PECULIARITIES OF RIBBON ION BEAM DYNAMICS IN THE UNDULATOR LINEAR ACCELERATOR

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3D dynamics in the linear undulator accelerator is studied using analytical methods. The Hamilton form of motion equations is carried out by means of a smooth approximation method. The comparison of analytically and numerically simulated results is provided.

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1. INTRODUCTION

The main task for an accelerator design is to improve the RF focusing efficiency and put forward new methods of bunching and focusing the high-intensity ion beams in low-energy linear accelerators. High-intensity ion accelerators can be used for neutron generators, nuclear power engineering, thermonuclear synthesis and other applications. In a conventional RF linac the beam is accelerated by the synchronous wave. The beam focusing and bunching for low-energy accelerator is the main problem. Today RFQ is a base RF accelerator which allows to have the maximum ion beam current (about to 150...200 mA). The further beam current increasing is a difficult task. One of the ways to increase the ion beam intensity in ion accelerators is to enlarge the beam cross-section (for example, to use a ribbon ion beam). The ribbon ion beam focusing and acceleration can be realized in periodical resonator. The design of RF linac for ribbon ion beam was called Ribbon Radiofrequency Focusing (RRF) accelerator and beam dynamics simulation results in such a system were considered in Ref. [1]. Another method of ion beam acceleration in the fields without a synchronous wave was suggested in [2]. In this case the accelerating force is to be driven by a combination of two non-synchronous waves (two RF undulators). The investigation of beam dynamics in the undulator accelerator could be provided using the analytical methods and numerical simulation.

2. MOTION EQUATIONS

The UNDULAC-RF can be realized using an interdigital H-type resonator including two rows of alternating-sighed electrodes connected to the vanes. To create the RF field space distribution it is necessary to provide the transverse focusing along the ribbon width using the special-form electrodes [3].

Let us assume the beam to interact with only two space harmonics of RF field. The investigation of beam dynamics in UNDULAC-RF by means of traditional analytical methods is very complicated because the acceleration and focusing are realized without synchronous space harmonic of RF field. This research could be done using of the smooth approximation technique. The field potential in the periodic resonator can be represented as a sum of the space harmonics:

$$U = \sum_p U_p(x,y)\sin\left(\int h_p dz\right)\cos(\omega t) . \qquad (1)$$

Here U_p are the p-th harmonic amplitudes, p=0,1,2…; h_p are the longitudinal wave numbers: $h_p = (\mu + 2\pi p)/D$, μ is the phase advance per period of structure D. The harmonics phase velocities are equal to $v_{ph,n} = \omega / h_n$. The particle acceleration and focusing are realized in the field of combined wave of two non-synchronous RF field harmonics. It is possible if the beam velocity is close to the combined wave phase velocity $v_c = 2\omega/(h_p + h_n)$. The analytical study of beam dynamics is not possible using the traditional methods, but may be done by means of an averaging method. A momentum and a coordinate of particles could be represented as a sum of a slow varying and rapid oscillating components: $\boldsymbol{p} = \overline{\boldsymbol{p}} + \widetilde{\boldsymbol{p}}$, $\boldsymbol{r} = \overline{\boldsymbol{r}} + \widetilde{\boldsymbol{r}}$. Using the averaging technique over rapid oscillations, as it was done in [2], the time-averaged equation of motion for a non-relativistic ion could be obtained. The smooth approximation can be provided if the fast oscillating component of the particle velocity is lower than slow varying one. The equation has the Hamilton form:

$$\frac{d^2\boldsymbol{R}}{d\tau^2} = -\frac{\partial}{\partial \boldsymbol{R}} U_{\text{ef}} , \qquad (2)$$

where U_{ef} is the effective potential function. Such a function depends only on the particle phase in the combined wave field and slow varying transverse coordinates. The UNDULAC-RF may be realized with the use of $\mu = 0$, $\mu = \pi$ RF field modes. Let us consider the undulator linac system using only basic (p=0) and first (p=1) non-synchronous RF field harmonics. The function U_{ef} for this case is equal

$$U_{\text{ef}} = \frac{\nu}{4}\left(k_0\boldsymbol{e}_0^2 + k_1\boldsymbol{e}_1^2\right) + \boldsymbol{e}_0 \cdot \boldsymbol{e}_1 \cdot \nu \times$$
$$\times \left(2\text{ch}(2h_\nu y)\cos\left(2\varphi_{\text{v,s}} + 2\psi\right) + 2\psi \cdot \sin\left(2\varphi_{\text{v,s}}\right)\right). \qquad (3)$$

Here $\boldsymbol{R} = [\rho, \eta, \psi]$, $\rho = \frac{2\pi}{\lambda}x$, $\eta = \frac{2\pi}{\lambda}y$, $\xi = \frac{2\pi}{\lambda}z$ are dimensionless coordinates; *x, y, z* are the slowly varying coordinates, λ is the wave length of; $\psi = \varphi_s - \left(\int h_s d\xi - \tau\right)$ is the particle phase, φ_s is the combined wave synchronous phase; $\tau = \omega t$;

$e^{p}_{x,y,z} = \dfrac{e\lambda E^{p}_{x,y,z}}{2\pi mc^2}$ are the dimensionless field harmonic amplitudes, $\boldsymbol{e_0}$ and $\boldsymbol{e_1}$ are the functions of transverse coordinates (the beam acceleration and focusing in UNDULAC can be realized using the longitudinal or transverse RF field); h^p_x and h^p_y are the transverse wave numbers, $\left(h^p_x\right)^2 + \left(h^p_y\right)^2 = \left(h^p_z\right)^2$. Coefficients are equal to k_0=1, k_1 =5/9, $\nu = 1/2$ for $\mu = 0$ mode and k_0=10/9, k_1=26/25, $\nu = 1$ for $\mu = \pi$ mode.

The 6D phase volume of the beam could be derived using the effective potential function. The acceleration, the transverse focusing conditions and transverse and longitudinal motions coupling could be studied using the effective potential function.

3. PARTICLE PHASE MOTION

The longitudinal motion equation in the combined wave field can be found from (3) if the phase velocity β_s closes to the velocity of particles:

$$\frac{d\beta_s}{d\tau} = e_0 e_1 \nu \cdot \sin\left(2\varphi_{\nu,s}\right) \;. \quad (4)$$

The rate of the energy gain in UNDULAC-RF is proportional to $\sin\left(2\varphi_{\nu,s}\right)$ but not $\cos\left(\varphi_{\nu,s}\right)$ as in conventional accelerators. This peculiarity provides formation of two bunches on one RF field period. The rate of energy gain in UNDULAC-RF with $\mu = \pi$ RF field mode is two times more as for $\mu = 0$ mode.

At first UNDULAC-RF with $\mu = \pi$ RF field is considered. The longitudinal wave numbers and phase velocities for RF field harmonics equal to $h_0 = \pi / D,\ h_1 = 3\pi / D$, $\beta_{0,s} = 2\beta_s$, $\beta_{1,s} = 2\beta_s / 3$ for this mode and wave number for combined wave $h_s = 2\pi / D$. It can be shown that the ratio of RF field harmonic amplitudes $\chi = E_1 / E_0$ influences on the ion dynamics. The vertical size of separatrixes for basic (curve 1) and first (curve 2) RF field harmonics and combined wave (curve 3) versus longitudinal coordinate *z* are presented in Fig.1. The figure is plotted using the following parameters: initial velocity of deuterium ions β_{in} =0.013, amplitude of basic RF field harmonics E_0 =150 kV/cm, $\chi = 0.7$, λ =1.5 m [4]. For large χ values ($\chi \geq 0.4$) the separatrices for the combined wave and first RF field harmonic can be overlapped and the ions can be recaught by the first RF field harmonic. The smooth approximation technique is not well suited for this case. The longitudinal ion velocity (curve 4) was calculated in the polyharmonic field. The smooth approximation for this case is also shown in this figure (curve 5). The oscillations of longitudinal velocity can be larger than the vertical separatix size. Such oscillations are observed for small values of the ratio of RF field amplitude ($\chi = 0.1$-0.4 [4]) and momentary value of particle velocity or phase may wind up into the separatrix of the first RF field harmonic. The smooth approximation can not be used in this case also. Such two cases are appeared with different χ and optimal χ value can be found for large current transmission realization.

UNDULAC-RF using $\mu = 0$ RF field has none of these peculiarities. The longitudinal wave numbers and phase velocities for RF field harmonics are equal to $h_0 = 0,\ h_1 = 2\pi / D,\quad \beta_{0,s} = \infty\ ,\ \beta_{1,s} = \beta_s / 2$ this case and $h_s = \pi / D$. The separatrix overlapping for combined wave and first RF field harmonic takes place only when $\chi \geq 0.7$. The fast oscillations of the longitudinal velocity are very large and with small χ values ($\chi = 0.1$-0.5) and can be two or three times larger than the vertical size of a combined wave separatrix. However the value of the longitudinal ion velocity never takes place inside the separatrix of the first RF field harmonic. The particle phase motion for UNDULAC-RF using $\mu = 0$ mode is illustrated in Fig.2 (curve 1 is vertical separatrix size for first RF field harmonic, curve 2 is the same for combined wave, curve 3 is an ion longitudinal velocity versus *z*). The figure is plotted with W_{in}=150 keV, E_0=150 kV/cm, $\chi = 0.6$.

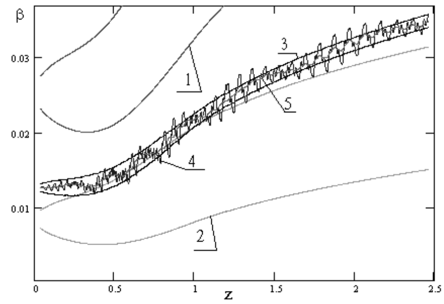

Fig.1. Separatrices for RF field harmonics and combined wave, longitudinal ion velocity versus longitudinal coordinate z ($\mu = \pi$ mode)

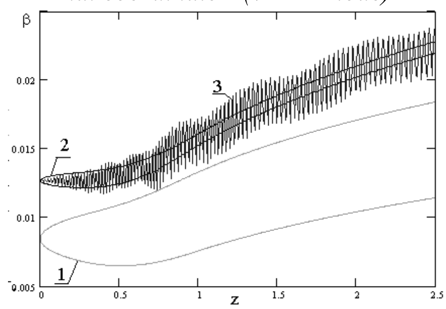

Fig.2. Separatrices for RF field harmonics and combined wave, longitudinal ion velocity versus longitudinal coordinate z ($\mu = 0$ mode)

4. TRANSVERSE BEAM MOTION

The existence of a minimum for U_{ef} in the beam frame is the necessary condition for the simultaneous transverse and longitudinal beam motion stability. The transverse beam motion in UNDULAC-RF can also be analyzed by means of the effective potential function. It was shown that the transverse focusing condition is performed independently on the χ value for UNDULAC-RF using $\mu = \pi$ mode field. It is the advantage of this type of undulator linac. The sections obtained by cutting of U_{ef} by the different planes are plotted in Fig.3, a) for this case (curve 1 is section versus particle phase ψ with transverse coor-

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dinates x=0 and y=0, curve 2 versus y with ψ =0, x=0, curve 3 versus x with ψ =0, y=0). The figure is plotted with W=150 keV, E_0=150 kV/cm, χ = 0.6, $\varphi_s = \pi/2$.

The amplitudes of RF field harmonics must be equal for effective transverse focusing in UNDULAC-RF using $\mu = 0$ mode field. The transverse focusing is provided by means of the first RF field harmonic only. However the transverse focusing can be obtained beyond axes with a lower value of ratio χ if the high beam quality is not necessary. The sections of U_{ef} for this type of the undulator linac are plotted in Fig.3, b) for example with χ = 0.6. The function U_{ef} has a local maximum in this case and the ion beam will be double-layer at the accelerator exit.

The frequencies of transverse and longitudinal oscillations may also be easily calculated using U_{ef} . It was discovered that the oscillations are stable in UNDULAC-RF with $\mu = \pi$ mode RF field and resonant effects for phase and transverse oscillations are not observed here. In UNDULAC-RF with $\mu = 0$ mode the oscillations can be non-stable.

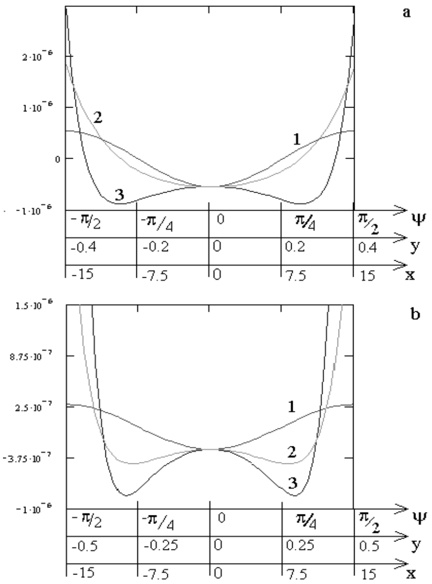

Fig.3. Effective potential function of UNDULAC-RF using the RF field with $\mu = \pi$ (a) and $\mu = 0$ (b) modes

The maximal accelerator channel aperture values and the transverse velocity can be calculated with the use of U_{ef} . These values define the transverse acceptance of accelerator channel. The longitudinal acceptance can be obtained as an area of separatrix for combined wave. The transverse acceptances for UNDULAC-RF with $\mu = \pi$ mode RF field equal to A_x=(60-80)π mm·mrad, A_y=(2…2.5)π mm·mrad, longitudinal acceptance A_φ = 20…25 keV·mrad. In UNDULAC-RF with $\mu = 0$ mode RF field these values are lower: A_x=60π mm·mrad, A_y=(1.5…2)π mm·mrad, A_φ=15…20 keV·mrad.

5. NUMERICAL SIMULATION OF BEAM DYNAMICS

The beam dynamics can not be studied completely using only the effective potential function. The function U_{ef} is obtained using a smooth approximation. The influence of particle phase and velocity oscillations can be studied thoroughly using the numerical simulation. A smooth approximation was verified by the numerical simulation of beam dynamics in both the full polyharmonic and averaged RF fields. Using the smooth approximation it was shown that the current transmission coefficient K_t for UNDULAC-RF is equal to 90…95% for the $\mu = \pi$ mode RF field and 85…90% for $\mu = 0$ mode. The investigation of beam dynamics in the polyharmonic field showed that K_t is lower for $\mu = \pi$ mode UNDULAC-RF and K_t=60…65%. The optimum value of χ ratio equals to 0.3-0.35 in this case. This result is consent with the analytical study. For $\mu = 0$ mode UNDULAC-RF K_t does not exceed 40%.

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ОСОБЕННОСТИ ДИНАМИКИ ЛЕНТОЧНОГО ИОННОГО ПУЧКА В ЛИНЕЙНОМ ОНДУЛЯТОРНОМ УСКОРИТЕЛЕ

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Было проведено аналитическое исследование трехмерной динамики в линейном ондуляторном ускорителе. С использованием метода усреднения по быстрым осцилляциям получено уравнение движения в форме Гамильтона. Проводится сравнение результатов аналитического исследования и численного моделирования динамики пучка.

ОСОБЛИВОСТІ ДИНАМІКИ СТРІЧКОВОГО ІОННОГО ПУЧКА У ЛІНІЙНОМУ ОНДУЛЯТОРНОМУ ПРИСКОРЮВАЧІ

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Було проведено аналітичне дослідження тривимірної динаміки в лінійному ондуляторному прискорювачі. З використанням методу усереднення по швидким осциляциям отримане рівняння руху у формі Гамільтона. Проводиться порівняння результатів аналітичного дослідження і чисельного моделювання динаміки пучка.