DISPERSION AND RADIATING ABILITY OF RELATIVISTIC ELECTRONIC GAS

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In the paper the dispersion and conditions of formation of nonequilibrium radiation in the relativistic electronic gas are considered. For a case of a high-density electron bunch in the cw-approach and a wave-zone unharmonic oscillator the general kind of transfer plane-parallel front of nonequilibrium radiation equation is obtained.

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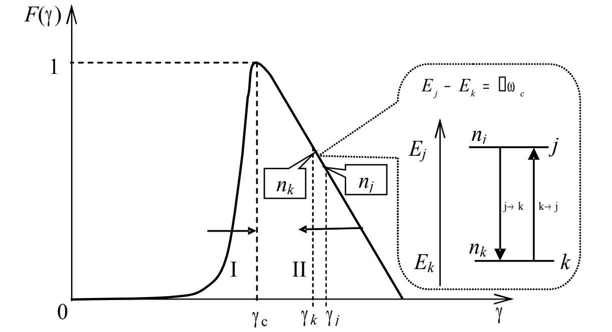
In FEL the question about superradiation of electron bunch in a working range of frequencies (the stimulated radiation of oscillators) is basic. Formation of a stimulated radiation field probably takes place only in the nonequilibrium system. Therefore it is necessary to define the mechanism of nonequilibrium radiation formation in the initially equilibrium system. As a rule, occurrence of stimulated (prime) radiation is considered, as a casual process. In paper [1] the mechanism of resonant amplification of slow-wave plasma fluctuations in the relativistic electronic bunch due to the interaction with the electromagnetic wave of the external source is theoretically investigated. It has been shown, that under conditions of exact resonance in the magnetized electronic bunch with a small density of volumetric charge there can be resonant amplification of slow plasma waves. In earlier work [2] the theoretical analysis of the response of excited oscillators with randomly distributed phases on casual electromagnetic perturbation has been carried out. As a result the conclusion has been made, that the electron bunch is unstable to the external electromagnetic perturbation when the essential nonlinearity of the equations of electron movement in a field of external wave takes place. Thus, field fluctuations that give a phase shift in the equation of electron movement can be sources prime radiations.

In [3] the general view of the transfer equation of wave package in boundless electronic gas in monovigour approximation is obtained. The equations received in the given paper, are fair for a case of a small density of the medium consisting from statistically independent diffusers. In work [4] the initial excitations of active medium, which are stochastic fluctuations of density charge in the bunch, forming the fields of nonequilibrium radiation are considered.

RADIATING INSTABILITY OF ELECTRONIC GAS

Let the bunch of relativistic electrons goes along an axis 0Z in periodic magnetic field *H (z)*. We shall set electron’s power distribution in a bunch of function *F* (γ), submitted on fig., where γ – the relativistic factor. Everyone electron is source of bremsstrahlung which intensity of its field is equal $E_{Bs\,i} = \left(2gDr_e^2\gamma_i H_0^2/3\right)^{1/2}$, where D is the period of magnetic field structure, g - average volumetric electron’s concentration in bunch, H_0-intensity of magnetic field on axis 0Z, $r_e = e^2/mc^2$ -classical radius of electron.



Function of power distribution in the electron bunch F (γ)

Let us consider the radiating instability in the bunch with a small electron density when $\omega_p << \omega_{Bs}$, where ω_p is the frequency of plasma fluctuations of electronic gas, ω_{Bs} is the frequency of bremsstrahlung.

In approach of the anharmonic oscillator the parameters of the system being considered are constant during T_0 (T_0 is the period of electron fluctuations in the magnetic field). We shall define the characteristic size of the area of radiating interaction.

If in the electron bunch the energy of the interchange by quanta $\hbar\omega_c$ takes place then from the principle of uncertainty ($\Delta l \Delta k = \pi$ where Δl is the size of wave package, Δk is the interval of wave numbers) it follows that $\Delta\omega'\Delta t' = \pi$, i.e.

$$\Delta\omega' = \frac{\pi c}{an(\omega_c)},$$

where a is the average distance between electrons, $n(\omega_c)$ is the factor of refraction of electronic gas at a frequency ω_c.

So as $a_{\min} = \lambda_c'^2/(2\Delta\lambda')$ it is necessary that $|\Delta\omega'| = \omega'_c\Delta\lambda'/\lambda'_c$. Hence, at $a < \lambda'/2$ it follows that the spectral range of power interchange channel is broadening up to $\lambda'_c \sim \Delta\lambda'$. Then we obtain the area of electron interaction by bremsstrahlung is the spherical layer with radius $r_0 = \lambda'/2$ and thickness $\delta = \Delta\lambda'$, i.e. shall take a note that secondary sources of radiation field and the centers of dispersion are concentrated in volume $V_s = 4\pi\left((r_0+\delta)^3 - r_0^3\right)/3$.

In system K' the electrons is not relativistic. Then we shall consider the radiation field in isotropic approach, $\Delta\omega << \omega_c$ and with fixed degree of polarization. Within the framework of the made assumptions, in spherical coordinates which center is connected to the leading center of i- electron, the change intensity of bremsstrahlung from radius is described by equation

$$dI(r) = -\left(\rho + \frac{2}{r}\right)I(r)dr,$$

where ρ is factor of proportionality which characterizes properties of environment and generally is function of *r*.

Let's define ρ for basic process in medium is scattering by free electrons of plainly-polarized wave own bremsstrahlung. In a general view, for one-photon processes ρ it's defined, how $\rho(r) = \sigma(r)\left(N_I(r) - N_{II}(r)\right)$ [6], where $\sigma(r)$ is the cross-section of dispersion plainly-polarized wave by free charges, $N_I(r)$ and $N_{II}(r)$ - the volumetric concentration of electrons in areas I and II (see fig. 1). In dipole approach ($\omega_{Bs} \approx \omega_c$)

$$\sigma = \frac{3}{2\pi}\lambda_c'^2 \sin\left(2\pi\frac{\Delta\gamma}{\gamma_c}\right), \qquad (1)$$

where λ_c' is the most probable for distribution $F(\gamma)$ wave-length of bremsstrahlung in system K'.

From equation (1) follows, that in monovigorous electron bunch the cross-section of resonant interaction of electrons with field own bremsstrahlung is smallest. Therefore formation of stimulated field radiations in electron's bunch is impossible for small density of bunch. For width of power distribution of electrons in a bunch $\Delta\gamma$ at which $\sin 2\pi\Delta\gamma/\gamma_c \sim 1$ spectral cleanliness of radiation $\lambda'/\Delta\lambda'$ does not allow to receive coherent (stimulated) radiation (coherence length $l_{coh} = \lambda'^2/\Delta\lambda' << a$). Absence of interactions between electrons in bunch with small density ($\omega_p << \omega_{Bs}$) and field of own bremsstrahlung for monovigorous bunch give the equality $n(\omega_c) = 1$.

Let us define the general requirements to dispersive properties of electronic gas at which process of formation of superradiation mode by excited oscillators in a bunch of electrons is most effective. We shall show that these properties can be received from task optimization function of parameters of magnetic field and electron bunch to intensity of radiation.

In approach of monovigorous bunch we shall consider, that parametrical interaction of radiations field of flat waves does not give collecting effect as the sum of phases of oscillator's fluctuations for the period T_o is not constant. Average time of synchronous fluctuations in ensemble of oscillators we shall define from condition of synchronism: $\omega_c\Delta t = \pi$. Then the considered ensemble of oscillators N_a is made in the volume, the limited spherical surface with radius equal to $r_a' = \lambda_c'/2$, and $N_a = gV_a = g\pi D^3/(6\gamma_c^3)$. Then, $\bar{E}_{Bs}^2 = E_{Bs}^2 N_a$.

In laboratory system of readout (the system K) for the lowest fashion of generation bremsstrahlung in unit volume of electron's bunch we have

$$I(\gamma_i) == \frac{2e^4 g}{3m^2c^3}\left(H_0^2\gamma_i^2 + \frac{\pi D^3 g}{6\gamma_i^3}E_{Bs}^2 \sin^2(\omega_i - \omega_c)t\right).$$

Function $I(\gamma_i)$ is not monotonous and has an extremum in point where takes place the equality

$$\frac{\omega_p^2}{\omega_i^2} = \frac{\sqrt{6}}{\sqrt{\pi}\,\sin(\omega_i - \omega_c)t}.$$

Whence it is easy to obtain factor of refraction of environment for radiation field $H(z)$ by frequency ω_i which extend along an axis 0Z

$$n_z(\omega_i) = \sqrt{1 + \frac{\sqrt{6}}{\sqrt{\pi}\,\sin(\omega_i - \omega_c)t}}. \qquad (2)$$

From (4) it follows in the direction of an axis 0Z environment is not transparent for the most probable for distribution $F(\gamma)$ of frequency ω_c bremsstrahlung. In dense electronic gas, at $\omega_p > \omega_i$, the field E_{Bs} is weakened as a result of resonant interaction with plasma fluctuations of environment (polarizing losses) and on distance $\sim\lambda_c'$ falls down up to zero [7]. Absorbing energy of the field E_{Bs}, electrons reemit it with displacement of a phase on $\pi/2$. Thus such process provide creation of nonequilibrium radiations field E_{pr}. As other channels of loss energy field E_{Bs} are absent, in monovigorous electron's bunch is possible to accept $E_{pr}(\omega_c) \approx E_{Bs}$, and the direction and degree of polarization of fields E_{Bs} and E_{pr} are identical. In the equation (2) the volume $\sin(\omega_i - \omega_c)t$ is the characteristic of linac. It has been earlier shown, that in high-dense electron's bunch occurs of broadening spectral range radiating of power interaction of oscillators. Therefore, at $N_a >> 1$, in time T_o the volume $\sin(\omega_i - \omega_c)T_o$ is not determined also the minimal value of parameter of refraction equally $n_{\min} \approx 1.5$. Then condition of existence of a field E_{pr} and transition of ensemble of oscillators in a mode of superradiation is $g = \sqrt{6}\gamma_c^2/\sqrt{\pi}D^2 r_e$

Let's consider the dynamics of separate electron in system K' in case of the high-density electron's bunch, when $\omega_p \sim \omega_c$. Then the cross-section of dispersion of fields $E_{pr}(\omega_c)$ by electrons of bunch is

$$\sigma = 3\lambda_c'^2\cos(\omega_i - \omega_c)T/(2\pi).$$

At $\Delta\gamma << \gamma_c$, we have $\rho = 3\lambda_c'^2(N_I - N_{II})/(2\pi)$.

For a case of the charge bunch high density having place in the linac such as LU-50 [8] with volumetric electron concentration in a bunch of about 10^{11} sm^{-3}, it is necessary to take into account the charge interactions.

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Achievement of condition $\omega_p \sim \omega_c$ can be connected both to a longitudinal grouping of electrons, and with cross-section focusing a bunch. However phase drift of electrons in a field of falling wave, resulting to longitudinal grouping, is energetically more favourable, than cross-section focusing of a bunch. Therefore we shall take into account only effect of a longitudinal grouping, and the cross-section size of bunch we shall count constant.

For one-dimensional dependence of parameter $\rho(r)$ from coordinate in electronic gas the change of this parameter is function of difference concentration of electrons in areas I and II (see fig.1) $A(r) = N_I(r) - N_{II}(r)$, which will defined by efficiency of process of phase drift of electrons in field of E_{pr}. Then

$$dA(r) = -\frac{\upsilon_{dr}(\omega_c) n_r(\omega_c)}{c\Delta\lambda'} A(r) dr,$$

where $\upsilon_{dr}(\omega_i) = \frac{\lambda'_i n_r(\omega_i)}{c} \frac{e}{m_e} E_{pr}$ is average speed of phase drift of electron for T_o.

In the cw-approach, at $\upsilon_{dr} n_r(\omega_c)/c << 1$, dependence $\rho(r)$ has the following form

$$\rho(r) = \frac{3}{2\pi} \lambda'^2_c A_0 \left(1 - \frac{\upsilon_{dr}(\omega_c)\gamma_c n_r(\omega_c)}{c\lambda'_c \Delta\gamma} r\right).$$

In system K from the differential equation of transfer of plane-parallel wave front of nonequilibrium radiation we obtain

$$dI_{pr}(z) = -\frac{3}{2\pi} \frac{D^2}{\gamma_c^2} A_0 \left(1 - n_r(\omega_c) \frac{\upsilon_{dr}\gamma_c}{cD\Delta\gamma} z\right) I_{pr}(z) dz$$

(transformation $\rho(r)$ to system K is carried out by replacement $r = z/\gamma_c$ and $\lambda'_c = D/\gamma_c$). Thus we receive dependence of intensity of nonequilibrium radiation from dimensionless coordinate $\xi = z/D$

$$I_{pr}(\xi) = I_0 \exp\left[-\frac{3A_0 D^3}{2\pi\gamma_c}\left(\xi - n_r(\omega_c)\frac{\upsilon_{dr}\gamma_c}{2c\Delta\gamma}\xi^2\right)\right],$$

where $I_0 = \pi D^3 g E_{Bs}^2 / (6\gamma_c^3)$ is the initial intensity of nonequilibrium radiation.

From general kind of equations for transfer of plane-parallel wave front of nonequilibrium radiation follows action of nonlinear term in exhibitor's parameter is directed to condition of balance, for which $I_{pr}(\xi) = I_0$.

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ДИСПЕРСИЯ И ИЗЛУЧАТЕЛЬНАЯ СПОСОБНОСТЬ РЕЛЯТИВИСТСКОГО ЭЛЕКТРОННОГО ГАЗА

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Рассмотрены дисперсия и условия формирования неравновесного излучения в релятивистском электронном газе. Для случая большой плотности электронного сгустка в квазистационарном приближении в волновой зоне ангармонического осциллятора получен общий вид уравнения переноса плоскопараллельного фронта неравновесного излучения

ДИСПЕРСІЯ І ВИПРОМІНЮВАЛЬНА ЗДАТНІСТЬ РЕЛЯТИВІСТСЬКОГО ЕЛЕКТРОННОГО ГАЗА

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Розглянуті дисперсія й умови формування нерівновагого випромінювання в релятивістському електронному газі. Для випадку великої густини електронного згустку в квазістаціонарному наближенні в хвильовій зоні ангармонічного осцилятора отримано загальний вид рівняння переносу плоскопараллельного фронту нерівновагого випромінювання.