RADIATION AND BEAM ENERGY SPREAD IN A CHANNEL WITH RANDOM ROUGNESS

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The energy spread of particles inside a short bunch propagating along the co-axial channel with small random axially symmetric wall perturbations is calculated. Although the radiation is small, its reaction on particles distributed along a bunch can create a spread of particle energy and leads to an uncontrolled growth on a transverse emittance of the transported beam.
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1 INTRODUCTION

Short-wave free electron lasers are very sensitive to transverse and longitudinal emittances of an electron beam. The problem of radiation of electron bunches inside a channel with small random perturbations of the wall has recently been discussed. Although the radiation is small, its reaction on particles distributed along a bunch creates a spread of particle energy and could lead to an uncontrolled growth of transverse emittance in the transported beam. Estimations [1 – 3] of the effect show its possible danger for practically reached uniformity of the wall surface. These estimations are based on the concept of effective beam-channel coupling impedance of a beam and a channel. Real part of the impedance is interpreted as the losses due to diffraction radiation, and imaginary part is calculated using Kronig-Kramers relations (see, for example, [4]). Radiation losses themselves are estimated for a statistical model of the surface. It seems that this approach is not quite correct, or, at any case, has a vague area of applicability. Actually, the use of coupling impedance as a coefficient of proportionality between a harmonic of the current of frequency ω and of wave number k and a harmonic of electric field with the same characteristics does mean the equality of phase velocities of these two wave processes. Then, the field acting on a test particle of the mono-energetic beam depends only on the position of the particle relative to the bunch and does not depend explicitly on time. Consequently, it results in linear growth of energy deflexion and transverse pulse with the distance z.

Waves propagating in the system with non-regular (random) pertuberations of the wall have continuous spectrum with respect to k, because reflections from individual perturbations are non-coherent. It means the absence of slow eigenwaves of the channel and non-stationary behaviour of the radiation field acting on the test particle even for a harmonically space-modulated beam. Hence, the variations of particle energy are non-regular and the conception of impedance looses its physical meaning. Therefore, we calculate it using the straight way of classical electrodynamics.

2 EVOLUTION OF FIELD ALONG NON-REGULAR SYSTEM

We consider longitudinally modulated electron beam of velocity βc and current density $j(r, z/\beta c-t)$ which is assumed to have multiplicative distribution:

$$j(r,z,t) = \beta c q(\omega)\psi(r)\exp[i\omega(\frac{z}{\beta c}-t)], \quad (1)$$

propagating co-axially along a round quasi-regular waveguide (see Fig. 1). Here, $q(\omega)$ is the Fourier-amplitude of the linear charge density, $\psi(r)$ is a normalised to unity transverse density distribution.

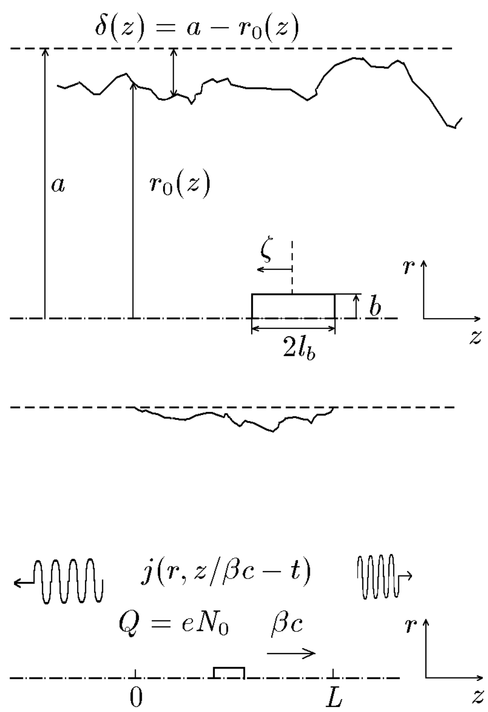

Fig. 1. Geometry and notations.

We limit ourselves the case of axially symmetric perturbations of the wall surface when only TM-waves can be excited. They influence the particle energy only and don't lead to transverse displacements. The deviations of the real radius of the channel $r_0(z)$ from an approximating radius a of a coaxial smooth cylinder are assumed small enough (in the sense discussed further).

Non-uniform wave equations must be completed by the following boundary condition:

$$E_z(r_0(z),z) \equiv -\frac{dr_0}{dz}E_r(r_0(z),z). \quad (2)$$

It means vanishing of a tangential component of the electric field on the perfectly conducting wall of the waveguide.

Following methodically [1, 2] we approximate the boundary condition by its expansion at $r = a$ using the smallness of $\delta(z) = a - r_0(z)$:

$$E_z(a,z) \approx \delta' E_r(a,z) + \delta \frac{\partial E_z}{\partial r}\Big|_{r=a} + \cdots, \quad (3)$$

where prime denotes a total derivative with respect to z. This approximation is reasonable if $|\delta'| << 1$, i.e., if perturbations at the surface are smooth enough. The area of its applicability was discussed in [1, 2]. Under the condition, the fields excited by the beam inside a regular wavegude of radius a can be substituted into the right part of (3). Because the radiation can't exist in a regular waveguide (except of transition radiation at ends of the waveguide) then the zero order field propagates with the velocity of the beam having periodicity $\propto \exp(ik_0z/\beta)$, where $k_0 = \omega/c$, and represents just a Coulomb's field in the laboratory frame. The method of its calculations is a standard one. It is necessary to use the expansion the field over the full system of eigenfunctions of the transverse Laplacian's operator (for the case, Bessel's functions of zero order with roots λ_n).

Taking into account this expansion the boundary condition (3) can be written as

$$E_z(a,z) = Aq(k_0)f(z)\exp(ik_0z/\beta), \quad (4)$$

where

$$f(z) = \delta' + \frac{ik_0\delta}{\beta\gamma^2}, A = \sum_n \frac{4\pi\beta^2\gamma^2\psi_n J_1(\lambda_n)}{\lambda_n^2\beta^2\gamma^2 + k_0^2a^2}. \quad (5)$$

The solution of the non-uniform equation for E_z with the non-uniform boundary condition (4) can be represented in the following form:

$$E_z(r,z) = E_z^0(r,z) + Aq(k_0)f(z)\exp\frac{ik_0z}{\beta} + E_z^1(r,z),$$

where $E^1_z(r,z)$ satisfies the non-uniform equation with zero boundary condition.

Note once more that $E^0_z(r,z)$ and $E^0_z(a,z)$ have no poles for real k_0 and, consequently, can't describe free propagating radiation field. The last is described by the term $E^1_z(r,z)$ and have to meet the demands of the Sommerfeld's principle (principle of causality). For the case: at the input of the waveguide (z=0) waves exist and propagate only in negative direction of z-axes; and at the output of the waveguide ($z = L$) waves propagate only in positive direction. This rule of the solution choice is valid and for $L \to \infty$ as well.

As far as $E^1_z(a,z)$= 0 we can find the solution of non-uniform equation for complex amplitudes of the radiation field with zero boundary condition for $E^1_z(r,z)$ in the form of the expansion over the same eigenfunctions $J_0(\lambda_m r/a)$.

$$E_s(z) = A_s^+(z)\exp(i\kappa_s z) + A_s^-(z)\exp(-i\kappa_s z) - \frac{2Aq(k_0)f}{\lambda_s J_1(\lambda_s)}\exp\frac{ik_0z}{\beta},$$

where

$$A_s^+(z)\exp(i\kappa_s z) + A_s^-\exp(-i\kappa_s z) = 0,\ \kappa_s = \sqrt{k_0^2 - \lambda_s^2}.$$

Taking into account above-mentioned principle of causality we can find the amplitudes of forward and backdirected waves.

Spatially and temporally modulated current of large enough frequency (higher than the cutoff frequency $\lambda_s c/a$) excites forward and backdirected modes of radiation. Low frequency components of the current (imaginary κ_s) do not excite radiation, and corresponding fields propagate together with the beam. These fields represent corrections of Coulomb's field more or less locally related to the current density. Note, that the correction themselves are calculated without taking into account boundary effects at z=0 and z=L. However this can be of importance only if $k_0a \approx \lambda_s$, when Coulomb's field is not yet screened by the walls.

The radiation field as distinct from Coulomb's field is defined by overall beam inside the waveguide, and grow (for forward wave) along it. However, because of random positions of small perturbations of the wall surface the phase of the field is random as well, and the radiation can be only partially coherent even a perfect regular arrangement of primary radiators i.e., of beam particles. The degree of the coherency is defined by correlation properties of a non-uniform surface.

3 PARTICLE ENERGY VARIATIONS DUE TO RADIATION FIELD

We find the energy variation $\Delta_s\gamma(\zeta)$ of a test particle placed at a lag distance ζ (ζ= 0 at the centre of a bunch) in longitudinal direction and at radial position r under the action of s-mode. We restore omitted term $\exp(-ik_0ct)$ in the field expression putting there $t=(z+\zeta)/\beta c$, and integrate $J_0(\lambda_s r/a)E_s(z)\exp(-ik_0z/\beta)$ along the waveguide (the influence of Coulomb's field of zero order and backdirected wave is neglected) and over k_0, i.e., the inverse Fourier transformation is to be performed to restore a real spatial/temporal structure of the bunched beam field:

$$\Delta_s\gamma = \frac{ieJ_0(\lambda_s r/a)}{mc^2\lambda_s J_1(\lambda_s)}\int_0^L \delta(L-\xi)G(\xi)d\xi,$$

where

$$G(\xi) = -\int_{-\infty}^{+\infty} Aq(k_0)\kappa_s \exp(-ik_0\zeta/\beta)\times \left\{1 + \frac{\kappa_s - k_0\beta}{\kappa_s - k_0/\beta}\left[\exp\left[i(\kappa_s - k_0/\beta)\xi\right] - 1\right]\right\}dk_0. \quad (6)$$

The behaviour of $G(\xi)$ is determined by an hierarchy of three characteristic lengths - the channel transverse radius a, the channel length L (or the distance $\xi = L$-z), and the bunch longitudinal size in the lab frame l_b. Apart from pure geometry this hierarchy depends essentially on the Lorentz factor γ. We can consider two limiting cases.

1) If the independent variable ξ satisfies the inequality $a\gamma/\lambda_s\xi >> 1$ then for the majority of the harmonics (with wave-numbers essentially lesser than $\lambda_s\gamma^2/a$ the phase slip at the total length is small. The corresponding exponent in (6) may be changed then for unity. Then $G(\xi) \approx G(0)$ is independent of ξ and the expression for the energy gain is simplified. Thus case can be considered as the "short channel" solution if the length of the channel $L << L^* = a\gamma/\lambda_s$.

2) If $\xi >> L^*$ even a smooth minimum of the function $\kappa_s - k_0/\beta$ plays certain selective role and quasi-resonant harmonic can be of importance. It can be showed that asymptotic behaviour of $G(\xi)$ for large ξ does not depend on ξ and for this case $G(\xi) = G(\infty)$.

4 RESULTS AND ESTIMATIONS

It follows from the consideration above that the total intensity and amplitude of the most effective harmonic depends on a spatial distribution of the bunch current. We shall restrict ourselves by the case of a transversally uniform ($0 < r < b$) single bunch of charge Q with Gaussian longitudinal distribution of half-width l_b

The estimates obtained above are valid for an arbitrary realisation of the perturbation $\delta(z)$ and contain certain information on the asymptotic phase of the radiation field. In the case of random perturbation this information is not of importance. Let us suppose that the mathematical expectation $\overline{\delta} = 0$. As far $G(\xi)$ does not change its value at the correlation length l_c the mathematical expectation $\overline{\Delta}_s$ vanishes as well. The role of a real characteristic of the energy diffusion is played by the diffusion coefficient $\langle\Delta^2\rangle$, which is a sum over transverse harmonics squared and averaged:

$$\left\langle \Delta^2 \right\rangle = \left\langle \left| \sum_s \Delta_s \gamma \right|^2 \right\rangle . \qquad (7)$$

The energy gain at the output of the channel can be written as

$$\left\langle \Delta^2 \right\rangle = C^2 G^{*2} \left\langle \delta^2 \right\rangle l_c L,$$

where $C = r_0 N_0/ba^2$, r_0 is the electron classical radius, N_0 is the number of electrons per bunch, and $G^* = G(0)$ or $G(\infty)$ are dimensionless functions (see Fig. 2) of the lag ζ (measured in channel radius a units) describing the distribution of energy gain along the bunch. Note that for the case $\langle\Delta^2\rangle \propto l_c L$ as it should be for a diffusion process.

Calculations were carried out for the set of parameters of LCLS: $\gamma = 30000$; $l_b/a = 0.002$, (for radius of the channel $a = 0.5$ cm the half-length of the bunch is $l_b = 10$ μm). The coefficient $C = 1.4\times10^3$ m^{-2} for $b/a = 0.1$ and bunch charge $Q = 1$ nC ($N_0 = 6.25\times10^9$). For the set of the parameters it follows that $L^* \approx 50$ m. Taking maximum values of dimensionless energy gains from Fig. 2 and choosing rather pessimistic parameters $(\langle\delta^2\rangle)^{1/2} \approx 5$ μm, $l_c = a$ and $L >> L^*$ we find the amplitude of energy oscillations $\Delta\gamma/\gamma \approx 5\times10^{-4}$.

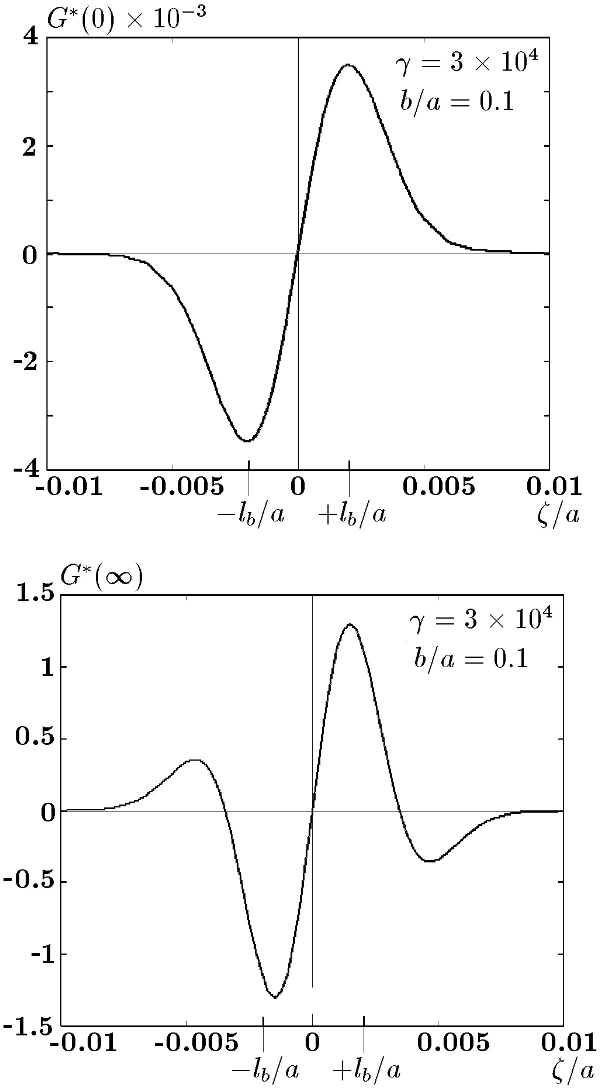

Fig. 2. Dimensionless part of particle energy variation. The dependencies are slightly asymmetric with respect to the centre of the bunch.

5 CONCLUSION

We consider the energy spread inside a bunch propagating along the channel with small random perturbations of the wall as not dangerous for the X-ray FEL's. From this point of view a more vital problems are the energy spread forced by self space charge of the bunch and angular spread forced by the radiation.

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