BRILLOUIN AND KYNETIC FLOWS IN A MAGNETRON DIODE

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Two classes of approaches have received the most attention to describe the important space charge dynamics in a magnetron are Brillouin flow and double-stream kinetic model. Precise analysis of electron dynamics in fully self-consistent kinetic equilibrium in a smooth-bore magnetron shows that for a given external magnetic field and a voltage there exists a multiplicity of natural equilibrium states, differing as to structure of electron trajectories and emission current density. The value of emission current density differs from one to other type of the equilibrium and can aspire to zero under the same condition of space charge limited flow due to a large number of revolutions of electrons around the cathode. The greater the number of revolutions the closer are the main parameters of the kinetic flow to Brillouin one. Work is supported by RFBR under grant 03-02-17301.

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1. INTRODUCTION

Despite the great number of works about magnetron operation, a detailed description of electron dynamics under strong space charge influence is complicated by the non-linear nature of a field-particles system. In particular, there is no satisfactory solution even to the problem of electron flow formation in a magnetron with a smooth anode (a coaxial magnetron diode). In the work presented here it is shown that within the framework of accepted kinetic descriptions of a coaxial magnetron diode, multiple steady states of electron flow are possible for a given diode geometry and the same set of external parameters (applied voltage and external magnetic field). These states are distinguished by the number of electron revolutions around the cathode and the current emitted from the cathode. Direct transition from kinetic flows to Brillouin flow as a limit is shown. Numerical simulation helps to investigate the dependence of steady state properties of electron flow on its history of formation. Comparison of analytical data and results of numerical simulation is made with the purpose to analyse the conditions of applicability for existing analytical models.

2. THEORETICAL MODELS

Usually it is supposed for a magnetron that at the initial stage a magnetically insulated axially symmetric rotating electronic flow is formed. As a rule, the description of the electron flow is based on two models: a hydrodynamic parapotential model, or a Brillouin flow [1,2], in which electrons rotate along the circular trajectories around the cathode [3,4], in which electrons move along cycloid trajectories, beginning and coming to the end on the cathode surface. It is necessary to emphasise that in the latter case it was always supposed that an electron makes a single revolution along a cycloid independent of the geometry of the diode (plane or cylindrical).

It is easy to show for a plane diode that at the top of the cycloid trajectory the radial velocity and electromagnetic force both are equal to zero. Thus it is possible to "connect" another descending (that usually is done), or ascending trajectory, then continuing them symmetrically up to the cathode, i.e. the top of trajectories in the plane diode is a point of solution branching.

But for a coaxial cylindrical diode it was shown [5,6] that artificial connecting of trajectories is impossible: the cylindrical metrics removes degeneration. And the structure of a kinetic flow differs in that the angular movement of electrons around an axis can significantly exceed 2π. The more the number of electron revolutions, the greater the time the electron stays in the diode gap, and there should be less emission current from the cathode surface. Thus, the steady state of an electron flow depends on the value of the emission current chosen (and an electric field on the cathode surface equal to zero corresponding to a space charge limited current; but the value of the emitted current is much less than the limiting current and, basically, can approach to zero).

3. BRILLOUIN (PARAPOTENTIAL) MODEL [1]

A system of units in which $c = e = m_e = 1$ is hereafter used. The electronic flow consists of circular trajectories with radii r filling completely or partially the gap between the cathode with radius r_k and anode with radius r_a: $r_k < r < r_e$, $r_e \le r_a$.

Outer (boundary) parameters - anode voltage ($\gamma_a - 1$) and (kept in a short pulse regime) average value of magnetic induction in the diode gap B_0 - are connected to parameters of an electronic flow by the relations

$$\ln\frac{r_e}{r_r} = \int_0^{A_e} \frac{dx}{\sqrt{(C^2 + x^2)(1 + x^2)}},$$

$$\gamma_a = \gamma_e + r_e E_e \ln\frac{r_a}{r_e},$$

$$r_a A_a = \frac{r_a^2 - r_k^2}{2} B_0 = r_e A_e + \frac{r_a^2 - r_k^2}{2} B_0$$

from which in that specific case $r_e = r_a$ (electronic flow occupies the whole anode-cathode space) is essentially only the last relation between an average magnetic induction and diode voltage:

$$(r_a^2 - r_k^2) B_0 = 2 r_a \sqrt{\gamma_a^2 - 1}.$$

4. KINETIC (EMISSIONING) MODEL [4]

This analytical model, supposing presence of an emission current from the cathode coming back on the cathode, is described by a system of equations for $\gamma(r)$ and $A(r)$:

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{d\gamma}{dr}\right)=\frac{f_0}{r}\frac{\gamma}{\sqrt{\gamma^2-1-A^2}},$$

$$\frac{d}{dr}\left(\frac{1}{r}\frac{d}{dr}rA\right)=\frac{f_0}{r}\frac{A}{\sqrt{\gamma^2-1-A^2}},$$

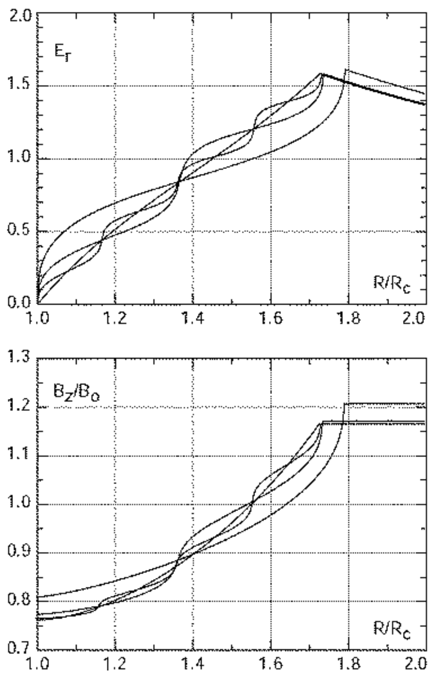
$$\frac{d\gamma}{dr}=E,\frac{1}{r}\frac{d}{dr}rA=B,$$

where the constant f_0 is proportional to density of cathode emission current. On the cathode $\gamma(r_k)=1$, $A(r_k)=0$, $E(r_k)=0$, $B(r_k)=B_k$, and the outer boundary of electron flow is at radius r_e, at which electrons turn back to the cathode and the radial pulse of electrons $p_r=\sqrt{\gamma^2-1-A^2}$ is equal to zero. In this model anode voltage and average value of magnetic induction are connected to parameters of an electronic flow by the relations:

$$\gamma_a=\gamma_e+r_eE_e\ln\frac{r_a}{r_e},$$

$$r_aA_a=\frac{r_a^2-r_k^2}{2}B_0=r_eA_e+\frac{r_a^2-r_k^2}{2}B_e.$$

The results of calculations on these models are shown in Fig. for the same external conditions (voltage and external magnetic flux) and with zero electric field on the cathode. The Brillouin smooth solution and three kinetic ones from a set of possible number of layers (n = 1,2,4) are shown. With increasing number of layers, emission current from the cathode tends to zero, electrons make more and more revolutions before coming back to the cathode, and a kinetic solution gradually approaches the Brillouin one. In the situation when magnetic field is much stronger than electric field, the Brillouin solution



Kinetic (n=1,2,4) and Brillouin models of a rotating beam (E(r) upper, B(r) lower) for the same external conditions (voltage and magnetic flux)

appears to be the only possible one. Thus the kinetic solution occurs only if the electric field on the cathode is not equal to zero. We note that in the inverted magnetron diode (with the anode on inner surface) only a single-layer kinetic solution exists, which points to the essential influence of the cylindricity on the electron flow within the diode. In the "plane" approximation for the kinetic model the criterion for restriction of number of layers by any value is not presented, and the usual choice of n=1 is in essence arbitrary.

Defining $p=\sqrt{\gamma^2-1-A^2}$ and introducing the new variable t the system can be written in the following form for numerical integration

$$\frac{d}{dt}r=p,\frac{d}{dt}\gamma=pE,\frac{d}{dt}(rE)=f_0\gamma,$$

$$\frac{d}{dt}(rA)=rpB,\frac{d}{dt}B=\frac{f_0}{r}A,\frac{d}{dt}p=\gamma E-AB+\frac{A^2}{r}$$

with initial conditions for t = 0:

$$r=r_k,\gamma=1,E=0,A=0,B=B_k,p=0.$$

Using the notations

$$x_0=r,x_1=\gamma-1,x_2=rE,x_3=rA,x_4=B,x_5=p$$

the final form of the system for numerical integration can be written as

$$x_0'=x_5,$$

$$x_1'=\frac{x_2x_5}{x_0},$$

$$x_2'=(1+x_1)f_0,$$

$$x_3'=x_0x_4x_5,$$

$$x_4'=f_0\frac{x_3}{x_0^2},$$

$$x_5'=\frac{x_1x_2+x_2+(x_3/x_0)^2-x_3x_4}{x_0}.$$

Initial step for numerical integration is calculated using the following expansion in series of a necessary high order obtained by means of MATHEMATICA 2.2:

$$x_0=r_k+\frac{f_0t^3}{6r_k}\left(1-\frac{B_k^2t^2}{20}-\frac{f_0t^3}{30r_k^2}\right),$$

$$x_1=\frac{f_0^2t^4}{8r_k^2}\left(1-\frac{B_k^2t^2}{18}\right),$$

$$x_2=f_0t\left(1+\frac{f_0^2t^4}{40r_k^2}\right),$$

$$x_3=\frac{f_0B_kt^3}{6}\left(1-\frac{B_k^2t^2}{20}+\frac{B_kt^3}{20r_k^2}\right),$$

$$x_4=B_k\left(1+\frac{f_0^2t^4}{24r_k^2}-\frac{f_0^2B_k^2t^6}{720r_k^2}\right),$$

$$x_5=\frac{f_0t^2}{2r_k}\left(1-\frac{B_k^2t^2}{12}\right).$$

5. TEMPORAL EFFECTS AND THEORETICAL MODELS

Discussion of effects observed in a magnetron diode had, as its basic purpose, to show that theoretical models describing one and the same situation, actually correspond to various physical conditions. We shall illustrate this by an example, comparing analytical results and numeric simulation realised with a PIC-code KARAT [7].

Modelling of particle emission in the KARAT code can be realised in two ways: (1) by setting the law for temporal change of the emitted current, the value of which can be less than, or more than the limiting current, the applied voltage being fixed (this situation corresponds to emission from photocathodes or external injection of a beam through the surface of the emitter); and (2) by setting the law for temporal rising of diode voltage to some constant value, the emission current being fixed at a value greatly exceeding the value limited by the space charge (this situation corresponds to the thermionic cathode).

Typical distribution functions of particles in these two cases are different. In the first case a single electron revolution in a cycloid is realised (symmetric two-peak distribution of electrons on a pulse p_r) without accumulation of charge in an accelerating gap; in the second case - symmetric distribution of particles on a pulse p_r with electron capture during voltage increase, growth of number of particles in the diode gap and multiturn dynamics of electrons.

6. DISCUSSION

Both theoretical models give about the same physical results. Recall that the equations describing Brillouin flow can be deduced on the basis of the same approach used for the kinetic model [8, 9]. Under conditions of conservation of full particle energy and canonical angular momentum

$$P_\theta = r(p_\theta + A) = const,$$

the next general set of equations can be derived:

$$(r\gamma^2 v_z^{\prime})^{\prime} + \gamma\left(\frac{v_\theta P_\theta^{\prime}}{v_z}\right)^{\prime} = 0,$$

$$(r\gamma^2 v_\theta^{\prime})^{\prime} - \frac{\gamma^2 v_\theta}{r} - r\gamma\left(\frac{P_\theta^{\prime}}{r}\right)^{\prime} = 0,$$

where the prime denotes d/dr.

For the case of a cathode surface coinciding with a magnetic flux surface we have $P_\theta = const$ and

$$r\gamma^2 v_z^{\prime} = const.$$

The constant equals to zero (i.e., $v_z = const$ in this case and we have the same equations as in [9]. Therefore, it is not surprising that a direct transition exists from the kinetic flow to a Brillouin one as a limit.

7. CONCLUSION

Precise analysis of electron dynamics in fully self-consistent kinetic equilibrium in the smooth-bore magnetron shows that for a given external magnetic field and a voltage there exists a multiplicity of natural equilibrium states, differing as to structure of electron trajectories and emission current density. The value of the emission current density differs from one to other type of the equilibrium and can aspire to zero under the same condition of the space charge limited flow due to the different number of revolutions of electrons around the cathode.

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БРИЛЛЮЭНОВСКИЙ И КИНЕТИЧЕСКИЙ ПОТОКИ В МАГНЕТРОННОЙ ПУШКЕ

А.В. Агафонов

В рамках аналитического подхода показано, что в коаксиальной магнетронной пушке возможно существование многозначных стационарных состояний пучка при заданных значениях внешних параметров (геометрия диода, напряжение на пушке и внешнее магнитное поле), отличающихся числом оборотов электронов вокруг катода и током, эмиттируемым с катода. Показана возможность предельного перехода от кинетического потока к потоку бриллюэновскому. Работа выполнена при поддержке гранта РФФИ 03-02-17301.

БРИЛЛЮЕНІВСЬКИЙ І КІНЕТИЧНИЙ ПОТОКИ У МАГНЕТРОННІЙ ГАРМАТІ

А.В. Агафонов

У рамках аналітичного підходу показано, що в коаксіальній магнетронній гарматі можливе існування багатозначних стаціонарних станів пучка при заданих значеннях зовнішніх параметрів (геометрія діода, напруга на пушку і зовнішнє магнітне поле), що відрізняються числом обертів електронів навколо катода і струмом, що емітується з катода. Показано можливість граничного переходу від кінетичного потоку до бриллюенівського потоку. Робота виконана за підтримкою гранта РФФД 03-02-17301.