DYNAMICS OF CHAOTIC WAVES UNDER WEAK NONLINEAR INTERACTION IN THE MAGNETIZED PLASMA WAVEGUIDE

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The chaotic dynamics of high-frequency (HF) electromagnetic wave decay into HF electromagnetic waves and LF plasma waves in the magnetized plasma waveguide has been investigated. In contrast to [1-3], where we have investigated the case when one LF wave takes part in the interaction, in this work we consider the case when in the interaction a few of LF waves take part. It is shown that criterion of dynamic chaos occurence [1, 2] not only allows to qualitatively find the parameter region, where the dynamics of weak nonlinear wave interaction is chaotic, but also rather correctly describes the boundary of transition to chaos.
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1 INTRODUCTION

The criterion of dynamic chaos occurrence at weak nonlinear wave interaction [1, 2] consists in the following. Energy transfer from one wave to another on their nonlinear interaction can have the nature of instability (with increment Γ). It was found that 4Γ is the half-width of the nonlinear resonance on weak nonlinear wave interaction. If it exceeds the distance between neighbouring wave resonances (which is equal to 2δ , for details see [1-4]), then the dynamics of weak nonlinear wave interaction is chaotic: $K \equiv 2\Gamma/\delta > 1$. We have shown that the modified decay is always chaotic [3]. In addition we have found out that the numerical parameter $Ch = 4\Gamma/\Omega - 1 > 0$ (Γ is a solution of the dispersion equation describing the modified decay, Ω is the dimensionless frequency of plasma wave) rather correctly describes the boundary of transition to chaos. Bellow we have investigated the chaotic dynamics of HF electromagnetic wave decay (frequency $\omega_{N,M}$, longitudinal wave number k_M , amplitude $A_{N,M}$) in the magnetized plasma waveguide (radius of a metal case a) into HF electromagnetic waves ($\omega_{n,l}, k_l, A_{n,l}$) and into LF plasma waves ($\omega_{k,p}, \kappa_p, \Phi_{k,p}$). It is shown that the criterion of dynamic chaos occurrence $Ch = 4\Gamma/\delta - 1 > 0$ (where Γ is the maximum increment giving the corresponding dispersion equation, and δ is the minimum among frequencies and their differences) not only allows to find qualitatively the parameter region where the dynamics of weak nonlinear wave interaction is chaotic, but also rather correctly describes the boundary of transition to chaos.

2 THE BASIC EQUATIONS AND RESULTS

A reduced set of equations, which describe the decay process under investigation, has a form [4]:

$$i\frac{dA_{N,M}}{dt} = \frac{\omega_p^2}{4\omega_{N,M}} \sum_{k,n;p,l} \alpha_N^{Nkn} \frac{n_{k,p}}{n_{eo}} A_{n,l} \exp(i\Delta_{n,l}^{N,M} t),$$

$$i\frac{dA_{n,l}}{dt} = \frac{\omega_p^2}{4\omega_{n,l}} \sum_{k;p} \alpha_n^{Nkn} \frac{n_{k,p}^*}{n_{eo}} A_{N,M} \exp(-i\Delta_{n,l}^{N,M} t), \qquad (1)$$

$$\left(\frac{d^2}{dt^2} + \omega_{k,p}^2\right) n_{k,p} = \frac{-\omega_p^2}{8\pi m} \sum_{n;l} \frac{\alpha_k^{Nnk} \kappa_p^2}{\omega_{n,l}\omega_{N,M}} A_{N,M} A_{n,l}^* \exp(-i\Delta_{n,l}^{N,M} t)$$

where $\omega_{k,p}^2 = \omega_p^2 \kappa_p^2 / (\kappa_p^2 + \kappa_{\perp k}^2)$, $\quad \Delta_{n,l}^{N,M} = \omega_{N,M} - \omega_{n,l}$,

$$\alpha_j^{Nnk} = \int_0^a J_o(k_{\perp N} r) J_o(\kappa_{\perp k} r) J_o(k_{\perp n} r) r dr \Big/ \int_0^a J_o^2(k_{\perp j} r) r dr .$$

It is supposed in (1), that the condition of spatial synchronism is fulfilled: $k_M = k_l + \kappa_p$. If in (1) the condition of spatial synchronism is fulfilled only for one triplet of wave vectors, for example for waves belonging to the first transversal mode (*N=1*, *n=1*, *k=1*) then from (1), taking into account the LF excitation on the second transversal mode, we have:

$$i\frac{d\varepsilon_{1,M}^{(i)}}{d\tau} = \rho_{1,p}\varepsilon_{1,l}^{(s)} \exp(i\Delta_{1,l}^{1,M}\tau) + \frac{\alpha_1^{121}}{\alpha_1^{111}} \rho_{2,p}\varepsilon_{1,l}^{(s)} \exp(i\Delta_{1,l}^{1,M}\tau),$$

$$i\frac{d\varepsilon_{1,l}^{(s)}}{d\tau} = \rho_{1,p}^*\varepsilon_{1,M}^{(i)} \exp(-i\Delta_{1,l}^{1,M}\tau) + \frac{\alpha_1^{121}}{\alpha_1^{111}} \rho_{2,p}^*\varepsilon_{1,M}^{(i)} \exp(-i\Delta_{1,l}^{1,M}\tau) \qquad (2)$$

$$\frac{d^2\rho_{1,p}}{d\tau^2} + \Omega_{1,p}^2 \rho_{1,p} = -\varepsilon_{1,M}^{(i)}\varepsilon_{1,l}^{(s)*} \exp(-i\Delta_{1,l}^{1,M}\tau),$$

$$\frac{d^2\rho_{2,p}}{d\tau^2} + \Omega_{2,p}^2 \rho_{2,p} = -\frac{\alpha_2^{112}}{\alpha_1^{111}} \varepsilon_{1,M}^{(i)}\varepsilon_{1,l}^{(s)*} \exp(-i\Delta_{1,l}^{1,M}\tau),$$

where introduced are dimensionless variables:

$$\varepsilon_{1,L}^{(i)} = \frac{A_{1,L}(\tau)}{A_{1,L}(o)}, \quad \rho_{1,p} = \frac{n_{1,p}}{n_{eo}} \left[\frac{\omega_p^2}{\kappa_p^2}\alpha_1^{111} \sqrt{\frac{\omega_{1,l}}{\omega_{1,L}}} \frac{\pi m n_{eo}}{2A_{1,L}^2(0)}\right]^{\frac{1}{3}}$$

$$\varepsilon_{1,l}^{(s)} = \frac{A_{1,l}(\tau)}{A_{1,L}(0)} \sqrt{\frac{\omega_{1,l}}{\omega_{1,L}}}, \quad \rho_{2,p} = \frac{n_{2,p}}{n_{eo}} \left[\frac{\omega_p^2}{\kappa_p^2}\alpha_1^{111} \sqrt{\frac{\omega_{1,l}}{\omega_{1,L}}} \frac{\pi m n_{eo}}{2A_{1,L}^2(0)}\right]^{\frac{1}{3}},$$

$$\tau = t/t^*, \quad \Omega_{1,p} = (\kappa_p / \kappa_{1,p})\omega_p t^*, \quad \Omega_{21,p} = (\kappa_p / \kappa_{2,p})\omega_p t^*,$$

$$\Delta_{1,l}^{1,L} = \left(\omega_{1,L} - \omega_{1,l}\right) t^*, \quad t^* = \left[\kappa_p^2 \frac{\omega_p^4}{\omega_{1,L}\omega_{1,l}^2} \frac{A_{1,L}^2(0)}{32\pi m n_{eo}} \left(\alpha_1^{111}\right)^2\right]^{-\frac{1}{3}}.$$

At $\Omega_{1,p} > \Omega_{2,p} >> 1$ from (2) one can obtain such reduced equations:

$$i\frac{d\varepsilon_{1,M}^{(i)}}{d\tau} = \rho_{1,p}\varepsilon_{1,l}^{(s)} + \alpha\rho_{2,p}\varepsilon_{1,l}^{(s)} \exp(i\delta\tau),$$

$$i\frac{d\varepsilon_{1,l}^{(s)}}{d\tau} = \rho_{1,p}^*\varepsilon_{1,L}^{(i)} + \alpha\rho_{2,p}^*\varepsilon_{1,M}^{(i)} \exp(-i\delta\tau), \qquad (3)$$

$$i\frac{d\rho_{1,p}}{d\tau}=\frac{\varepsilon^{(i)}_{1,M}\varepsilon^{(s)*}_{1,l}}{2\Omega_{1,p}},\quad i\frac{d\rho_{2,p}}{d\tau}=-\beta\frac{\varepsilon^{(i)}_{1,M}\varepsilon^{(s)*}_{1,l}}{2(\Omega_{1,p}-\delta)}\exp(-i\delta\tau),$$

which describe the HF electromagnetic wave decay into the HF electromagnetic wave and the LF plasma wave in the magnetized plasma waveguide, which is excited by the LF wave (last equation in (3)). In (3) $\Delta^{1,M}_{1,l}=\Omega_{1,p}$, $\alpha=\alpha^{121}_{1}/\alpha^{111}_{1}$, $\beta=\alpha^{112}_{2}/\alpha^{111}_{1}$, $\delta=\Omega_{1,p}-\Omega_{2,p}$. At the linear stage, when $|\varepsilon^{(i)}_{1,M}|=1$, from (3) it is easy to obtain the dispersion equation:

$$\omega^{2}=-\frac{1}{2\Omega_{1,p}}-\frac{\omega}{\omega+\delta}\frac{\alpha\beta}{2(\Omega_{1,p}-\delta)}. \qquad (4)$$

The result of the numerical solution of dispersion equation (4) at $\delta\in]0;2.5]$, $\Omega_{1,p}\in[4;6.5]$, α=0.187, β=0.433 is represented in Fig. 1. The dependence of Γ on δ and $\Omega_{1,p}$ is shown with the help of the map of lines of an identical level. It is easy to see that the increment is maximum when *δ≈0*. It monotonically decreases with growth of δ and verges towards the value $\Gamma=1/\sqrt{2\Omega_{1,p}}$. In Fig. 2 the map of the parameter $Ch=4\Gamma/\delta-1$ is presented as a function of δ and $\Omega_{1,p}$. The set of equations (3) was numerically solved for different values of δ and $\Omega_{1,p}$. Fig. 3 shows the map of the maximum Lyapunov index, according to parameters of δ and $\Omega_{1,p}$, obtained by solving numerically Eq. (3). The numerical value of the maximum Lyapunov index is represented by tints of grey colour. Correspondence between the tints of grey and the numerical value of the maximum Lyapunov index can is seen in Fig. 4. Comparison of Fig. 2 and 3 shows that the criterion $Ch>0$ defines adequately well the parameter region where the dynamics of system (3) is chaotic. In this parameter region, where system (3) is close to the integrated one (region δ~0), despite that the criterion is fulfilled, the grade of system randomness is decreased. At δ=0 system (3) is integrated and its dynamics is regular. Such behaviour of nonintegrated systems is not unusual. So, on interaction of wave-particle type, in the phase space there are islands of stability near which the particle dynamics has a strong regular component, and neighboring trajectories diverge with time not exponentially fast, but proportionally to $t^{-\xi}$, where $\xi\sim 1$.

Let us turn to system (2). At the linear stage of decay it is easy to obtain the dispersion equation:

$$(\omega^{2}-\Omega^{2}_{1,p})(\omega+\Delta^{1,M}_{1,l})=1+\alpha\beta\frac{\omega^{2}-\Omega^{2}_{1,p}}{\omega^{2}-\Omega^{2}_{2,p}}. \qquad (5)$$

From (5) it is seen that if we neglect the LF excitation of decay on the second transversal mode, then we obtain the dispersion equation $(\omega^{2}-\Omega^{2}_{1,p})(\omega+\Delta^{1,M}_{1,l})=1$ which was investigated in [3]. Particularly, in [3] it is shown that the modified decay ($\Delta^{1,M}_{1,l}\sim 0,\Omega^{2}_{1,p}<<1$) always is stochastically instable. In this case in the criterion $Ch=4\Gamma/\delta-1>0$: Γ is the maximum increment which gives (5), and $\delta=\min(\Omega_{1,p},\Omega_{2,p},|\Omega_{1,p}-\Omega_{2,p}|)$. The result of the numerical solution of dispersion equation (5) at $\Delta^{1,M}_{1,l}\in[0;10]$, $\Omega_{1,p}\in[0;10]$, $\Omega_{2,p}=5.0$, α=1.0, β=1.0, is represented in Fig. 5. Dependence of Γ on $\Delta^{1,M}_{1,l}$ and $\Omega_{1,p}$ is shown with the help of the map of lines of an identical level. From Fig. 5 one can see that the instability takes place when the conditions of the synchronism $\Omega_{1,p}=\Delta^{1,M}_{1,l}$, $\Omega_{2,p}=\Delta^{1,M}_{1,l}$ are fulfilled. In the parameter region, where $\Omega_{1,p}\approx\Omega_{2,p}\approx\Delta^{1,M}_{1,l}>>1$, system (2) may be reduced.

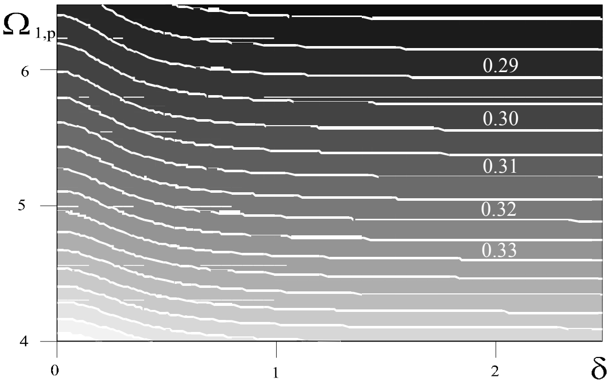

Fig. 1. Map of increment.

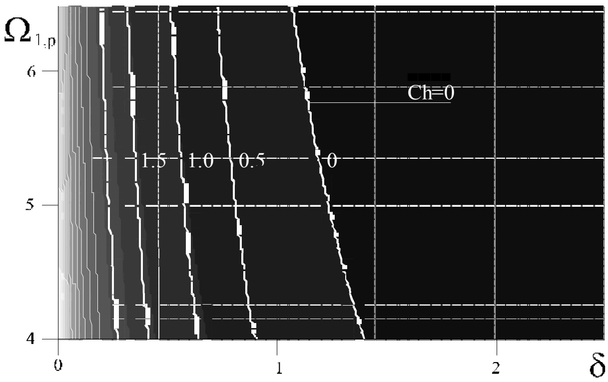

Fig. 2. Map $Ch=4\Gamma/\delta-1$.

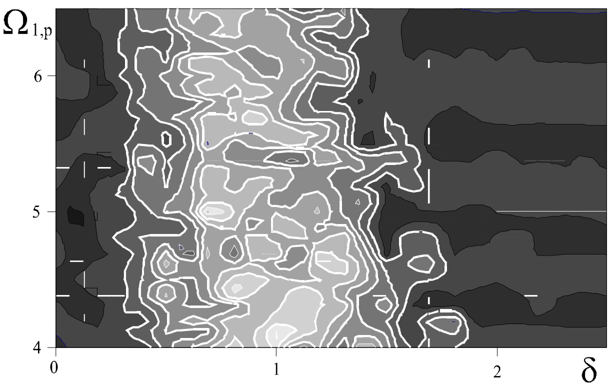

Fig. 3. Map of maximum Lyapunov index.

As a result we obtain system (3). This case was investigated earlier. Let us consider the case when the frequency of one or both LF waves can not be assumed as high, i.e. when the system gets into the region of modified decay. In Fig. 6, 7 shown are the maps of the maximum increment Γ and the stochasticity parameter $Ch > 0$ depending on $\Delta^{1,M}_{1,l} \in [0.2;5.0]$ and $\Omega_{1,p} \in [0.4;5.0]$ at $\Omega_{2,p} = 0.2$. From Fig. 7 it is seen that the parameters region, where the dynamics (2) is stochastic, is increased in comparison with the case of modified decay nonexcited by the additional LF wave (see also [3]). The map of the maximum Lyapunov index is shown in Figs. 8, 9. Comparing Fig. 7 and Fig. 8 one can see that in the parameter region where the criterion $Ch > 0$ is fulfilled the maximum Lyapunov index is more than zero, and the dynamics of the weak nonlinear wave interaction is chaotic.

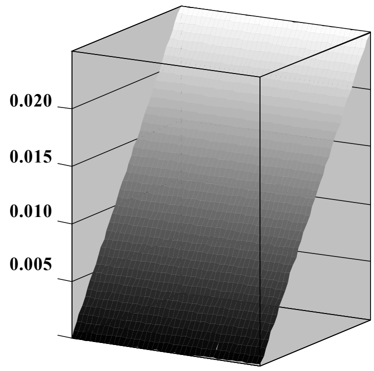

Fig. 4. Correspondence between the tints of grey and the numerical value of maximum Lyapunov index.

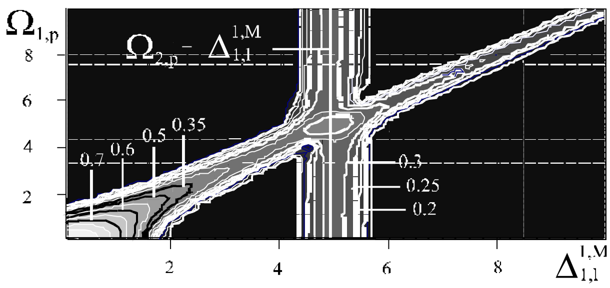

Fig. 5. Map of increment Γ. $\Omega_{2,p} = 5.0$.

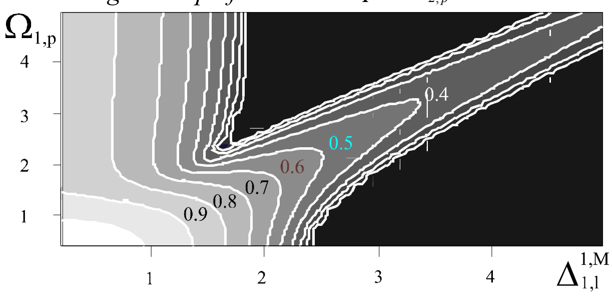

Fig. 6. Map of increment Γ. $\Omega_{2,p} = 0.2$.

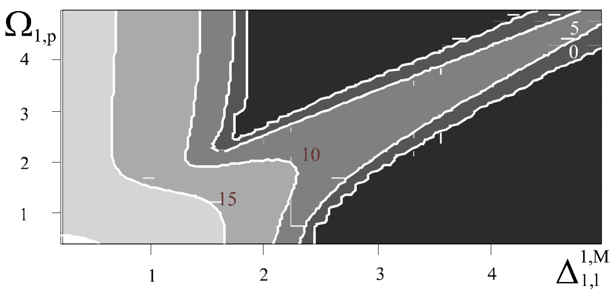

Fig. 7. Map of parameter $Ch > 0$. $\Omega_{2,p} = 0.2$

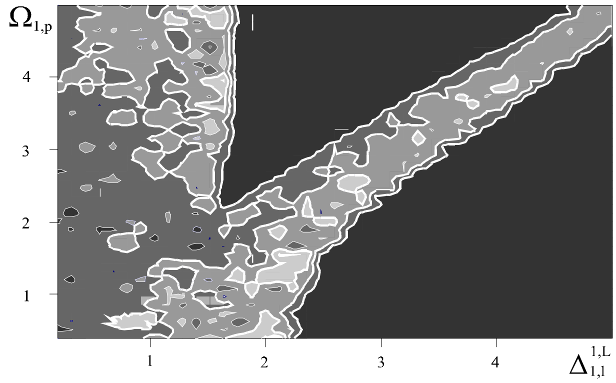

Fig. 8. Map of maximum Lyapunov index.

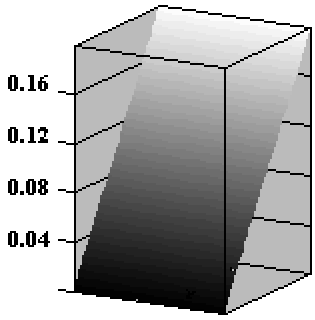

Fig. 9. Correspondence between the tints of grey and the numerical value of maximum Lyapunov index.

3 CONCLUSIONS

We have formulated the conditions under which the decay of the electromagnetic wave, propagating in the magnetized plasma waveguide, into the HF electromagnetic wave and LF plasma wave is stochastic. It is shown that the criterion of the dynamic chaos occurence $Ch = 4\Gamma/\delta - 1 > 0$, where Γ is the maximum increment giving the corresponding dispersion equation, and δ is the minimum quantity among frequencies and their differences, not only allows to find qualitatively the parameter region, where the dynamics of weak nonlinear wave interaction is chaotic, but also describes rather correctly the boundary of transition to chaos.

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