrons that occur in an acceleration system based on linearly polarized waves.

5. INTERACTION OF A PARTICLE WITH AN IMPULSE OF AN ELECTROMAGNETIC WAVE

Let's consider a motion of a charged particle in a field of an impulse of a plain electromagnetic wave characterized by a vector potential:

$\vec{A} = \vec{A}(\omega t - \vec{k}\vec{r}) \equiv \vec{A}(\psi)$. The equation, which describes motion of particles in such a field in dimensionless variables $\tau \equiv \omega t$, $k \equiv k/k_1$, $k_1 \equiv \omega/c$, $\beta \equiv v/c$ $p \equiv p_1/m_0c$, (v-velocity of a particle, p-its impulse), $A \equiv ek_0A_1/m_0c\omega$, becomes:

$$\frac{d\vec{p}}{d\tau} = -\frac{\partial\vec{A}}{\partial\psi}\left(1-\vec{k}\vec{\beta}\right) - \vec{k}\left(\vec{\beta}\frac{\partial\vec{A}}{\partial\psi}\right). \qquad (6)$$

It is convenient to add to (6) the equation for the energy:

$$\frac{d\gamma}{d\tau} = -\left(\vec{\beta}\frac{\partial\vec{A}}{\partial\psi}\right). \qquad (7)$$

From the equations (6) and (7) it is easy to find a following integral:

$$\vec{p} + \vec{A} - \vec{k}\gamma = \text{Const} = \vec{p}_0 + \vec{A}_0 - \vec{k}\gamma_0. \qquad (8)$$

In a case of a pure transverse wave: ($k_\parallel = 1, k_\perp = 0$, $[k_\parallel E] = [k_\parallel H] = [k_\parallel A] = 0$), the equations (6)-(8) are completely integrable in a laboratory coordinates. The solution is:

$$p_\parallel - p_{\parallel 0} = \frac{\left(\vec{A}-\vec{A}_0\right)^2 + \vec{p}_{\perp 0}^{\,2}}{2\gamma\dot{\psi}}; \quad \vec{p}_\perp - \vec{p}_{\perp 0} = \left(\vec{A}-\vec{A}_0\right); \qquad (9)$$

$$r_\parallel - r_{\parallel 0} = \frac{p_{\parallel 0}(\psi-\psi_0)}{\gamma\dot{\psi}} + \frac{1}{2\left(\gamma\dot{\psi}\right)^2}\int_{\psi_0}^{\psi}\left[\left(\vec{A}-\vec{A}_0\right)^2 + \vec{p}_{\perp 0}^{\,2}\right]d\psi; \qquad (10)$$

$$\vec{r}_\perp - \vec{r}_{\perp 0} = \frac{-1}{\gamma\dot{\psi}}\int_{\psi_0}^{\psi}\left[\left(\vec{A}-\vec{A}_0\right) - \vec{p}_{\perp 0}\right]d\psi.$$

From (9), (10), one can see that the particles are dragged by the field of a wave. Their longitudinal impulses oscillate, but keep their direction, and a longitudinal coordinate is determined by an integral of a non-negative function. The most interesting and important feature characterizes dynamics of particles in a field of a circularly polarized impulse. The interaction of particles with a wave in this case does not depend on their initial location. An entrainment of particles in a longitudinal direction occurs and this entrainment happens on a spiral trajectory. Moreover, the longitudinal momentum of all particles follows the shape of the field envelope.

Fig.2 illustrates these features and shows temporal dependence of particles momentums obtained from numerical solution of equation (6). The components of an electromagnetic impulse are defined by a relation: $\partial A_x/\partial\psi = A_0\exp[-\beta(\psi-\psi_0)^2]\cdot\cos\psi$, with $A_0 = 3$, $\beta = 0.01$, $\psi_0 = 50$. If the particles of a bunch have initial energy $\gamma_0 = 10$ then they gain energy of $\gamma \approx 100$ on a distance of 0.4 cm. In addition, these particles do not disperse in transverse direction. Such laser impulse is convenient for an acceleration purposes.

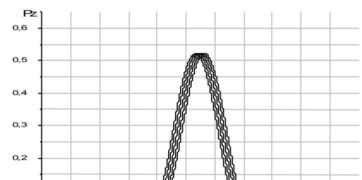

Fig.2. Dependence of longitudinal momentum of particle from its initial phase: 0, π/4, π/2, 3π/4, π at $A = 3$

Above we have considered dynamics of charged particles in a field of an intensive electromagnetic wave and an opportunity to use it for the purposes of acceleration. However, some features of this dynamics can be useful for other applications too. Let's indicate an opportunity to use these results for excitation of intensive short-wave coherent radiation. For example, if there is a cluster of particles in a wave, then the charge of this cluster will be in N – times higher then the charge of one particle (N – number of electrons in the cluster). If cluster size is smaller than a length of wave, in which it moves, all particles emit together and the radiation is coherent. The intensity of radiation from such a bunch will be proportional to N^2. Usual number of electrons in a cluster is ~10^8…10^{10}. Therefore, it is possible to use such clusters for the purposes of excitation of intensive short-wave coherent radiation.

6. ROLE OF FRICTIONAL FORCE AT INTERACTION OF ADIATION WITH CHARGED PARTICLES

When charged particles are in a field of electromagnetic waves then they are move with acceleration. Thus there is a radiation. The particles lose energy. There is a radiation friction. As any frictional force radiation friction will brakes, in most cases, charged particles. The force of radiation friction promptly grows with energy of particles (~γ^4). Therefore this friction restricts energy, which can be received particles. The maximal energy can be estimated by equating accelerating forces to forces of radiation friction. So, for example, in work [6], studied acceleration of electrons by a field of a laser radiation, the authors have equated force of radiation friction to accelerating forces (forces of high-frequency pressure). In result they have found, that in a field of a laser radiation the electrons can not gain energy large, than 200 MeV ($\lambda \sim 1\mu k$).

In the present section we shall show, that the frictional force, including forces of radiation friction, can promote power transmissions from an external laser field to accelerated particles. Let's mark, that for the first time on an unusual role of frictional force at a motion of charged particles in a field of an intensive electromagnetic wave was pointed in work [7].

Let's consider most simple model, which can describe dynamics of a charged particle in a field of a laser radiation. Let particle is moving in a field of a homogeneous flat electromagnetic wave. Field of this wave we shall present as:

$$\vec E = \mathrm{Re}\{\vec E_0 \exp(i\Psi)\};\ \vec H = \mathrm{Re}\{[\vec k \vec E]/k_0\},$$

Here $\Psi = \omega \cdot t - \vec k \cdot \vec r$, $\quad \vec E_0 = \vec\alpha \cdot E_0$, $\quad \vec\alpha = \{\alpha_x, i\alpha_y, \alpha_z\}$, $k_0 \equiv \omega/c$.

Except this field we shall consider that there is frictional force, which brakes a particle. In the beginning we shall consider model, in which we shall not point concrete the nature of these forces. Then, we shall consider concretely forces - radiation friction. Equation of motion of a charged particle in dimensionless variables: $\vec p = \vec p/mc$, $\tau = \omega t$, $\vec r = k_0 \vec r$, $\vec k = \vec k/k_0$, $\quad \varepsilon = eE_0/mc\omega$, $\tau = \omega \cdot t$ is possible to write as:

$$\frac{d\vec p}{d\tau} = \mathrm{Re}\{((1-\vec k\vec v)\vec\varepsilon + \vec k(\vec v\vec\varepsilon))\exp(i\Psi) - \mu\vec v\}. \quad (11)$$

This equation differs from investigated in [2] and from (2) only by presence of a frictional force. From (11) it is possible to receive the following relation:

$$\frac{d}{dt}\{\vec p - \vec k\gamma + \mathrm{Re}(i\vec\varepsilon\exp(i\Psi))\} = -\mu[\vec v - \vec k v^2]. \quad (12)$$

If the friction absent $(\mu = 0)$, then the expression in curly brackets represents integral of the equation (11). To simplify represented below formulas, we shall consider, that the interaction of a particle with wave happens in vacuum; that the wave is linearly polarized and is going along an axis z, i.e. we shall consider (count), that

$\vec\varepsilon = (\varepsilon, 0, 0)$; $\vec\alpha = (1,0,0)$, $\vec k = (0,0,1)$. In this case vector equation (11) can be essentially simplified:

$$p_x' = \varepsilon \cdot \cos\Psi - \mu(p_x/I),$$

$$p_z' = (p_x \cdot \varepsilon/I) \cdot \cos\Psi - \mu(p_z/I),$$

$$p_y' = 0,\ I' = -\mu[1 - (1/\gamma \cdot I)]. \quad (13)$$

Such quantities and labels here are introduced: $I = \gamma - p_z$; $I' \equiv dI/d\tau$.

In system (3) first three equations completely self-consistent. The last, fourth equation, is a corollary these three. Let's note that in absence of frictional force the equations (13) are completely integrated. At presence of friction system (13) is convenient to rewrite as:

$$\rho_x' = -(\mu/I)[\rho_x + \varepsilon \cdot \sin\Psi];$$

$$\rho_z' = -(\mu/I)[\rho_z + (p_x^2/2\gamma I^2) - (p_x/I)]. \quad (14)$$

Where $p_x = \varepsilon \cdot \sin\Psi + (p_{x,0} - \varepsilon \cdot \sin\Psi_0) + \rho_x$,

$p_z = (p_x^2/2I) + (p_{z,0} - p_{x,0}^2/2I_0) + \rho_z$.

The obtained above equations are rigorous. From these equations it is visible, that new variable ρ_x and ρ_z, and also quantity I vary slowly ($\mu << 1$). The analysis of the equations (14) shows, that asymptotically $p_x = \varepsilon \cdot \sin\Psi + (p_{x,0} - \varepsilon \cdot \sin\Psi_0)$. At realization of an inequality $\gamma \cdot I \geq 1$ the presence of frictional force always gives in acceleration of charged particles. Let's mark, that at the zero starting conditions this inequality is always fulfilled. Besides at $\varepsilon >> 1$ always $\gamma \cdot I \geq 1$. If $\varepsilon < 1$, it is possible to find conditions, at which a frictional force will lead to brake particles.

So, for example, at $p_{x,0} = 0$, $p_{z,0} = 3$, $\varepsilon \sim 0.5$, the quantity $\gamma \cdot I$ will be less unity ($\gamma \cdot I < 1$). Thus the particles are broken by frictional force. As an example in Fig.3 the dependence of a longitudinal impulse of a particle on time is given at $\varepsilon = 3$, $\mu = 0.01$, $p_{x,0} = p_{z,0} = 0$. It is visible, that the quantity of a longitudinal impulse monotonically increased. In a Fig.4 the case is given, when the frictional force brakes particles ($\varepsilon = 0.2$, $\mu = 0.01$, $p_{x,0} = 0$, $p_{z,0} = 2$).

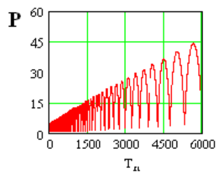

Fig.3

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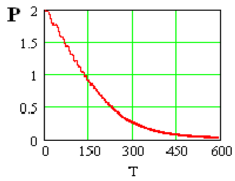

Fig.4

It is necessary to note, that in all cases at small strengths of a field ($\varepsilon << 1$) the frictional force brake fast particles ($p_z >> 1$).

Let's consider now role of radiation friction forces. We shall interest with large strength of fields ($\varepsilon \geq 1$). Therefore we can be restricted to a case of a relativistic motion. For this case the dimensionless force of radiation friction can be presented as [8]:

$$\vec{F}_f = \frac{\omega}{\Omega}\left(F_{ik}\cdot u^k\right)\cdot\left(F^{mn}\cdot u_n\right)\cdot\vec{v}\,, \qquad (15)$$

where F_{ik} – tensor of an electromagnetic field; u^k – four-vector of velocity; v – three-dimensional vector of velocity, "frequency" $\Omega_e = 3mc^3/2e^2 = 1{,}8\cdot 10^{23}$ s^{-1}.

In our case we have only two components of an electromagnetic field (E_x, H_y). Taking into account, that $E_x = H_y$, and also, that the four-vector of velocity in our notation looks like $u^k = (\gamma, p)$, $u_n = (\gamma, -p)$, the force of radiation friction can be presented by the following expression:

$$\vec{F}_f = -\frac{\omega}{\Omega}\cdot\varepsilon^2\cdot I^2\cdot\frac{p}{\gamma}\cdot\cos^2(\psi)\,. \qquad (16)$$

From the formula (16) it is visible, that in this case coefficient μ already is a complicated function of time. However qualitative analysis of quantity I can be carried out, to the similarly previous case. From here follows, that influence of friction force is qualitatively similarly to the previous case. Thus, dynamics of particles in a field of a laser radiation has the important feature, which allows to use frictional force for arising of efficiency of power transmission from a wave to particles.

7.1. DYNAMICS OF PARTICLES AT PRESENCE OF A CONSTANT MAGNETIC FIELD

The motion of particles in a field of an electromagnetic wave qualitatively varies at presence of an external magnetic field. The most important difference is that fact, that in a field of an intense electromagnetic wave the nonlinear cyclotron resonances are overlapped. In result the local instability arise. The charged particles stochastically accelerated by a field of an electromagnetic wave.

The analysis of dynamics of particles in this case show, that the dynamic chaos is alternated. It means that the casual ejections arise seldom. However the more seldom these casual ejections appear, the more intensity it is.

The characteristic of such process can be the moments M_n. Under it, the higher moment, the more it is (see Fig.5). The analytical and numerical examinations of dynamics of particles show namely such dependence. As an example in Fig.6 the evolution a cross impulse (P) from time represented at $\varepsilon = 0.5$, $P(0) = 1$. Besides it is possible to show, that the moments grow in time and the higher moments grow faster lowest. It means existence super diffusion.

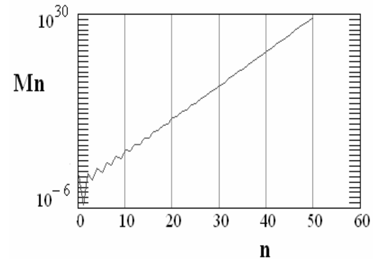

Fig.5

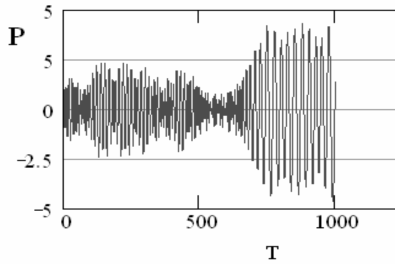

Fig.6

7.2. DYNAMICS OF LEADING CENTERS

In stationary values homogeneous external magnetic fields dynamics of leading centers of charged particles is expressed feebly. She is determined only by nonlinear effects. In most cases this dynamics can be neglected. Such situation arise, for example, in masers on cyclotron resonances or when we are doing analyze of high-frequency instruments, which operation grounded on cyclotron resonances. However dynamics of leading centers in a homogeneous constant magnetic field and in a field even of a homogeneous flat electromagnetic wave has a series of features, which can be useful for the application. In particular, we shall show lower, that in such fields it is possible to create condition, at which the leading centers of one isotope will move in one direction, and other isotopes will move in an opposite direction. Such feature of a motion of leading centers can be useful for an isotope separation. Let's mark, that this feature of dynamics of leading centers was found together with Stepanov K.N.

The full set of the equations, which describe a motion of charged particles in our case, coincides with set equations (2) of first part.

Let's mark, that only the conditions that are close to ionic cyclotron resonances will interest. It means that the frequency of an external electromagnetic wave is close to ionic cyclotron frequency. In this case the parameter of a wave force can be rewritten as the relation of an electric field intensity of an electromagnetic wave to intensity of a constant magnetic field ($\varepsilon = E/H$). Practically always this parameter is small. Therefore, having used a smallness of this parameter, it is possible to receive the following system of the shortcut equations, which is valid in a neighborhood of ionic cyclotron resonances

$$\dot{\theta}_1 = \left[\omega_H\left(1-\frac{V^2}{2}\right)-1\right]-\frac{\varepsilon}{2V}\left(5|V^2-1|\right)\sin\theta_1;$$

$$\dot{V} = -0.5|\varepsilon|\cos\theta_1;\ \dot{\xi} = -2|\varepsilon||V|\sin\theta_1,$$

where – $\omega_H = q|H/M|c|\omega$ dimensionless ionic cyclotron frequency.

From these equations it is visible, that the first two equations represent completely closed system of the equations concerning variable V and θ_1. After the solution of this pair of the equations, there is easily to find solution, defining dynamics of leading centre ξ :

$$\theta_1 = (\omega_H - 1)\tau;\ V = V(0) - \frac{\varepsilon}{2(\omega_H-1)}\sin\theta_1;$$

$$\langle\xi\rangle \approx \varepsilon^2\cdot\tau/2(\omega_H-1);\ \langle V\rangle = V(0).$$

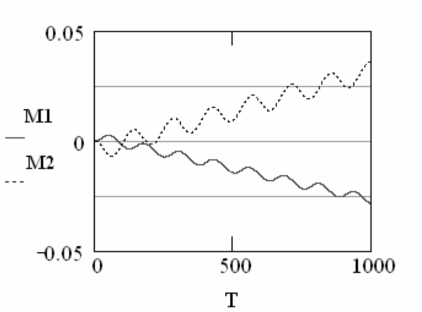

Fig.7.

Fig. 7 Evolution of leading center at and

Here angular brackets designate an average on a phase θ_1. From expression for average coordinate of leading centre $\langle\xi\rangle$ follows, that it is possible to choose such frequency ω, that the particles with one mass will move in one direction ($\omega_H > 1$), and particle with other mass ($\omega_H < 1$) – in the other direction. In Fig.7 dependence of evolution of leading center at ε=0.001 and μ=0.05 on time are represented. It is visible, that leading centers of particles with different masses are moving in opposite directions.

8. CONCLUSION

Dynamics of charged particles in fields of intensive electromagnetic waves can be useful both for the purposes of acceleration and for making new sources of intensive radiation.

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ДИНАМИКА ЗАРЯЖЕННЫХ ЧАСТИЦ В ПОЛЕ ИНТЕНСИВНЫХ ЭЛЕКТРОМАГНИТНЫХ ВОЛН

В.А. Буц

Изложены результаты аналитического и численного исследований особенностей динамики заряженных частиц в поле интенсивных электромагнитных волн и электромагнитных импульсов. Найдены условия, при которых возможно эффективное ускорение заряженных частиц полем высокочастотного и лазерного излучения в вакууме. Показано, что силы трения могут способствовать ускорению частиц. Обнаружены хаотические режимы, в которых все моменты растут во времени. Более того, высшие моменты растут быстрее низших. Это означает наличие супердиффузии. Показано, что существуют условия, при которых ведущие центры частиц с различными массами движутся в разных направлениях.

ДИНАМІКА ЗАРЯДЖЕНИХ ЧАСТОК У ПОЛІ ІНТЕНСИВНИХ ЕЛЕКТРОМАГНІТНИХ ХВИЛЬ

В.О. Буц

Викладені результати аналітичного та чисельного дослідження особливостей динаміки заряджених часток у полі інтенсивних електромагнітних хвиль та електромагнітних імпульсів. Знайдені умови, при яких є можливість ефективно прискорювати заряджені частки полем високочастотного та лазерного випромінювання у вакуумі. Доведено, що сили тертя можуть допомагати прискоренню часток. Знайдені хаотичні режими, в яких усі моменти зростають у часі. Більш того, вищі моменти зростають швидше ніж нижчі. Це означає наявність супердиффузії. Доведено, що існують умови, при виконанні яких ведучі центри часток с різними масами рухаються у різних напрямках.