SIMULATION OF WAKEFIELDS EXCITED BY A TRAIN OF ELECTRON BUNCHES IN RECTANGULAR DIELECTRIC WAVEGUIDE

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Wake field excitation by a sequence of electron bunches in rectangular dielectric waveguide of finite length is investigated for acceleration with high gradient electric field. Characteristics of wake field for parameters of planned in NSC KIPT experiments are determined.

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1. INTRODUCTION

Recently several papers appeared, which are devoted to acceleration of electrons by wakefield in dielectric filled waveguide [1-3]. Increased interest to wakefields in dielectric is connected with the fact that short charged bunches excite simultaneously a great many of radial harmonics of the dielectric waveguide that leads to compression of wakefields in longitudinal direction and to formation of narrow peaks of field on the axis of system with intensity much greater, than amplitude of one radial harmonic [4].

In papers [4,5] regular sequences of bunches with rather small charge, fields from which would sum coherently, were offered to use for achieving intensive accelerating fields. Experimentally it easier to create such sequence, than a single bunch with big charge. Nevertheless, this method faces with the following difficulties.

Firstly, for providing of effective coherent summing of multimode fields it is necessary to provide equidistance of the next resonance frequencies of the waveguide [4]. However in the cylindrical waveguide with partial dielectric filling the requirement of equidistance of excited frequencies is fulfilled only approximately.

Secondly, the pattern of field excitation in the dielectric waveguide with finite length qualitatively differs from the idealized model of infinite waveguide. Good approximation for describing of waveguide of finite length without reflections on the output is the semi-infinite waveguide. The solution of wake problem in semi-infinite waveguide has shown, that when driving bunches excite only traveling forward wave, "removal" of excited oscillations after bunches with group velocity [6] occurs. Therefore in waveguide without reflections at excitation of wakefield from a sequence of big number of bunches only a part of this sequence will be effective.

With the purpose to bypass the mentioned difficulties in Brookhaven [1] and NSC KIPT experiments on excitation of wakefields in the rectangular dielectric waveguide are planed. For theoretical substantiation of planed experiments we carried out the research of effects of longitudinal finiteness on excitation of wakefields.

2. WAKEFIELD IN THE SEMIINFINITE RECTANGULAR DIELECTRIC WAVEGUIDE

Let's consider a rectangular metal waveguide with width b ($0 \le x \le b$) and height d ($0 \le y \le d$). The waveguide is filled by homogeneous dielectric with permittivity ε. In longitudinal direction the waveguide occupies area $0 \le z < \infty$. From the end $z=0$ it is short-circuited by a metal wall. We suppose, that monoenergetic point electron bunch moves with constant velocity v_0 along the axis of waveguide to the end face of waveguide. Distribution of charge density and current density of such bunch is:

$$\rho = Q_b \delta\left(x-x_0\right)\delta\left(y-y_0\right)\delta\left(t-t_0-z/v_0\right)/v_L,\ j_z = v_0 \rho ,$$

where Q_b is bunch charge, t_0 is time of bunch arrival to the waveguide, x_0, y_0 are transverse coordinates of bunch.

Having solved a wave equation with boundary conditions on metal walls of a waveguide we shall obtain expression for a longitudinal electric field as the sum of Cherenkov radiation E_z^{cher} and the transition radiation E_z^{trans} [7]:

$$E_z\left(t,x,y,z,t_0,x_0,y_0\right) =$$
$$E_z^{cher}\left(t,x,y,z,t_0,x_0,y_0\right) + E_z^{trans}\left(t,x,y,z,t_0,x_0,y_0\right),$$

$$E_z^{cher}\left(t,x,y,z,t_0,x_0,y_0\right) =$$
$$-\frac{16\pi Q_b}{bd\varepsilon}\sum_{k,l}\sin\left(\frac{\pi k}{b}x_0\right)\sin\left(\frac{\pi l}{d}y_0\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\times$$
$$\cos[\omega_{kl}(t-t_0-z/v_0)]\left[\Theta(t-t_0-z/v_0)-\Theta(t-t_0-z/v_g)\right]$$

$$E_z^{trans}\left(t,x,y,z,t_0,x_0,y_0\right) =$$
$$-\frac{16\pi Q_b}{bd\varepsilon}\sum_{k,l}\sin\left(\frac{\pi k}{b}x_0\right)\sin\left(\frac{\pi l}{d}y_0\right)\sin\left(\frac{\pi k}{b}x\right)\sin\left(\frac{\pi l}{d}y\right)\times$$
$$\left\{\left[\theta\left(t-t_0-z/v_{ph}\right)-\theta\left(t-t_0-z/v_{gr}\right)\right]\times\right.$$
$$\sum_{m=1}^{\infty}\left(-1\right)^m\left(r_1^{2m}-r_2^{2m}\right)J_{2m}\left(y\right)+\theta\left(t-t_0-z/v_{gr}\right)\times$$
$$\left.\left[J_0\left(y\right)+\sum_{m=1}^{\infty}\left(-1\right)^m\left(r_1^{2m}+r_2^{-2m}\right)J_{2m}\left(y\right)\right]\right\},$$

where $\omega_{kl}^2 = [(\pi k/b)^2 + (\pi l/b)^2]/(\varepsilon/c^2 - 1/v_0^2)$, J_0 - cylindrical functions, $\theta(x)$ - Heaviside function and

$$r_1 = \sqrt{\frac{t-t_0-z/v_{ph}}{t-t_0+z/v_{ph}}}\times\sqrt{\frac{1-v_{ph}/v_0}{1+v_{ph}/v_0}},\ v_{ph} = c/\sqrt{\varepsilon},\ ,$$

$$r_2 = \sqrt{\frac{t-t_0-z/v_{ph}}{t-t_0+z/v_{ph}}}\times\sqrt{\frac{1+v_{ph}/v_0}{1-v_{ph}/v_0}},\ v_{gr} = c^2/v_0\varepsilon .$$

Vavilov - Cherenkov wakefield with the account of "damping wave» is nonzero at $\left(t-t_0\right)v_{gr} \le z < \left(t-t_0\right)v_0$. Within this area the envelope of cherenkov signal is constant. Value v_{gr} is group velocity of synchronous with bunch electromagnetic wave. The plane $z^{gr} = \left(t-t_0\right)v_{gr}$ is back front of wakefield. This front moves after a bunch with group velocity.

The field of transitional radiation exists in area $0 \le z < (t - t_0)\, v_{ph}$. Value v_{ph} is the greatest velocity of electromagnetic signal propagation in the dielectric waveguide, with this velocity the fastest high-frequency part of transitional signal – so-called "precursor" is propagated.

For a bunch of finite sizes or for a sequence of bunches the expression for wakefield is fulfilled by integration on transverse coordinates and on moments of entrance of elementary charges.

In Fig.1-Fig.2 results of calculations for the following parameters are presented: $b = 4.3 cm$; $d = 8.6 cm$; the charge of a single bunch $Q_b = -0.32 nC$; energy– $4\ \mathrm{MeV}$; transverse sizes of a bunch- $b_0 = 1.0 cm$ $d_0 = 1.0 cm$; $\varepsilon = 2.83$.

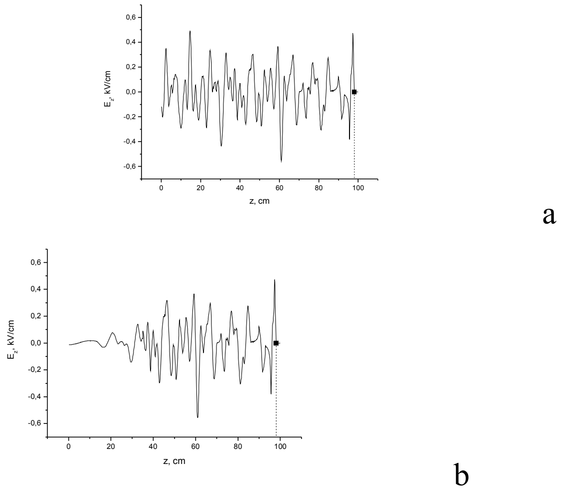

Fig. 1. Wakefield excited by a single bunch in the semi-infinite dielectric waveguide at t=3.29 ns:
(a) - cherenkov field, (b)- full field. The label and dash line show bunch location

The single bunch excites a plenty of transverse harmonics (we consider 50 harmonics on x and 50 harmonics on y). As frequencies of excited harmonics are not divisible ratio with the lowest eigen frequency the longitudinal structure of the field has the irregular character even without taking into account transitional radiation. Presence of transition radiation especially complicates the field pattern as it contains the whole continuous spectrum of frequencies, beginning from a cut-off frequency. The field excited by a bunch flies out from a system with a group velocity and near to input of system the amplitude of the field is close to zero.

When in the semi-infinite waveguide the sequence of bunches is injected with repetition rate $f = 2886 MHz$, which is equal to lowest eigen frequency of structure, the longitudinal structure of the field qualitatively changes. It becomes regular with the narrow peaks following with period of bunch train. I.e. the sequence of bunches "cuts out" from a spectrum excited by a single bunch only frequencies multiple to of bunch repetition rate. And, as follows from comparison of curves on Fig. 2, the transition field destroys insignificantly regular structure of the field. "Removal" of excited oscillations with group velocity reduces in restriction of maximum quantity of bunches which give the contribution to growth of amplitude of the field [6,7]. On length of the system $L = 100\ cm$ this bunch quantity is 17. Additional injection of bunches will not reduce in increase of amplitude of the field at given length of structure.

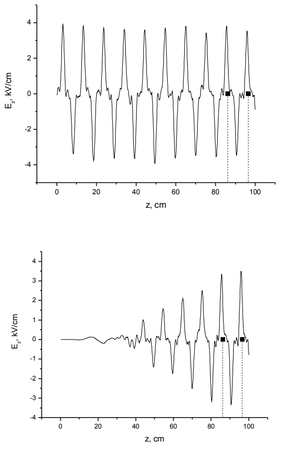

Fig. 2. Wakefield excited by a sequence of 17 bunches in the semi-infinite dielectric waveguide at t=8.43 ns:
(a)- cherenkov field, (b)- full field. Labels and dash lines show locations of the last 2 bunches of a sequence

3. WAKEFIELD IN RECTANGULAR DIELECTRIC RESONATOR

Let's suppose, that output end of waveguide $z = L$ as well as input end is closed by the metal gride, transparent for particles. Then the statement of problem of the previous section passes in determination of the longitudinal electric field excited by a point electron bunch in the rectangular dielectric resonator. We will suppose, that the height of the resonator considerably exceeds its width, therefore we will neglect dependence of excited fields on coordinate y. Having solved wave equation with boundary conditions analogous to the previous section and a source we obtain expression for longitudinal electric field

$$E_z = E_0 \sum_{m=1}^{\infty} \sum_{l=0}^{\infty} \delta_l \omega_{10} \frac{\cos(k_l z)}{\omega_{ml}^2 - \omega_l^2} \left\{ \left[\frac{\omega_{ml}^2 - k_l^2 c^2/\varepsilon}{\omega_{ml}} \sin[\omega_{ml}(t - t_0)] - \right.\right.$$
$$\left. \frac{\omega_l^2 - k_l^2 c^2/\varepsilon}{\omega_l} \sin[\omega_l (t - t_0)] \right] \theta(t - t_0) -$$
$$(-1)^l \left[\frac{\omega_{ml}^2 - k_l^2 c^2/\varepsilon}{\omega_{ml}} \sin[\omega_{ml}(t - t_0 - L/v_0)] - \right.$$
$$\left.\left. \frac{\omega_l^2 - k_l^2 c^2/\varepsilon}{\omega_l} \sin[\omega_l (t - t_0 - L/v_0)] \right] \theta(t - t_0 - L/v_0) \right\} G_m(x, x_0),$$

where $E_0 = -8\pi Q_b v_0 / aL\varepsilon\omega_{10}$, $\omega_{ml}^2 = [\kappa_m^2 + k_l^2] c^2/\varepsilon$, $\omega_l = k_l v_0$, $\kappa_m = \pi m/b$, $k_l = \pi l/L$; function δ_l is equal 1 if $l = 0$ it is equal 2 if $l \ne 0$; $G_m(x, x_0) = \sin(\kappa_m x)\sin(\kappa_m x_0)$.

As seen from full field will consist of the field of space charge (corresponding to frequencies ω_l) and the fields, excited by a bunch in the resonator on frequencies ω_{ml}. After exit of particles from the resonator the field of space charge as follows from , disappears. It should be noted, that at the condition $\omega_{ml} = \omega_l$ the corresponding items in the sum become dominant. The indicated condition is exactly cherenkov radiation in delay medium. Then these resonant items may be treated as cherenkov radiation in the dielectric resonator, and the rest of the field as transition radiation on both boundaries. Let's note, that radiation of a charged particle in the vacuum rectangular resonator is first considered in [8], and in the cylindrical vacuum resonator in [9]. In these cases the condition of cherenkov radiation is not fulfilled. The resonant case is of interest for our researches.

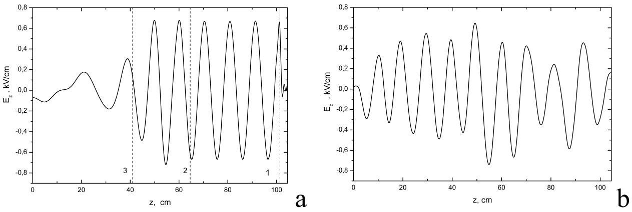

Fig. 3. Wakefield excited by a single bunch in the flat dielectric resonator: (a) - at t=3.415ns, (b) – at t=99ns

Let's choose for the calculations parameters of experiment for the setup in NSC KIPT: charge of a single bunch $Q_b = -0.32nC$, energy– 4 MeV, $b = 4.3cm$, bunch repetition rate 2850 MHz, $\varepsilon = 2.509$, $L = 104.51cm$. For such sizes the resonant condition is fulfilled for numbers of longitudinal and transverse harmonics at ratio $l = 5m$.

On fig.3-fig.4 outcomes of calculations are presented allowing in sums for 151 longitudinal harmonics and 1 transverse harmonic.

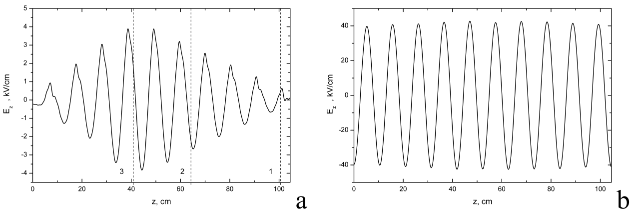

Fig. 4. Wakefield excited by a sequence of 100 bunches in the plane dielectric resonator: (a) – at t=3.415 ns, (b) - at t=99.942ns

In Fig.3 the longitudinal structure of the field excited by a single bunch is presented. Before bunch exit from the resonator (t=3.415 ns) structure of the field corresponds to structure of the field in semi-infinite waveguide, the amplitude of the field decreases from the position of group front to the enter of the resonator (line 1 shows the location of a bunch, 2 – phase front, 3 – group wave front). At long times, after a multiple reflection of group wave front from the both end-walls of the resonator, levelling of amplitude of the field along its length occurs (see the graph for t=99 ns).

The longitudinal structure of the field created by a sequence of 100 bunches in the dielectric resonator is presented on Fig. 4. The upper figure corresponds to the moment of time when the first bunch of sequence is near to the output of the resonator (line 1 - the location of the first bunch, 2 – phase wave front from the first bunch, 3 – group wavefront from the first bunch). The amplitude of the field grows from a head of sequence to the position of the group wave front, excited by the first bunch, and then decreases to the enter of the resonator. Wakefield in the resonator qualitatively and quantitatively coincides with the field in semi-infinite waveguide up to exit of the first bunch from the waveguide (see Fig. 2). At major times, after exit of all bunches from the resonator, the homogeneous distribution of amplitude is established in the waveguide.

Comparing Fig. 2 and Fig. 4 it follows, that in the resonator it is possible to excite wakefield with the amplitude considerably exceeding amplitude of the field in the semi-infinite waveguide. At that the regularity of oscillations is conserved.

4. ACKNOWLEDGEMENTS

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МОДЕЛИРОВАНИЕ ВОЗБУЖДЕНИЯ КИЛЬВАТЕРНОГО ПОЛЯ ПОСЛЕДОВАТЕЛЬНОСТЬЮ ЭЛЕКТРОННЫХ СГУСТКОВ В ПРЯМОУГОЛЬНОМ ДИЭЛЕКТРИЧЕСКОМ ВОЛНОВОДЕ

Н.И. Онищенко, Г.В. Сотников

Исследовано возбуждение кильватерного поля электронными сгустками и их последовательностью в прямоугольных диэлектрических волноводах конечной длины: полу бесконечном волноводе и резонаторе. Определены характеристики кильватерного поля для параметров планируемого в ННЦ ХФТИ эксперимента.

МОДЕЛЮВАННЯ ЗБУДЖЕННЯ КИЛЬВАТЕРНОГО ПОЛЯ ПОСЛІДОВНІСТЮ ЕЛЕКТРОННИХ ЗГУСТКІВ В ПРЯМОКУТНОМУ ДІЕЛЕКТРИЧНОМУ ХВИЛЕВОДІ

М.І. Онищенко, Г.В. Сотников

Досліджені процеси збудження кільватерного поля електронними згустками та їх послідовністю в прямокутних діелектричних хвилеводах скінченої довжини: напів обмеженого хвилеводу та резонаторі. Визначені характеристики кільватерного поля для параметрів запланованого в ННЦ ХФТІ експерименту.