ELECTRIC BARRIER FORMATION IN KIND OF CONNECTED DIP AND HUMP OF ELECTRIC POTENTIAL NEAR ECR POINT

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The self-consistent formation, observed in experiments, of the solitary barrier for plasma electrons and ions has been analytically described.
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THERMAL BARRIER FORMATION FOR PLASMA ELECTRONS IN ECR POINT

In [1] the formation of the thermal barrier for plasma particles was observed near the point of electron cyclotron resonance (ECR) in inhomogeneous magnetic field. In this paper two mechanisms of similar thermal electric barriers formation for electrons and ions of the plasma and plasma flow in ECR points on ends of the magnetized cylindrical trap are described (see fig. 1).

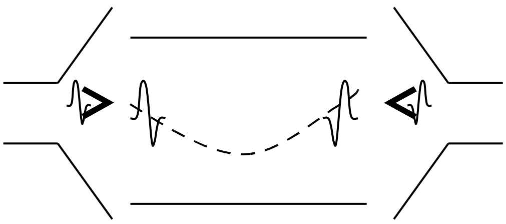

Fig. 1. Scheme of the thermal electrical barriers formation for plasma electrons and ions in ECR points on ends of the magnetized cylindrical trap

We consider the case of inhomogeneous magnetic field. Namely the magnetic field is minimum in the center of the cylindrical trap and it grows to the ends of the cylindrical trap. Near ECR point the transversal electron velocity $V_{\perp o}$ is increased. At electron motion from the system they are reflected from the magnetic barrier back. Further the electrons move inside the system. In the inhomogeneous magnetic field the electron transversal velocity $V_\perp(z)$ decreases $V_\perp(z)=V_{\perp o}(H(z)/H_o)^{1/2}$, but the electron longitudinal velocity $V_\|(z)$ increases $V_\|(z)=V_{\perp o}(1-H(z)/H_o)^{1/2}$. It results in average electron velocity. Growth of electron velocity leads to that electrons in the area, in which they are penetrated, form noncompensated negative charge δn_e. According to Poisson equation it is the perturbation of the dip of the electric potential, from which the plasma electrons are reflected. Growth of the electron longitudinal velocity with respect to the ions near ECR point provides nonequilibrium state. The reflection of the electrons with nonequilibrium distribution function from the dip of the electric potential leads to growth of the dip's amplitude. So these current-carrying electrons excite the electric potential dip with amplitude φ_o on an ion mode with velocity V_c, close to zero, and are reflected from it. From Vlasov equation for electrons and hydrodynamic equations for ions one can derive evolution equation, describing this dip. Really we use slow evolution of the dip for its description. In zero approximation, taking into account that the resonant electrons are reflected from the dip, one can derive from Vlasov equation the expression for electron distribution function

$$f_e=f_{oe}[-(V^2-2e(\varphi\pm\Delta\varphi)/m_e)^{1/2}\pm V_\|],\ V\gtrless A(\varphi)\mathrm{sign}(z) \quad (1)$$

$A(\varphi)=[2e(\varphi_o+\varphi)/m_e]^{1/2}$.

We use the normalized values: $\phi\equiv e\varphi/T_e$, $N_-\equiv n_{o-}/n_{o+}$, $N_e\equiv n_{oe}/n_{o+}$, $Q_\pm=q_\pm/e$, $V_{s\pm}=(T_e/M_\pm)^{1/2}$. We normalize x on Debye radius of electrons r_{de}, $V_\|$ on V_{the}, time t on plasma frequency of positive ions ω_{p+}^{-1}, velocity of solitary perturbation V_c on ion-acoustic velocity $(T_e/M_+)^{1/2}$. T_e, is the temperature of electrons, n_{o-}, n_{o+} are unperturbed densities of negative and positive ions, $q_\pm$ is the charge of positive and negative ions.

Integrating (1), one can derive the electron density in first approximation on V_o

$$n_e\approx n_{oe}\exp(\phi)[1-(2\Delta\phi/\sqrt{\pi})\int_o^\beta dx\,\exp(-x^2)-$$
$$-2V_\|(2/\pi)^{1/2}\int_o^\beta dx\,(x^2-\phi)^{1/2}\exp(-x^2)] \quad (2)$$

Far from the dip the plasma is quasineutral $n_e(z)\big|_{z\to\infty}=n_e(z)\big|_{z\to-\infty}=1$. From here one can derive, using (2), the expression for potential jump near the dip

$$\Delta\phi=V_\|(2/\pi)^{1/2}(1-\exp(-\phi_o))/[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\,\exp(-x^2)] \quad (3)$$

From hydrodynamic equations for ions one can obtain for perturbations of densities of positive and negative ions

$$n_{i\pm}=n_{\pm NL}+n_{\pm\tau},\ n_{\pm NL}=n_{o\pm}/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{1/2}, \quad (4)$$
$$\partial n_{\pm\tau}/\partial z=\pm 2(\partial\varphi/\partial t)(n_{o\pm}q_\pm/M_\pm V_c^3)\times$$
$$\times[1-(\pm q_\pm)\varphi/M_\pm V_c^2]/[1-(\pm q_\pm)2\varphi/M_\pm V_c^2]^{3/2}$$

Substituting (2), (4) in Poisson equation one can derive nonlinear evolution equation

$$\partial^3_z\phi+\{Q_+^2V^2_{s+}(1-2\phi Q_+V^2_{s+}/V_c^2)^{-3/2}(1-\phi Q_+V^2_{s+}/V_c^2)+$$

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$$
\begin{aligned}
&+Q_-^2N_-V^2_{s-}(1+2\phi Q_-V^2_{s-}/V_c^2)^{-3/2}(1+\phi Q_-V^2_{s-}/V_c^2)\}2\partial_t\phi/V_c^3\\
&+(\partial_z\phi/V_c^2)\{Q_+^2V^2_{s+}(1-2\phi Q_+V^2_{s+}/V_c^2)^{-3/2}+\\
&+Q_-^2N_-V^2_{s-}(1+2\phi Q_-V^2_{s-}/V_c^2)^{-3/2}\}-\\
&-\{\exp(\phi)-\mathrm{sign}(z)V_{\|}(2/\pi)^{1/2}\{(\phi_o/(\phi_o+\phi))^{1/2}\exp(-\phi_o)-\\
&-\int_{\sqrt{-\phi}}^{\sqrt{\phi o}} dy(1-2y^2)\exp(-y^2)/(y^2+\phi)^{1/2}+\\
&+(1-\exp(-\phi_o))[1-(2/\sqrt{\pi})\int_o^{\sqrt{\phi o}} dx\exp(-x^2)]^{-1}\times\\
&\times[\exp(-\phi_o)/(\phi_o+\phi)^{1/2}+2(\phi_o+\phi)\exp(-\phi_o)+\\
&+4\int_{\sqrt{-\phi}}^{\sqrt{\phi o}} dy\, y(y^2+\phi)^{1/2}\exp(-y^2)]/\sqrt{\pi}\}\}\partial_z\phi=0
\end{aligned}
\tag{5}
$$

From nonlinear equation (5) one can show that the dip propagates with the slow velocity $V_c\approx0$. From (5) one can get also the growth rate of the dip small amplitude

$$
\begin{aligned}
&\gamma_{nl}\approx\omega_{p+}(V_{\|}/V_{the})^{3/2}(q_+/e)(n_+/n_e)^{1/2}\{1+\\
&+[1/3-(n_e/n_+)(e/q_+)](e\varphi_o/T_e)(\pi/2)^{1/2}(V_{the}/2V_{\|})\}
\end{aligned}
\tag{6}
$$

One can see that the dip is formed at ratio of electron current to thermal velocity $V_{\|}/V_{the}$ larger than threshold. The threshold decreases at decreasing of ratio of electron and positive ion densities n_e/n_+ and equal zero at $n_e/n_+<q_+/3e$. The threshold is maximum at $n_e/n_+=1$.

BARRIER FORMATION FOR PLASMA IONS IN KIND OF ELECTRIC POTENTIAL HUMP NEAR ECR POINT

As the electrons are reflected from the dip and the flow ions pass with V_{o+} through the dip freely, noncompensated volume charge of ions is formed after the dip, in which field the ions are slowing down and reflected. This volume charge forms perturbation of the electric potential hump. The ion flow enhances this hump of the electric potential. We describe the quasistationary properties of the hump, neglecting the nonequilibrium condition. Taking into account the nonequilibrium condition leads to the hump's excitation, in other words to growth of the hump's amplitude. Further we will show, that the electric potential hump is almost nonmobile in space.

In linear approximation the perturbation excitation by plasma flow, propagating relative to negative and motionless positive ions, is described by the following dispersion ratio:

$$
1+1/(kr_{de})^2-\omega^2_{p+}/(\omega-kV_{o+})^2-\omega^2_{p-}/\omega^2-\omega^2_{pq}/\omega^2=0. \tag{7}
$$

Here ω, k are the frequency and wavevector of the perturbation; $\omega_{p\pm}$ are the plasma frequencies of the negative and flow's positive ions; ω_{pq} is the plasma frequency of the positive motionless ions; r_{de} is the Debye radius of electrons; V_{o+} is the velocity of the positive ion flow.

From (7) one can see that one can select the flow velocity such

$$
\begin{aligned}
&V_{ph}=\omega/k\approx(V_{o+}/2^{4/3})[(n_-m_+q_-^2/\, n_+m_-q_+^2)+\\
&+(n_{+q}q_{+q}^2/\, n_+q_+^2)]^{1/3}<<V_{s+},\\
&\lambda=2\pi/k=2\pi r_{de}/(V_{s+}^2n_+q_+^2/V_{o+}^2n_ee^2-1)^{1/2}>>r_{de},
\end{aligned}
\tag{8}
$$

that the perturbation is almost motionless, that is $V_{ph}<<V_{s+}$. $V_{s+}=(T/m_+)^{1/2}$ is the ion-acoustic velocity of the flow positive ions. Here n_-, m_-, q_- (n_+, m_+, q_+) are the density, mass and charge of the negative (positive) ions.

From (7) one can derive the growth rate of the perturbation excitation:

$$
\begin{aligned}
&\gamma=(1.5)^{1/2}(V_{o+}/r_{de})[(n_-m_+q_-^2/n_+m_-q_+^2)+\\
&+(n_{+q}q_{+q}^2/n_+q_+^2)]^{1/3}(V_{s+}^2q_+/V_{o+}^2e-1)^{1/2}.
\end{aligned}
\tag{9}
$$

On non-linear stage of the instability development the electric potential perturbation φ represents the solitary hump of finite amplitude φ_o.

The distribution function of nontrapped electrons, that is arranged outside the separatrix, looks like:

$$
f_e(v)=[n_{oe}/V_{te}(2\pi)^{1/2}]\exp(e\varphi/T_e-m_ev^2/2T_e). \tag{10}
$$

For trapped electrons, that is for electrons located inside the separatrix, the distribution function does not depend on energy because of an adiabaticity of the evolution.

Integrating the electron distribution function on velocity, we receive following expression for electron density:

$$
n_e=(n_o/(2\pi)^{1/2})(2/T)^{3/2}\int^{\infty}_o d\varepsilon(\varepsilon+e\varphi)^{1/2}\exp(-\varepsilon/T). \tag{11}
$$

From hydrodynamic equations for positive ions it is possible to receive following expression for their density:

$$
n_+=n_{o+}/[1-2q_+\varphi/m_+(V_{o+}-V_h)^2]^{1/2}. \tag{12}
$$

Here V_h is the velocity of the solitary perturbation.

As a result from (11), (12) and Poisson equation we have equation for the spatial distribution of the electric potential of the perturbation of any amplitude:

$$
\phi''=(2/\sqrt{\pi})\int^{\infty}_o da e^{-a}(a+\phi)^{1/2}-1/(1-2Q\phi/v_{oh}^2)^{1/2}. \tag{13}
$$

$Q=q_+/e$, $\phi=e\varphi/T$, «'»$=\partial/\partial x$, $x=z/r_{de}$, $v_{oh}=(V_{o+}-V_h)/V_{s+}$.

From the condition $\phi'|_{\phi=\phi o}=0$ and (13) we obtain the hump velocity, v_{oh}:

$$
\begin{aligned}
&v_{oh}^2/Q=(A-2)^2/2(A-2-\phi_o),\\
&A=(8/3\sqrt{\pi})\int^{\infty}_o da e^{-a}(a+\phi)^{3/2}.
\end{aligned}
\tag{14}
$$

In the approximation of small amplitudes from (13), (14) we receive:

$$
v_{oh}^2\approx Q,\ \ L\approx[15\sqrt{\pi}/4(1-1/\sqrt{2})]^{1/2}\phi_o^{-1/4}. \tag{15}
$$

If V_{o+} close to $(q_+/e)^{1/2}V_{s+}$, then the perturbation is approximately motionless.

Taking into account the small densities negative and motionless positive ions, we obtain from the Poisson equation the evolution equation:

$$
2\omega^2_{p+}\partial^3\varphi/\partial t^3/(V_{o+}-V_h)^3=-(\omega^2_{p-}+\omega^2_{pq})\partial^3\varphi/\partial z^3. \tag{16}
$$

From (16) the growth rate of the non-linear perturbation amplitude is followed:

$$\gamma_{NL}\approx\omega_{p+}(e\varphi_o/T)^{1/2}[(n_{o-}m_+q^2_-/n_{o+}m_+q^2_+)+ +(n_{oq}q^2_{+q}/n_{o+}q^2_+)]^{1/3} . \quad (17)$$

ELECTRON MECHANISM OF BARRIER FORMATION FOR PLASMA IONS NEAR ECR POINT

Let us consider the mechanism of the electric potential hump formation by plasma electrons near the dip of the electric potential.

The potential jump accelerates ions. Hence on first front of the electric potential dip the ion density becomes smaller. If into this region the electron flow penetrate with electron velocity only a little smaller than electron thermal velocity, hence on the first front of the potential dip the electron drift velocity becomes more than electron thermal velocity due to flow continuity law. Due to Bunemann mechanism interaction of electron flow with this region an electric potential hump is excited.

Let us describe a solitary perturbation in type of electric potential hump. We will show that it represents a nonlinear perturbation on a slow electron-sound mode. As it is slow, resonant electrons can be trapped by such perturbation.

From Vlasov equation the expression for perturbation of electron distribution function follows. Integrating which on velocities in case of small amplitudes of the solitary perturbation φ_o one can derive the expression for perturbation of electron density

$$\delta n'=\partial_t\varphi[y+(1-2y^2)(1-R(y))/y]+ +\varphi'R(y)+\varphi\varphi'[1-y^2+(3/2-y^2)(R(y)-1)] \quad (18)$$

$$R(y)=1+(y/\sqrt{\pi})\int_{-\infty}^{\infty}dt\exp(-t^2)/(t-y),\ y=V_h/V_{th}\sqrt{2}$$

Here point means derivation on time, and prime is a spatial derivation. V_h, φ are velocity and potential of soliton. $\phi=e\varphi/T_e$. Substituting (18) in Poisson equation, one can derive an equation, describing spatial distribution of potential:

$$(\phi')^2 =\phi^2R(y)-[1+(2y^2-3)R(y)]\phi^3/6 \quad (19)$$

Let us determine approximately the soliton width from (19): $\Delta x=(48T_e/\varphi_o)^{1/2}$. The soliton width decreases with amplitude growth.

In case of large amplitudes, $e\varphi_o/T_e>1$, from Vlasov equation we have the expression for electron distribution function $f=f_o[(u^2-2e\varphi/m)^{1/2}+V_h\mathrm{sign}(u)]$ for $|u|=|V-V_h|>(2e\varphi/m)^{1/2}$. Here f_o is Maxwell distribution function.

Thus we obtain the equation for the soliton shape

$$(\phi')^2= =-\phi+(2/\sqrt{\pi})^{1/2}\int_{-\infty}^{\infty}dt(t-y)^2\exp(-t^2)\{[1+\phi/(y-t)^2]^{1/2}-1\} \quad (20)$$

From (20)

$$\Delta x=[2e\varphi_o/T_e(\sqrt{2}-1)]^{1/2} \quad (21)$$

we conclude that the soliton width grows with φ_o. Therefore, it is necessary to take into account electrons, trapped by the soliton field. Assuming distribution of their density as $n_{tr}(x)=n_2\exp[e\varphi(x)/T_{tr}]$, we derive similarly to (21), that width and velocity of the soliton grow with amplitude growth (in difference from case of small amplitudes of the solitary perturbation).

Thus, at a neglect of ion mobility this solitary perturbation is stationary and electron one. However at taking into account of ion mobility it is necessary to expect occurrence of slow growth of the perturbation’s amplitude, as a result of Bunemann instability development. In the following order of the theory of disturbances from (18) one can derive the correction of the next order to a spatial derivative from electron density

$$n_{e1}'=\partial_t\varphi[y+(1-2y^2)(1-R(y))/y] \quad (22)$$

This expression, as follows from a spatial derivative from Poisson equation, should be equal to a spatial derivative from ion density perturbation n_i'. n_i' is possible to find in linear approximation from ion hydrodynamic equations

$$\partial^2_t n_i=(m_e/m_i)\phi'' \quad (23)$$

Equating the second time derivative from (10) and first spatial derivative from (23), we obtain

$$\partial^3_t\phi=(6m_e/m_i)\phi''' \quad (24)$$

The solution of (24) we search as

$$\phi(x,t)=\phi_o(t)\eta[x-\int_{-\infty}^{t} dt_1\delta v_o(\phi_o(t_1))] , \quad (25)$$

$\eta(x)$ is quasistationary shape of the perturbation, assuming, that $\partial_t\phi_o(t)=\gamma\phi_o(t)$. In (25) the change of soliton velocity with change of its amplitude is taken into account.

Substituting $\partial_t\phi$ through $\gamma\phi-\delta v_h\phi'$, we obtain from (24)

$$\gamma\approx(m_e/m_i)^{1/3}\phi_o^{1/2}$$

REFERENCES

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