DIFFUSION OF THE EROSION PRODUCTS OF COPPER ELECTRODES FROM ELECTRIC ARC CHANNEL

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The diffusion of vapour derived from electrodes is investigated for low erosion rates. In the case of low temperatures, the problem is solved analytically for free-burning and wall-stabilized electric arcs. For a wider temperature range, the problem is solved by numerical simulation. The temperature profile for the free-burning electric arc was obtained using a Gaussian approximation of experimental results.
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INTRODUCTION

A free-burning electric arc between vaporizing electrodes is a suitable source of high-density plasma. Investigations of such arcs are relevant to the development of modern switching devices. The material that is vaporized due to the current enters the gap between the electrodes. Since the metal vapour atoms have low ionization energy in comparison with atoms of the ambient gas (noble gases or air, as a rule), ionization of the metal vapour contributes substantially to the electron density. As a result, the metal vapour plays a key role in the processes of heat, mass and charge transfer in the plasma. Thus, the basic parameters of the electric arc are largely determined by the vaporized electrode material. As a rule the content of electrodes vapour material doesn't exceed (1-10)%, so the diffusive mode of transport should be prevail. This kind of mode is considered in the present paper.

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The radial distribution of the copper concentration in the arc is determined from the continuity equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\overline{J_{Cu}}\right) = w_{Cu}\,, \tag{1}$$

where $\overline{J_{Cu}}$ – sum of diffusion fluxes of copper components and w_{Cu} – mass production rate of copper per unit volume, which is approximated by either a step function distribution:

$$w_{Cu} = \begin{cases} w_0\,, r \le r_s \\ 0, r > r_s \end{cases} \tag{2}$$

or a Gaussian distribution:

$$w_{Cu}=k_w w_0 exp(-r^2/r_s^2), \tag{3}$$

where r_s is the approximation constant. The correction factor $k_w=\{1-exp[-(r_w)_n^2]\}^{-1}$, where $(r_w)_n=r_w/r_a$, (the indices w and a correspond to the adsorbing wall and the arc channel) is included in order to allow equal copper fluxes into the arc when both expressions (2) and (3) are used. As $(r_w)_n$ increases, k_w decreases exponentially towards the value k_w=1; when $(r_w)_n$ >2, the value of k_w can be approximated as 1 with satisfactory accuracy. In both cases, $w_0=2G/(\pi r_a^2L)$, where G – erosion rate and L – electrode separation.

The distribution of copper along the electric arc is considered to be homogeneous, because the rate of copper transport is determined by the diffusion coefficient, which is larger within the arc channel due to the high temperature. As shown in [1], the result of calculations are practically independent of the value of r_s for $r_s<r_a$. We therefore use $r_s=r_a$.

We apply the approach derived in [2] to calculate the copper diffusion flux. In this approach, the multicomponent diffusion coefficients D_{ij} are combined into one combined ordinary diffusion coefficient $\overline{D^x{}_{Cu-N_2}}$, which describes the diffusion of all copper components with respect to all nitrogen components. The expression for the diffusive mass flux in the usual case of low values of electrode vapour fraction in the arc plasma then becomes [3]

$$\overline{J_{Cu}} = -\rho\, \overline{D^x{}_{Cu-N_2}} \cdot d\overline{c_{Cu}}/dr \tag{4}$$

where ρ – mass density of mixtures and $\overline{c_{Cu}}$ – sum of mass fraction of copper components.

The calculations according to technique described in [2] show that for low temperatures (up to 7000 K), the combined ordinary diffusion coefficient is practically independent of the concentration of the electrode material vapour, and can be approximated by a simple power dependence of the form

$$\overline{D^x_{Cu-N_2}} = D_0\left(T/T_0\right)^{n_D}. \tag{5}$$

where T – temperature, n_D – constant of approximation and the index 0 corresponds to the arc's center. This allows a simple analytical solution for the radial distribution of the mole fraction of copper vapour to be derived [3]. The system of equations (1)-(5) is closed by the equation of state

$$n=p_a/(k_BT) \tag{6}$$

where n – number density, p_a – atmospheric pressure, k_B – Boltzmann constant, and the boundary conditions

$$d\overline{x_{Cu}}/dr\Big|_{r=0} = 0\,, \qquad \overline{x_{Cu}}\Big|_{r=r_w} = 0 \tag{7a,b}$$

where $\overline{x_{Cu}}$ – sum of mole fraction of copper components.

For the Gaussian radial temperature profile of the arc considered in [1,4]

$$T(r_n)=T_0exp(-ar_n^2), \tag{8}$$

where a is a empirical constant and $r_n=r/r_a$, the solution of (1) together with (2), (4)-(8) is:

$$\overline{x_{Cu}}(r_n) = \begin{cases} p_1\begin{bmatrix} Ei\left(\beta\,(r_w)_n^2\right) - Ei(\beta) + \\ exp(\beta)/\beta - exp\left(\beta\, r_n^2\right)/\beta \end{bmatrix}, & r_n \le 1 \\ p_1\left[Ei\left(\beta\,(r_w)_n^2\right) - Ei\left(\beta\, r_n^2\right)\right], & 1 < r_n \le (r_w)_n \end{cases}, \tag{9}$$

where $Ei(x)$ is the Cauchy principal value integral ($Ei(x)=-Ei(1,-x)$ for $x<0$, where $Ei(1,x)=\int_1^\infty e^{-xt}/t\,dt$ is the exponential integral) [5], $\beta=\alpha(n_D-1)$, $p_1=Gk_BT_0/(2\pi Lp_aD_0m_{Cu})$.

If the combined ordinary diffusion coefficient is approximated with satisfactory accuracy by the parabolic dependence (5), where $n_D=2$, the equation (1) together with (3) can be solved analytically for all areas at once, which gives the following result:

$$\overline{x_{Cu}}(r_n)=k_w p_1\left[\begin{array}{l} Ei\left(1,\,(r_w)_n^2(1-\alpha)\right)-Ei\left(1,\,r_n^2(1-\alpha)\right) \\ +\,Ei\left(\alpha (r_w)_n^2\right)-Ei\left(\alpha\, r_n^2\right)\end{array}\right] \qquad (10)$$

For a wall-stabilized electric arc, in which the product of erosion of the electrodes is deposited on the surface of the wall, it is possible to find the solution of equation (1) based on the double-layer quasi-channel model of the arc [6]. In this case, we specify w_0 in the form of a "step", as in equation (2), and write the temperature dependence of the thermal conductivity as $\lambda(T)=\lambda_w(T/T_w)^{n_\lambda}$, where n_λ is the constant.

The solution is:

$$\overline{x_{Cu}}(r_n)=\begin{cases} D+\dfrac{p}{\xi A}\left(1-Ar_n^2\right)^{\xi}, & r_n\le 1 \\ \dfrac{2p}{\xi B}\left[\left(B\ln\left(\dfrac{(r_w)_n}{r_n}\right)+C\right)^{\xi}-C^{\xi}\right], & 1<r_n\le (r_w)_n \end{cases}, \qquad (11)$$

where

$$\xi=(n_\lambda-n_D+2)/(n_\lambda+1),\quad A=1-\left(T_1/T_0\right)^{n_\lambda+1},$$
$$B=2\left(T_0^{n_\lambda+1}-T_1^{n_\lambda+1}\right)\left(T_1^{n_\lambda+1}-T_w^{n_\lambda+1}\right)/\left(T_0T_1\right)^{n_\lambda+1},$$
$$C=\left(T_w/T_0\right)^{n_\lambda+1},\quad T_1=T(r_n=1), \qquad (12)$$
$$D=\frac{2p}{B\xi}\left[\left(B\ln(r_w)_n+C\right)^{\xi}-C^{\xi}\right]-\frac{p}{A\xi}(1-A)^{\xi}.$$

If the temperature of the arc plasma between the copper electrodes is greater than 7000 *K* [7], the combined ordinary diffusion coefficient has a complicated dependence on both the temperature and the copper concentration. For the $\overline{x_{Cu}}<10\%$ in the interval 300-12000 *K*, we can approximate the dependence with satisfactory accuracy by the following analytic expression:

$$\overline{D^{x}_{Cu-N_2}}\left(T,\overline{x_{Cu}}\right)=\frac{2}{\left(1+e^{0.18/T}\right)}\frac{C_1\left(\overline{x_{Cu}}\right)T^{1/12}}{\left(T-C_2\left(\overline{x_{Cu}}\right)\right)^2+C_3\left(\overline{x_{Cu}}\right)}, \qquad (13)$$

with the temperature T is in *eV*, where

$$C_1=2.41+3.01\cdot\overline{x_{Cu}}-167\cdot\left(\overline{x_{Cu}}\right)^2+2240\cdot\left(\overline{x_{Cu}}\right)^3,$$
$$C_2=0.60+3.51\cdot\overline{x_{Cu}}-55.0\cdot\left(\overline{x_{Cu}}\right)^2+310\cdot\left(\overline{x_{Cu}}\right)^3, \qquad (14)$$
$$C_3=0.054+1.60\cdot\overline{x_{Cu}}-30.1\cdot\left(\overline{x_{Cu}}\right)^2+161\cdot\left(\overline{x_{Cu}}\right)^3.$$

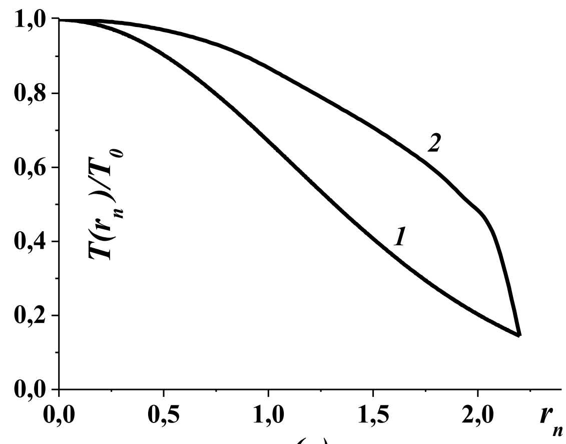
As a result of a single integration of equation (1), taking into account (3)-(5) with boundary condition (7a), the first-order integral equation for determining the copper concentration in the arc was obtained:

$$\frac{d\overline{x_{Cu}}}{dr_n}+\frac{p_2T(r_n)}{r_n\left(\overline{D^{x}_{Cu-N_2}}/D_0\right)}\cdot\left[1-\exp\left(-r_n^2\right)\right]=0, \qquad (15)$$

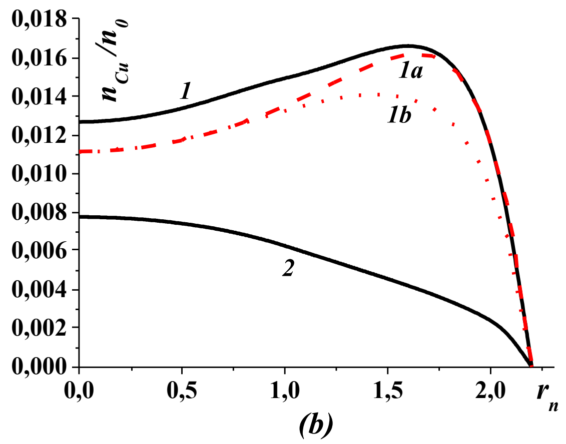
where $T(r_n)=T(r)/T_0$ and $p_2=k_GGk_BT_0/(\pi Lp_am_{Cu})$, with boundary condition (7b).

III. RESULTS AND DISCUSSION

Fig.1 shows examples of calculations for the arc between copper electrodes at atmospheric pressure when the temperature is low. The parabolic approximation for the dependence of the combined ordinary diffusion coefficient on temperature (5) was used with D_0~45 cm^2/s. The value $n_\lambda=5/2$ was assumed for the exponent in the temperature dependence of the thermal conductivity. The measured temperature profile reported in [8] for L=0.8 *cm* and the electric current of arc I=3.5 *A* was approximated by a Gaussian profile (8) with parameters T_0=6700 *K* and α=0.4. The erosion rate for a current 3.5 *A* was taken to



(a)



(b)

Fig.1. Radial normalized temperature profiles $T(r_n)/T_0$ (a) and copper concentration n_{Cu}/n_0 ($n_0=p_0/[k_BT_0]$) (b) for a Gaussian temperature distribution (curves 1, 1a, 1b) and for the temperature profile obtained from a self-consistent solution of the thermal conductivity equation (curve 2). (1) – Solution for a step distribution of w_{Cu} and n_D=2 (1a) – Solution for Gaussian distribution of w_{Cu} and n_D=2, (1b) – solution of differential equation (16) together with the combined ordinary diffusion coef-

ficient in the form of (13). $(r_w)_n$=2.2

be, in accordance with [9], equal to 56 $\mu g/s$ (in calculating this value, the mean duration of the arc was assumed to be 100 *ms*).

From the results, presented in *Fig.1* it is possible to conclude the following. The somewhat large values of cooper mole fraction $\overline{x_{Cu}}$ obtained from the "step" distribution of the cooper production rate w_{Cu} (curve *1*), compared with those obtained for the Gaussian distribution of w_{Cu} (curve *1a*), are explained by the greater localization of the copper production in the arc channel in the first case. Curve *1b* shows the solution of equation (15) with the diffusion coefficient in the form of (13) for the same parameters. The small differences between the curves *1a* and *1b* arise because of deviations from the parabolic dependence in the approximation of the combined ordinary diffusion coefficient at low temperatures.

For determining copper vapour distribution over a wide temperature range, equation (15) together with boundary condition (7b) was solved numerically for the specified Gaussian temperature profile $T(r_n)$:

$T(r_n)=(1-T_w/T_0)exp(-\alpha r_n^2)+T_w/T_0$,

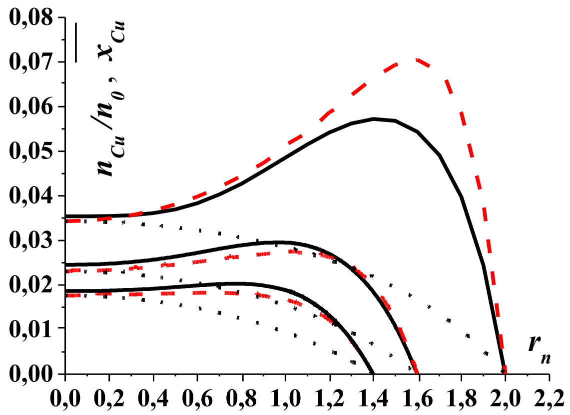


Fig.2. Radial profiles of normalized concentration n_{Cu}/n_0 ($n_0=p_0/[k_BT_0]$) (full curves) and copper mole fraction $\overline{x_{Cu}}$ (dotted curves) for a Gaussian temperature distribution and for various positions of the adsorbing wall. Corresponding values of n_{Cu}/n_0 calculated with a parabolic temperature dependence of the combined ordinary diffusion coefficient are shown by broken curves

For I=30 *A* the erosion rate equals 216.7 $\mu g/s$ [9]. The results of calculations for three positions of the adsorbing wall are presented in *fig.2*. If $(r_w)_n>2$, the copper mass fraction becomes greater than the 10% limit assumed in this paper. For the calculations, the following values of parameters were used: L=0.8 *cm*, T_0=8500 *K* and α=0.6 [7]. The corresponding solutions obtained with parabolic temperature dependence of the combined ordinary diffusion coefficient are shown by broken curves. While these provide an estimate the copper content in the arc plasma at low temperature, they give incorrect values for high temperatures because the diffusion coefficients are overestimated.

It is possible to conclude from results presented that as the distance from arc to the adsorbing wall increases, then the maximum copper concentration on the periphery of the vapour diffusion region increases. This is due to the lower value of the combined ordinary diffusion coefficient at low temperatures, and because of the fact that the density of gas increases with decreasing temperature at constant pressure. No significant increase in the copper vapour mole fraction is observed as the distance from the arc axis increases. This is contrary to the interpretation of the experimental results presented in reference [10]. These results, as was remarked in [11], can be explained in terms of the non-equilibrium state of the plasma at the arc periphery due to the influence of resonance radiation transfer.

The results of calculations of copper concentration for different temperature profiles (*fig.1b*) show significant, but mainly small, differences. In [12] the spatial distribution of the self-absorption of the 510.5 *nm* spectral line in a free-burning arc between copper electrodes were investigated by laser diagnostic methods. According to the results obtained, the copper concentration profile represented by curve *2* in *fig.1b* is more suitable than the others. This suggests that the temperature profile represented by curve 2 in *fig.1a* is more realistic than the widely-used Gaussian profile [13], and shows that it is necessary to choose the temperature profile at the periphery of arc very carefully

IV. CONCLUSION

Using papers [1-3] as starting points, we have solved the problem of determining the electrode vapour distribution in free-burning electric arcs over a wide temperature range using the combined diffusion coefficient formulation to treat diffusion. We have shown that the calculated concentration of the electrode vapour in the arc depends strongly on the temperature profile that is assumed, and to a much smaller extent on the distribution of the copper production rate near the electrode.

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