MOMENT ABERRATIONS IN MAGNETO-ELECTROSTATIC PLASMA LENSES (COMPUTER SIMULATION)

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1 INTRODUCTION

As it is known, the questions of intense ion beam focusing are important both for a problem of controlled thermonuclear synthesis, and for other areas of science and engineering: physics of high energies, accelerating engineering, radiating technologies, implantation metallurgy and so on (see, e.g., [1, 2]). Essential feature of intense ion beams is that for the blow-up avoidance, their charge should be compensated by electrons during transportation and focusing. In this case an application of plasmaoptical focusing systems is expedient, which development was begun by A.I. Morozov with the employees [3, 4], and recently basically is developed by the group of A.A. Goncharov [5–8]. Additional benefits of such systems are large focusing force, basic opportunity of geometrical aberration elimination, and possibility of wide-aperture (non-paraxial) beam focusing. Currently the question consists in improvement of these devices, reduction of aberrations, increase of efficiency and focusing force.

The quality of the charged particle focusing is defined by various aberrations: geometrical, depending from radius and angle of the particle injection (in [4] and in the subsequent works on plasmaoptics, concerning focusing of parallel beams, they refer to as spherical aberrations, or aberrations of wide beam); chromatic aberrations, connected with a longitudinal momentum of particles; at presence of magnetic fields to them aberrations connected with azimuthal movement of particles are added (in the plasmaoptics [4] they refer to as moment aberrations, as they are connected to the angular moment of a particle concerning an axis). Here it is pertinently to note, that in traditional electron optics, basically in connection with electronic microscopy, a little another terminology is accepted [9-11]: chromatic aberrations have the same sense, the term "moment aberrations" is not used, and geometrical aberrations are subdivided on eight kinds (one of which is the spherical aberration of a dot source), and three of them depend on the value and direction of a magnetic field and consequently refer to as anisotropic aberrations.

Theoretically the moment aberrations were studied in [4] by an example of compensated ion beam (CIB) focusing in so-called "A-system", and also under CIB recuperation (both was the plasmaoptical systems). In the case of plasma electrostatic lenses (or rather magneto-electrostatic ones, see below), moment aberrations were considered in the experimental works [5, 6]. In these works the conclusion was drawn that in the absence of spherical aberrations the relatively low coefficient of compression can be explained by the influence of unremovable in principle moment aberrations associated with a finite azimuthal swirl of fast beam particles in the magnetic field of a plasma lens.

In this work moment aberrations in the plasma magneto-electrostatic lenses are considered in more detail with the use of the computer modeling. For solution of the problem we have developed a special computer code – the model of plasmaoptical focusing device, allowing to display the main parameters and operations of experimental sample of a lens [5–8], to simulate the moment and geometrical aberrations and give recommendations on their elimination.

2 STATEMENT OF THE PROBLEM

In the previous work [12] with the use of computer simulation we have considered in detail Morozov plasma lens, in which the magnetic field was created by a single current ring. In order to locate the base electrodes (which introduce electric potentials into the plasma) near the central plane of the lens, in the experimental works [5–8] the magnetic field configuration with oppositely wound three short solenoids was used. In this work such a field configuration is simulated by three turns with opposite currents. Besides, unlike [12], the azimuth motion of focused particles, leading to moment aberrations, is taken into account. The case of axial symmetry is considered.

As it is known, in case of analytical investigation of a Morozov lens it is assumed that the magnetic surfaces coincide with equipotential lines of the electric field. So, magnitude and spatial distribution of the electric field in the plasma is determined completely by the magnetic field geometry and boundary conditions, which are set as a continuous distribution of the potential $\Phi(R, z)$ on a cylindrical surface of radius R, bordering on plasma. Thus it is supposed, that the ion beam propagation area is filled with plasma, density and other characteristics of which are sufficient for space charge neutralization of the beam and creation of required focusing fields. In practice [5–8], the electrical potentials are entered in the plasma discretely, by means of n ring "basic" electrodes of a radius R located in the central part of the lens, between the separatrices, where the longitudinal magnetic field passes through a zero (in our case -2.8 cm $< z <$ 2.8 cm). The magnetic surfaces at the left and right of the central area are considered as grounded.

The magnetic field of a current ring J with the radius a_c and the coordinate l along the z axis is described by the azimuthal component of the vector potential (see,

[13], § 4):

$$A_\varphi = \frac{4J}{ck}\sqrt{\frac{a_c}{r}}\left[\left(1-\frac{k^2}{2}\right)K(k)-E(k)\right]$$
$$k^2 = \frac{4a_c r}{(a_c+r)^2+(z-l)^2} \qquad (1)$$

where c is the velocity of light, K и E are the complete elliptic integrals of the first and second kind. The equation of a magnetic surface which is the exact integral of the equation of the magnetic force line in the case of axial symmetry is (see [13], § 3):

$$\Psi(r,z) \equiv rA_\varphi(r,z) = const \qquad (2)$$

(Ψ is so called magnetic flux function [4]).

We have calculated the topography of magnetic surfaces for various current relations in a central and two side rings (J_c and J_s, respectively; they are the reverse currents; the central ring is located at $z = 0$ and the side rings are located at $z = \pm 5$ cm). Further the field line topography at $J_c = -1.5\,J_s$ was used, which gives the satisfactory distribution over the lens volume.

3 BASIC EQUATIONS

In a Morozov lens the electric potential in the plasma Φ changes from one magnetic surface to another that is expressed mathematically as

$$\Phi(r,z) = F(\Psi). \qquad (3)$$

The components of electric and magnetic fields can be written as

$$B_r = -\frac{1}{r}\frac{\partial\Psi}{\partial z};\; B_z = \frac{1}{r}\frac{\partial\Psi}{\partial r}, \qquad (4)$$

$$E_r = -\frac{\partial\Phi}{\partial r} = -\frac{dF}{d\Psi}\frac{\partial\Psi}{\partial r} = -r\frac{dF}{d\Psi}B_z, \qquad (5)$$

$$E_z = -\frac{\partial\Phi}{\partial z} = -\frac{dF}{d\Psi}\frac{\partial\Psi}{\partial z} = r\frac{dF}{d\Psi}B_r. \qquad (6)$$

Substituting these expressions in the equations of motion for a cylindrical system of coordinates, we receive:

$$\frac{dV_r}{dt} = \frac{q}{M}B_z\left(\frac{1}{c}V_\varphi - r\frac{dF}{d\Psi}\right) + \frac{V_\varphi^2}{r}, \qquad (7)$$

$$\frac{dV_z}{dt} = \frac{q}{M}B_r\left(r\frac{dF}{d\Psi} - \frac{1}{c}V_\varphi\right), \qquad (8)$$

$$\frac{dV_\varphi}{dt} = \frac{q}{Mcr}\left(-V_z\frac{\partial\Psi}{\partial z} - V_r\frac{\partial\Psi}{\partial r}\right) - \frac{V_r V_\varphi}{r}, \qquad (9)$$

where c is the velocity of light, q, M and V are the charge, mass and velocity of the ion respectively (in this case the calculations were carried out for protons).

In case of injection into a lens of the homogeneous, parallel to an axis, monoenergetic ion beam, the entry conditions look as follow:

at $t = 0$ $V_z = V_0$, $V_r = V_\varphi = 0$, $z = z_i$ $(z_i < 0)$, $r = r_i$, (10)

where z_i is the coordinate of the injector output face, and the ion injection radius r_i is set from zero up to size somewhat smaller than the radius of base electrodes R, which in its turn is less than the radius of current rings a_c.

In this case we have set the boundary conditions as a potential distribution on the radius in a plane of the central coil $\Phi(r, 0)$, which, if necessary, can be represented as the potential distribution on a cylindrical surface $\Phi(R, z)$.

On the basis of formulas (1)-(10), simulation of ion trajectories in a plasma lens was made, to investigate the dependence of moment and geometrical aberrations on the lens parameters and position of the ion injector. Thus, as it will be seen below, the connection of moment aberrations with the law of conservation of a generalized angular moment of an ion relative to the axis z is established.

4 SIMULTANEOUS FOCUSING OF IONS BY ELECTRIC AND MAGNETIC FIELDS IN A MOROZOV LENS

As a rule, aberrations are calculated by a method of perturbation of paraxial particle trajectories [4, 9–11]. With the use of computer modeling we have a possibility to consider in the nonparaxial approximation the case of strong magnetic fields and large-aperture ion beams, at which a significant moment and geometrical aberrations take place. In this case the Morozov lens should be considered as a magneto-electrostatic one, because ion focusing in this case is caused by the action of both electric, and (to a lesser degree) magnetic fields; besides, moment aberrations occur through the action of a magnetic field.

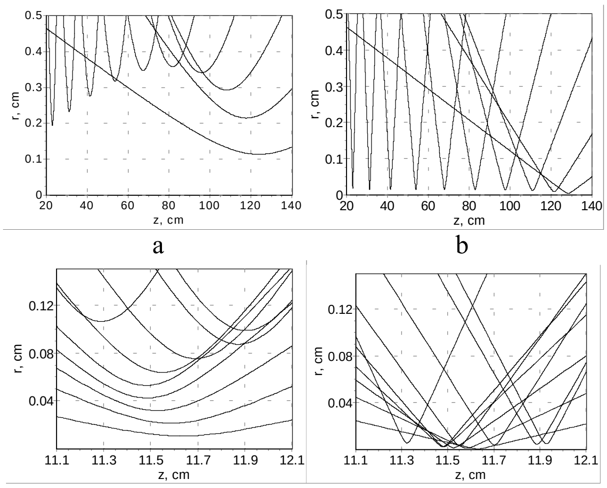

Fig. 1.

As an example we shall consider the focusing by a Morozov lens with the following parameters: radius of a proton beam is 5.5 cm, radii of current rings are 6.5 cm, current in the central ring is 30 kA (in the case of a short coil it corresponds to $3{\cdot}10^4$ ampere-turns). In Fig. 1 the proton trajectories are shown in the focus area for two locations of an proton injector of: $z_i = -20$ cm (a, c) and $z_i = -50$ cm (b, d) in two cases: (a, b) when the electric field is switched off (it is the case of a "pure" magnetic lens), and (c, d) at presence of the electric field optimized on a minimum geometrical aberrations.

The influence of moment aberrations consists in that the particles can not cross the lens axis; geometrical aberration influence consists in that the particles cross the axis in different points. As can be seen from Fig. 1, the location of the injector practically does not influence on geometrical aberrations, and moment aberrations decrease when the injector is moving back from the lens. It will be shown below, that this behavior can be explained within the framework of the law of conservation of generalized angular moment of a charged particle.

The electric field is picked up in view of action of a magnetic field and has the following potential distribution in a plane of the central ring:

$$\Phi = 1.5\, r^2 - 0.0232\, r^4 - 5\cdot 10^{-5}\cdot r^6. \qquad (11)$$

When the magnetic field changes, the focusing is appreciably worsened, therefore the Morozov lens in this case should be considered as a magneto-electrostatic one with moment and geometrical aberrations inherent to it.

5 DEPENDENCE OF MOMENT ABERRATIONS ON FOCUSING CONDITIONS AND LENS PARAMETERS

It is possible to consider an action of different kinds of aberrations as independent from each other, if they are small enough (see. [11], chapter 5.7). Let us consider the dependence of moment aberrations on focusing conditions with the parameters accepted in the preceding section.

In Fig. 2 the dependence of the minimum radius r of a proton trajectory in the focus area versus the injector coordinate z_i is represented at the fixed current of the central ring $J_c = 30$ kA (a) and versus the current in the central ring J_c at the fixed location of the ion injector $z_i = -15$ cm (b) for various values of the injection radius R. As can be seen from this figure, moment aberrations grow when a radius of injection increases, an injector is moved into the area of a stronger longitudinal magnetic field, and also a magnetic field of a lens rises. When the injector moves away from the current rings, moment aberrations tend to zero.

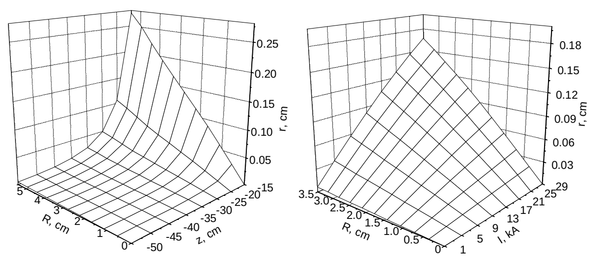

Fig. 2.

In some limiting cases, behavior of moment aberrations can be predicted on the basis of the input equations. To this effect, the equation of azimuthal motion of an ion (9) after some transformations can be presented as:

$$m\frac{d(rV_\varphi)}{dt} = -\frac{e}{c}\frac{d\Psi}{dt}, \qquad (12)$$

wherefrom after integration we obtain the expression known in electronics as the theorem of Bush:

$$rV_\varphi + \frac{e}{mc}\Psi = C_0 = const \qquad (13)$$

If the particles pass through an axis, then $C_0 = 0$.

From (13) it follows, that a sufficient condition of absence of moment aberrations at particle beam convergence into a focus is the absence of initial azimuthal velocities and the equality to zero of a magnetic field in the injection area. For lenses used in experiments [5–8], these conditions are fulfilled, hence it is necessary to search for other reasons of not quite good focusing in these experiments. One of such reasons can be not optimum distribution of focusing fields on volume of a lens.

6 PLASMA LENS AND APPLICATION OF ION BEAMS

As follows from above mentioned theses, in magneto-electrostatic plasma lenses the moment aberrations are easily eliminated by placing an ion injector in a zero magnetic field. However, they can be used, e.g., for a uniform ion exposition of samples in the focus area (about of part of square centimeter).

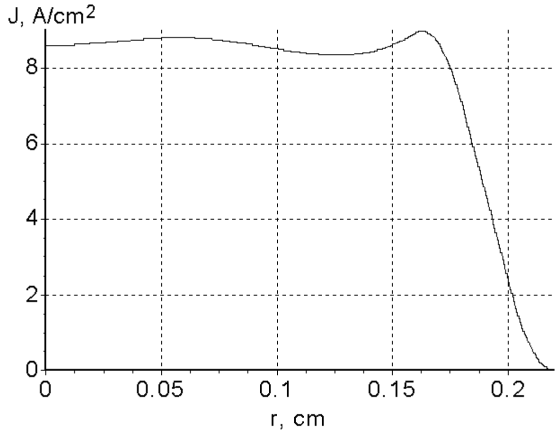

Fig. 3.

Placing an output face of an ion injector at a certain distance from a lens (or placing its in the special solenoid with an adjustable longitudinal magnetic field), it is possible to achieve in a focal plane the step-like distribution of a current density on radius. In particular, such distribution of $j(r)$ is shown in Fig. 3 for the following parameters: radius of a beam is 3.5 cm, coordinate of an injector of ions is $z_i = -15$ cm, current in the central ring is $J_c = 30$ kA, potential distribution along the radius in the central ring plane (in the GS units) $\Phi = 1.5\, r^2 - 0.023\, r^4$. In this case the ion current density in the focal plane is $j \cong 9$ A/cм^2, it is distributed rather regularly on the radius from 0 up to 0.17 cm and 340 times as much as initial one. In such a mode the focused beam remains laminar, i.e., the trajectories of ions are not crossed, and the ion radius in the focal plane is proportional to the ion radius in the plane of injection. However, it should be noted that within the focal plane the longitudinal ion velocity is rather non-uniform, mainly because of a radial movement and to a lesser degree because of azimuthal movement of ions. To reduce the ion velocity spread in the location of the sample, it is necessary to increase a focal length by reducing polynomial factors that represent $\Phi(r)$, e.g., see (11).

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