# EXCITATION OF WAKE FIELD AND TRANSITION RADIATION IN A SEMI-INFINITE WAVEGUIDE BY A RELATIVISTIC ELECTRON BUNCH

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The theoretical investigations and simulation are performed for the emission of a relativistic electron bunch during the injection into a semi-infinite vacuum/dielectric waveguide. The power and the frequency spectrum of high intensity pulse of ultra-wideband transition radiation, excited by a finite size electron bunch in a vacuum waveguide, are calculated numerically. It is shown that in a dielectric waveguide the short pulse of Cherenkov wake field drifts behind the relativistic bunch with the group velocity.

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## 1. INTRODUCTION

At present, much attention is focused on the problem of generation of short intense electromagnetic pulses whose frequency spectra width is comparable with their mean frequency, the so-called "ultra-wideband" (UWB) pulses [1, 2]. The current interest to the problem of pulsed emission of high-power electromagnetic signals arises primarily due to their application in UWB radiolocation.

Along with traditional methods of generation of short UWB pulses based on the use of UWB antennas (TEM-horns, spiral and biconical antennas, etc. [3]) the intense pulsed relativistic electron beams (IREBs) can be used. For the most efficient generation of electromagnetic UWB pulses the non-resonant (impact) mechanisms of IREBs radiation should be used, such as charging of a rod antenna by REB [4] or impact excitation of TEM-horn antenna by IREB [5]. High-power UWB pulses can also be the result of coherent transition radiation of pulsed IREB [6].

Here we theoretically investigate the excitation of UWB transition radiation during the injection of a short-pulsed IREB into a semi-infinite circular cross-section waveguide whose entrance (through which the beam is injected) is short-circuited by a conducting diaphragm. In such a waveguide, the large dispersion of electromagnetic waves is the peculiarity of the transition radiation, so that the shape of a UWB pulse of transition radiation propagating in a waveguide will deform permanently.

In the case of dielectric filling the intense Cherenkov wake field excitation can take place. It can be applied for wake field acceleration of charged particles [7,8] or for radiation sources [9,10]. In the case of short-pulsed IREB the interference of big number of radial modes leads to the essential peaking of the wake field with formation of narrow spikes of alternative sign [11]. The spatial structure of excited field is determined by the spatially limited Cherenkov field and transition radiation field.

## 2. NON-RESONANT WIDEBAND EMISSION IN A SEMI-INFINITE WAVEGUIDE

We consider a semi-infinite ($0 \le z < \infty$) cylindrical metal waveguide of radius b which is filled with a homogeneous dielectric with permittivity ε. The waveguide input end (z = 0) is short-circuited by a metal wall transparent to relativistic electrons. An axisymmetric monoenergetic electron bunch is injected through the metal wall and moves with a constant velocity $v_0 < c/\sqrt{\varepsilon}$ along the symmetry axis of the waveguide (the *z* axis). We start with the determination of the field of an infinitely short and infinitely thin charged ring with the charge density

$$\rho = \frac{-eN}{2\pi r_0 v_0}\delta(r - r_0)\delta(t - t_0 - \frac{z}{v_0}) \,, \qquad (1)$$

where -e is the charge of an electron, N is the number of electrons in the ring, $v_0$ is the ring velocity, $r_0$ is the ring radius, and $t_0$ is the time at which the ring enters the waveguide. Solving Fourier-transformed Maxwell's equations with allowance for the boundary condition $E_r = 0$ at the end metal wall, we obtain the following expression for the radial electric field of an axisymmetric E -wave:

$$E_r(t, r, z, t_0, r_0) = \frac{2Ne}{\pi b\varepsilon v_0} \times \sum_n \frac{\omega_{0n}^2 J_0(\lambda_n r_0/b) J_1(\lambda_n r/b)}{\lambda_n J_1^2(\lambda_n)}\{I_{2n} - I_{1n}\}, \qquad (2)$$

$$I_{1n} = \int_{-\infty}^{\infty} d\omega \frac{\exp[-i\omega t + i\omega(t_0 + z/v_0)]}{(\omega - i\omega_{0n})(\omega + i\omega_{0n})} \,, \qquad (3)$$

$$I_{2n} = \int_{-\infty}^{\infty} d\omega \frac{\exp[-i\omega\tau + i\xi\sqrt{\omega^2 - \alpha_n^2}]}{(\omega - i\omega_{0n})(\omega + i\omega_{0n})} \,, \qquad (4)$$

where $\tau = t - t_0$, $\xi = z\sqrt{\varepsilon}/c$, $\alpha_n = \lambda_n c/(b\sqrt{\varepsilon})$, $\omega_{0n} = \lambda_n v_0/(b\sqrt{1 - \varepsilon v_0^2/c^2})$, and $\lambda_n$ is the *n*-th root of the Bessel function $J_0$.

Integral (3) describes the Coulomb field of a charge moving in an infinite waveguide. Integral (4) corresponds to free oscillations of a cylindrical waveguide and describes the transition radiation. The exact analytic solution of integral similar to (4) was obtained in [12]. In [13] the saddle point technique was used in order to obtain the approximate solution of integral (4) under condition of Cherenkov resonance. Below we applied the method proposed in [12].

In order to calculate the integral (4) we used a sequence of substitutions: $p = -i\omega$,

$\zeta = (\sqrt{p^2 + \alpha_n^2} - p)/\alpha_n$, and $w = -\zeta/\beta$, where $\beta = \sqrt{(\tau - \xi)/(\tau + \xi)}$. After these conform transformations we passed from the integration along the real axis to the integration along the closed circular contour. This allows us to separate the integral form of the Bessel functions. Finally [14] we obtain the expression for the total electric field (2) of a thin annular electron bunch (1) in the form of the superposition of the Coulomb field of a moving charge and the transition radiation field:

$$E_r(t,r,z,t_0,r_0) = E_r^{coul} + E_r^{trans}, \quad (5)$$

$$E_r^{coul}(t,r,z,t_0,r_0) = -\frac{2Ne}{b^2\varepsilon\sqrt{1-\varepsilon v_0^2/c^2}} \times \sum_{n=1}^{\infty}\frac{J_0(\lambda_n r_0/b)J_1(\lambda_n r/b)}{J_1^2(\lambda_n)} \times \{\vartheta_1 \exp[\omega_{0n}(t - t_0 - z/v_0)] + \vartheta_2 \exp[-\omega_{0n}(t - t_0 - z/v_0)]\}, \quad (6)$$

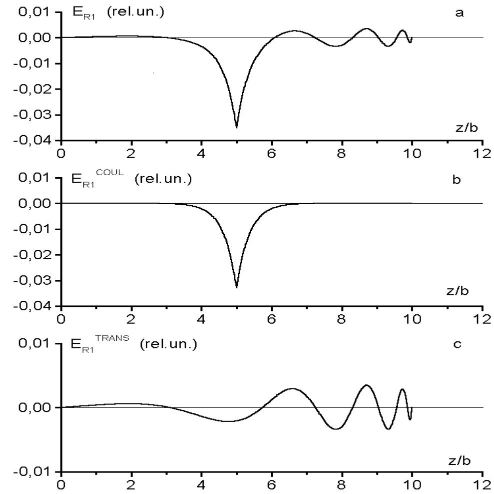
$$E_r^{trans}(t,r,z,t_0,r_0) = \frac{4Ne}{b^2\varepsilon\sqrt{1-\varepsilon v_0^2/c^2}} \times \sum_{n}\frac{J_0(\lambda_n r_0/b)J_1(\lambda_n r/b)}{J_1^2(\lambda_n)} \times \{\vartheta_1 \sum_{m=0}^{\infty}(q_1^{2m+1} - q_2^{2m+1})J_{2m+1}(y_n) + \vartheta_2 \sum_{m=0}^{\infty}(q_1^{2m+1} + q_2^{-2m-1})J_{2m+1}(y_n)\} - E_r^{coul}, \quad (7)$$

where $\vartheta_1 = 1$ if $z_0 \le z < z_{pr}$, else $\vartheta_1 = 0$; $\vartheta_2 = 1$ if $0 \le z < z_0$, else $\vartheta_2 = 0$; $z_0 = (t - t_0)v_0$ is the position of the ring-shaped bunch (1), $z_{pr} = (t - t_0)v_{pr}$ is the position of the precursor of transition field, $v_{pr} = c/\sqrt{\varepsilon}$ is the maximal velocity of the EM perturbation propagation in the dielectric waveguide; $q_{1,2} = \sqrt{\frac{(t - t_0 - z\sqrt{\varepsilon}/c)(c \mp \sqrt{\varepsilon}v_0)}{(t - t_0 + z\sqrt{\varepsilon}/c)(c \pm \sqrt{\varepsilon}v_0)}}$,

$y_n = (\lambda_n c/b\sqrt{\varepsilon})\sqrt{(t - t_0)^2 - z^2\varepsilon/c^2}$.

The quasistatic (6) and the transition radiation (7) components are non-zero in the region $z < z_{pr}$. For $t > t_0$ neither Coulomb nor transition radiation fields enter the region $z > z_{pr}$. The fastest and the highest-frequency component of the electromagnetic signal is the precursor, which propagates with the velocity $v_{pr}$ [15]. Since the bunch propagation velocity is $v_0 < v_{pr}$, the field overtakes the bunch. A qualitative pattern of the propagation of a transition radiation pulse is illustrated in Fig. 1. The bunch generates an UWB transition radiation pulse, whose shortest wavelength components are the components of the precursor. The oscillation amplitude in the pulse decreases toward the precursor, so that the total field vanishes at the point $z = z_{pr}$.

For simulations we chose an electron bunch of radius $a$ and length $L_b$ with Gauss density profile. The characteristics of the transition radiation signal near the waveguide entrance are illustrated in Fig. 2.



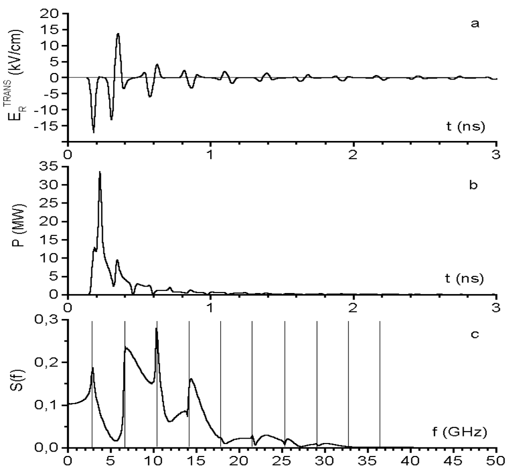

a

b

c

*Fig.1. The first harmonic: a – total, b – coulomb, c – transition field: $tc/b = 10$, $r/b = 1$, $v_0/c = 0.5$, $\varepsilon = 1$*

The electromagnetic pulse near the waveguide input end is characterized by a large field amplitude (15 kV/cm), high peak power (about 33 MW), and short duration (less than 1 ns). The spectrum of this signal is broadband, it has sharp narrow peaks at frequencies close to the critical frequencies $f_n = \lambda_n c/(2\pi b\sqrt{\varepsilon})$ of the waveguide. The presence of the low-frequency ($f < f_1$) part of the spectrum can be explained by the fact that the transition radiation signal has propagated only a short distance and, therefore, did not have enough time to form completely.




a

b

c

*Fig.2. The characteristics of the transition radiation: a – field, b – power, c – spectrum.* $z = 4cm$, $r = 1cm$, $b = 4cm$, $\varepsilon = 1$, $v_0 / c = 0.9$, $L_b = 2cm$, $a = 0.5cm$

## 3.WAKE-FIELD EXCITATION IN A SEMI-INFINITE DIELECTRIC WAVEGUIDE

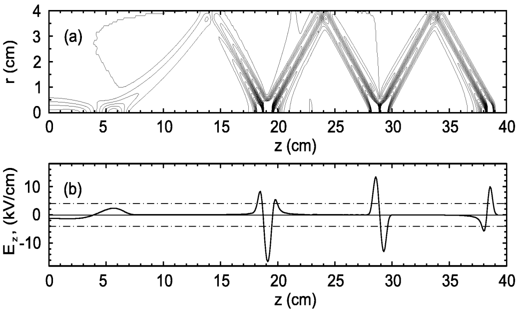
In the case of Cherenkov resonance ($v_0 > c / \sqrt{\varepsilon}$) the field, excited by the thin charged ring (1) can be written [16], similar to [13], in the form of superposition of spatially limited Cherenkov radiation field and transition radiation field:

$$E_z(t,r,z,t_0,r_0) = E_z^{cher} + E_z^{trans} \ , \qquad (9)$$

$$E_z^{cher}(t,r,z,t_0,r_0) = \frac{4Ne}{b^2\varepsilon}\sum_n \frac{J_0(\lambda_n r_0 / b) J_0(\lambda_n r / b)}{J_1^2(\lambda_n)} \times$$
$$\times\, \vartheta(z, z_{gr}, z_0) \cos\left[\tilde{\omega}_{0n}(t - t_0 - z / v_0)\right] \ , \qquad (10)$$

$$E_z^{trans}(t,r,z,t_0,r_0) = \frac{4Ne}{b^2\varepsilon}\sum_n \frac{J_0(\lambda_n r_0 / b) J_0(\lambda_n r / b)}{J_1^2(\lambda_n)} \times$$
$$\times \{\vartheta(z, z_{gr}, z_{pr}) \sum_{m=1}^{\infty} (-1)^m (\tilde{q}_1^{2m} - \tilde{q}_2^{2m}) J_{2m}(y_n) +$$
$$+ \vartheta(z, 0, z_{gr}) [J_0(y_n) + \sum_{m=1}^{\infty} (-1)^m (\tilde{q}_1^{2m} + \tilde{q}_2^{-2m}) J_{2m}(y_n)]\} \qquad (11)$$

$\vartheta(z, z_1, z_2) = 1$ if $z_1 \le z < z_2$, else $\vartheta(z, z_1, z_2) = 0$; $z_{gr} = (t - t_0) v_{gr}$ is the position of the group wavefront, $v_{gr} = c^2 / \varepsilon v_0$ is the group velocity of the resonance wave; $\tilde{\omega}_{0n}^2 = -\omega_{0n}^2$, $\tilde{q}_{1,2}^2 = -q_{1,2}^2$.




a

b

*Fig. 3. The topography of the field $E_Z$: (a) – the level curves of the field $E_Z$, (b) – the respective profile of $E_Z$ at r=0. Level curves are drawn with a step of 0.4 kV/cm in the range from –4 kV/cm to 4 kV/cm. Dash-dot lines in figure (b) mark the limits of this range: tc/b=10, b=4 cm, ε=2.6, $L_b$=1 cm, a=0.5 cm, $v_0$/c=0.9798, $Q_b$=1.6 nC*

In Fig.3(a) with the help of level curves the 2D (in the plane z-r) picture of distribution of longitudinal electric field excited by relativistic electron bunch with gauss profile is represented. The position of "group wavefront" is $z_{gr}$ = 15.6 cm, the coordinate of precursor is $z_{pr}$ = 24.8cm, and the bunch coordinate is $z_0$ 38.7 cm. In the region $z_{pr}<z<z_0$ the intense Cherenkov wake wave exists. Structure of this wave is formed as a result of periodic reflections of Cherenkov cone from the sidewalls of waveguide. In the region $0<z<z_{pr}$ the transition radiation field superimposes on the Cherenkov field. Weak transition oscillations in the precursor region 20 cm < z < 23 cm still can be noticed against the intense Cherenkov field. Behind $z_{gr}$ the field is small and its structure is different from the one of Cherenkov wave. Note the high amplitude and small width of field spikes in Fig.3(b). $E_z$ is maximal at the waveguide's axis, where the waves, reflected from sidewalls, are focusing.

## 4. CONCLUSION

When a charged bunch enters a semi-infinite cylindrical waveguide, it generates a transition radiation. If the Cherenkov resonance condition is not satisfied, the excited electromagnetic field is a superposition of the quasistatic field of a moving charge and the transition radiation field. The fastest component of the field (the precursor) propagates with the velocity $c / \sqrt{\varepsilon}$, which is higher than the bunch velocity.

The spectrum of the transition radiation signal is broadband and contains peaks corresponding to several radial modes. The peaks in the spectrum occur at frequencies somewhat higher than the corresponding critical frequencies of the waveguide. The transition radiation signal near the waveguide entrance is characterized by high peak power and short duration.

If the Cherenkov resonance condition is satisfied, the excited field consists of spatially limited Cherenkov radiation field and transition radiation field. Accounting of the boundary leads to the appearing of the effect of wake field's drift after the leading bunch with the group velocity of resonance wave. This results in limitation of intense wake field region in the longitudinal direction.

## REFERENCES


1. H. F. Harmuth, *Nonsinusoidal Waves for Radar and Radio Communication* (Academic Press, New York, 1981; Radio i Svyaz', Moscow, 1985).
2. L. Yu. Astanin and A. A. Kostylev, *Foundations of Super-Broadband Measurings* (Radio i Svyaz', Moscow, 1989).
3. N. V. Zernov and G. V. Merkulov. // *Zarubezhn. Radioelektron.*, №9, 95 (1981).
4. M. I. Gaponenko, V. I. Kurilko, S. M. Latinskij, et al. // *VANT, Ser. Yad.-Fiz. Issled.* **11**, 151 (1997).
5. V. A. Balakirev, M. I. Gaponenko, A. M. Gorban', et al. // *VANT, Ser. Fiz. Plazmy*, 118 (2000).
6. V. A. Balakirev and G. L. Sidel'nikov. // *Zh. Tekh. Fiz.* **69**, 90 (1999).
7. W. Gai, P. Schoessow, B. Cole, et al. // *Phys. Rev. Lett.* **61**, 2756(1988).
8. I. N. Onishchenko, V. A. Kiseljov, A. K. Berezin, et al., in *Proc. of the Particle Acc. Conf., New York, 1995* (IEEE, New York, 1995), p. 782.
9. V. Kiseljov, A. Linnik, V. Mirny, et al., in *Proc. of the 12th Int. Conf. on High-Power Particle Beams,*

*Haifa, Israel, 1998* (IEEE, Haifa, 1998), Vol. 2, p. 756.
10. T. B. Zhang, T. C. Marshall, J. L. Hirshfield. // *IEEE Trans. Plasma Sci.* **26**, 787(1998).
11. T. B. Zhang, J. L. Hirshfield, T. C. Marshall, and B. Hafizi. // *Phys. Rev. E* **56**, 4647 (1997).
12. N. G. Denisov. // *Zh. Eksp. Teor. Fiz.* **21**, 1354 (1951).
13. E. L. Burshtein and G. V. Voskresenskij. // *Zh. Tekh. Fiz.* **33**, 34(1963).
14. V. A. Balakirev, I. N. Onishchenko, D. Yu. Sidorenko, G. V. Sotnikov. // *Zh. Tekh. Fiz.* **72**, 88(2002).
15. L. Brillouin. // *Ann. Phys. (Leipzig)* **44**, 203 (1914).
16. V. A. Balakirev, I. N. Onishchenko, D. Yu. Sidorenko, G. V. Sotnikov. // *Zh. Eksp. Teor. Fiz.* **120**, 41 (2001).