COMPARISON OF PLASMA AND VACUUM CORPUSCULAR LENSES BY PARAMETERS DETERMINING THEIR FOCUSING STRENGTH

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For three types of electrostatic lenses the focusing length is proportional, accordingly, to the second-, or first-, or half-power of the parameter representing the relation of a charged particle accelerating potential to a lens potential. Accordingly, for magnetic lenses the focusing length is proportional to the second-, or first-, or half-power of the parameter representing the relation of the particle kinetic momentum to the electromagnetic component of its canonical momentum.

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1. INTRODUCTION

For axial-symmetric lenses, focusing is effect of either second, or first order, i.e. the optical force is proportional to the second or first degree of focusing fields (e.g., see [1, 2]). The second order lenses are traditional electrostatic or magnetic thin lenses used in the beam optics. The first order lenses of increased force have either extrinsic charges (in case of electrostatic lenses) or extrinsic currents (in case of magnetic lenses). The focusing length of electrostatic lenses is proportional to the second or first degree of parameter representing the relation of a charged particle source potential to a lens potential. In this work a similar parameter for magnetic lenses is considered: that is the relation of the particle kinetic momentum to the electromagnetic component of its canonical momentum. Furthermore, besides of these first and second types of lenses, there is third type. As was shown in Ref.[3], for long uniform plasma electrostatic and magnetic lenses with extrinsic charges or currents, the focusing length is proportional to the square root of the mentioned parameters. Further reduction of the focusing length is achieved by imposing a profiled magnetic field compressing the focusing channel on a measure of the beam focusing.

2. VACUUM ELECTROSTATIC LENSES

In the vacuum electrostatic lenses the focusing is a difference effect for focusing and defocusing electric field. Therefore for all these lenses focusing is effect of second order [1, 2].

The simplest example of the second order focusing gives a vacuum electrostatic lens in the form of the thin ring (with radius a) charged to a certain potential φ. In this case the focusing distance is as follows (see [2]):

$$L_f = 128(U/\varphi)^2 a/3\pi \,, \tag{1}$$

where U is the accelerating potential.

3. THIN ELECTROSTATIC LENSES WITH EXTRINSIC CHARGES

The situation is different, if a lens has extrinsic charges. In this case the electric field is everywhere focusing. Therefore the focusing be the first order effect, and the focusing force is much greater because it is proportional to first power of the parameter φ/U ($\varphi \ll U$) [1]. These electrostatic lenses of increased force concern to the second type, e.g., wire mesh lens (e.g., see [4]), Gabor electron lens [5,6], Morozov plasma lens [2,7], and other. In particular, D. Gabor proposed the magnetron lens with side loop cathode. That lens was studied experimentally in the work [8] (see Fig. 1) where ion beams with energy up to 80 keV were focused.

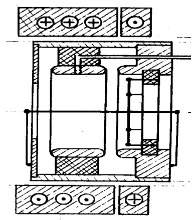

Fig. 1. View of the magnetron lens [8]

In those experiments the main theoretical relations for the magnetron lens were confirmed:

$$H_{cr} = 6.74\sqrt{\Phi_a}/R\,, \quad f = \frac{U}{\Phi_a}\frac{R^2}{L}\,,$$

where U is the ion acceleration potential (Volt), H_{cr} is the critical value of the magnetic field (Oersted), R, L, and Φ_a are the radius, length, and anode potential of the magnetron lens, respectively.

For uniform Gabor lens the focusing electrostatic force has the form $F_e = -2\pi neqr$, where n is the electron density. In case of the thin Gabor lens, in the impulse approximation [5,6], the focusing length is:

$$L_f = Mv^2/2\pi neql = Ua^2/\varphi(a)l\,, \tag{2}$$

where M, q, and v are the mass, charge, and velocity of ions; a is the radius of the electron lens, $\varphi(a)$ is the potential at $r=a$, l is the length of the lens (it is supposed that $a \ll l \ll L_f$).

In the Morozov lens the electric potentials are inserted into plasma by a set of the ring electrodes. In this case the system of "charged" magnetic surfaces are created in the plasma lens volume. The focusing length of the thin Morozov lens is [2, 7]:

$$L_f = aU/2\varphi_0\Theta \,, \tag{3}$$

where a is the lens radius, φ_0 is the lens potential, Θ is the geometrical factor (Θ~1).

4. EXTENDED ELECTROSTATIC LENSES WITH EXTRINSIC CHARGES

4.1. The long Gabor lens

In this case the equation for the ion motion is:

$$r'' + k_G^2 r = 0\,, \quad k_G^2 = 4\pi neq/Mv^2\,, \tag{4}$$

The expressions for ions trajectories and focusing distance are:

$$r = r_0 \cos k_G z\,,\ L_f = \frac{\pi}{2}\sqrt{\frac{Mv^2}{2\pi neq}} = \frac{\pi a}{2}\sqrt{\frac{U}{\varphi(a)}}\,. \qquad (5)$$

For a lens of length $l<L_f$: $L_f = l + k_G^{-1} ctg(k_G l)$, whence at $k_G l <<1$ it can be received the expression (2) for a thin lens: $L_f=(k_G l)^{-1}$.

4.2. Long plasmaoptic focusing devices of Morozov type in the uniform and non-uniform magnetic field

In the case of long plasmaoptic focusing device of Morozov type, the ring electrodes can be placed near by the lens faces at the lateral surface, i.e. at the input and output of the magnetic force lines (see Fig. 2); so, the geometrical aberrations are reduced to minimum. The equation of the focusing ion motion has the form [9]:

$$r'' + k_M^2 r = 0\ ,\ k_M^2 = 2q\varphi_0 / Mv^2 a^2\,, \qquad (6)$$

where *M, q,* and *v* are the mass, charge, and velocity of ions; *a* is the radius of the boundary magnetic surface, φ_0 is its potential. The expressions for ion trajectories, and focusing distance in the lens are:

$$r = r_0 \cos(k_M z)\,,\ L_f = \pi\,(2k_M)^{-1}\,, \qquad (7)$$

where r_0 is the radius of the ion injection.

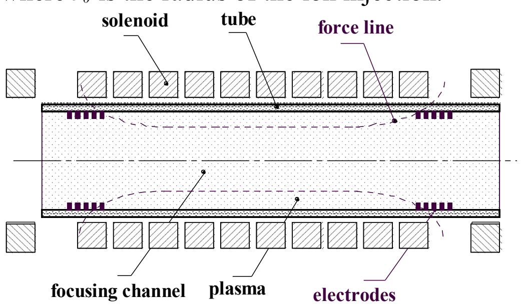

Fig. 2. Uniform Morozov lens

For uniform long solenoid, the focusing distance not depends from ion injection radius, i.e. focusing of the wide aperture ion beams is possible. If the length of the lens $l < L_f$, then the focusing distance $l_f = l + k_M^{-1}\mathbf{ctg}(k_M l)$. For “thin” lens ($k_M l << 1$) the focusing distance $l_f = (k_M^2 l)^{-1}$.

For non-uniform long solenoid, we can study the problem of the external magnetic field increasing (from the lens entrance to its end) by such a manner that the radius of the determined (so named "boundary") magnetic surface can coincide with the focused ion beam radius, on the whole lens length (see Fig. 3).

In this way, the efficiency and force of the lens are increased sufficiently. The problem is being solved at the paraxial approximation. In this case the equation of the magnetic surfaces is as follows:

$$a^2(z) = a_0^2 B_z(0)/B_z(z)\,, \qquad (8)$$

where $a(z)$ is the magnetic surface radius, $B_z(z)$ is the longitudinal magnetic field on the axis.

With account of (8), we receive the equation for focused ion motion [9]:

$$r'' + r k_M^2 B_z(z)/B_z(0) = 0\,,$$
$$k_M^2 = 2q\varphi_0 / Mv^2 a_0^2\,, \qquad (9)$$

where a_0 is the initial radius of the boundary magnetic surface at z = 0, and φ_0 is its potential.

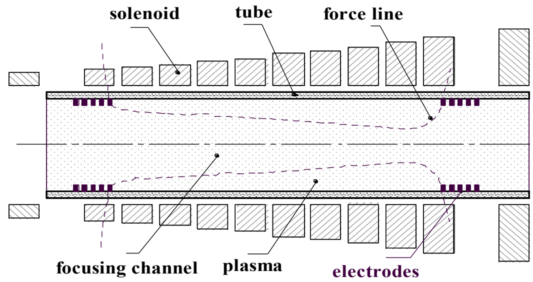

Fig. 3. Non-uniform Morozov lens

During the ion focusing and compression of the focusing channel by the magnetic field, some ions (with large injection radius) can move partly out of the focusing channel. To put all ions in the focus, it is needed an optimization of the magnetic field distribution. The magnetic surface that limits the focusing channel is determined from the condition that its radius (a_0) coincides with the radius of the focused beam (R). The functions $R(z)$ and $B_z(z)$ are determined from the equation:

$$R'' + K / R = 0\,, \text{where } K = 2q\varphi_0 / Mv^2\,. \qquad (9')$$

In the real experiment the current channel compression leads to the certain value R_g (not equal to zero) that corresponds to the lens end coordinate z_g. At this place the current channel is finished (by a wire mesh or metallic foil). Later on the inertial ion focusing in the focal spot takes place. In the case of the parallel ion beam injection, this coordinate is defined as the expression:

$$z_f = \sqrt{\frac{\pi}{2K}} R_0 \Phi_0 \left(\sqrt{2\ln\frac{R_0}{R_g}}\right) + \frac{R_g}{\sqrt{2K\ \ln(R_0/R_g)}} \qquad (10)$$

To conclude this section, we note that, as was mentioned in [1], the focusing length $L_f \propto U/\varphi$ in thin electron and plasma lenses is much shorter than $L_f \propto (U/\varphi)^2$ in vacuum ones. Formulas (5),(7),(10) show that, in extended electron and plasma lenses, the focusing length obeys the dependence $L_f \propto (U/\varphi)^{1/2}$ and thus is even shorter.

5. VACUUM MAGNETOSTATIC LENSES

The simplest example of the second order magnetic focusing gives a vacuum lens in the form of the thin ring of radius a_0 with current *J*. In this case the focusing distance is (see, e.g., [2]):

$$f = \frac{16}{3\pi^3}\frac{Mc^2}{q^2}\frac{WRc^2}{J^2}a_0\,, \qquad (11)$$

were *c* is the light velocity, *q, M* and $W = MV^2/2$ are charge, mass and kinetic energy of the ion.

As a parameter, it is expedient to use in the expression for focusing distance the relation of the particle kinetic momentum to the electromagnetic component of its canonical momentum, i.e., MV/P_{EM}.. (For the circular current $P_{EM}=qA_\varphi/c$, where A_φ is the projection of the magnetic vector-potential that is proportional to *J*.)

Instead *A* it is expedient to use expressions with the same number of dimension: *J/c* or $H_E L_E$, where H_E and L_E are the effective magnetic field and dimension of the lens. For example, the formula (11) can take the form:

$$f = \frac{8}{3\pi^3} \frac{(MV)^2}{(qJ/c^2)^2} R_0 . \quad (12)$$

As it was stated (e.g., see [1]), in the vacuum magnetic lenses the focusing force is proportional to $V_z H_r H_z$, where H_r and H_z are the magnetic field components. So, for all these lenses focusing is the second order effect.

6. THIN MAGNETIC LENSES WITH EXTRINSIC CURRENTS

The situation is different, if a thin lens has extrinsic currents (e.g., plasma magnetic lens [10,11,6], parallel wires lens [12], magnetic horn [13], lithium and parabolic lens [14], others). In this case the magnetic field is everywhere focusing. Therefore the focusing be the first order effect, and the focusing force is much greater because it is proportional to first power of the parameter MV/P_{EM}.. For example, the focusing distance for the thin current carrying plasma lens is:

$$f = \frac{MV}{2qJ/c^2} \frac{b^2}{l} \quad (13)$$

where J is the lens longitudinal current, b and l are the radius and length of the current channel. The focusing distance for the parabolic lens has the similar form:

$$f = \frac{MV}{4qJ/c^2} \frac{R_0^2}{L_0} , \quad (14)$$

R_0 and $2L_0$ are the main radius and length of the lens.

7. EXTENDED UNIFORM AND NON-UNIFORM MAGNETIC LENSES

Let us consider the problem of ion beam focusing by the extended plasma magnetic lenses. We investigate the case that the current radius is determined by the external non-uniform longitudinal magnetic field (see Fig. 4). The problem is solved at the paraxial approximation; the equation of the magnetic surfaces is as (8).

We assume that in the case of the strong magnetic field the electrons, which transport the current in plasma, are moving along the cylindrical magnetic surfaces enclosed one into another. The boundary conditions are: at z=0, $a(0) = b$, where b is the radius of an electrode that supply the current in the plasma (e.g., it is the inner electrode of the plasma gun). As a result, the equation for the focused ion trajectories will take the form [3]:

$$r'' + k^2 r B_z(z)/B_z(0) = 0 , \; k^2 = 2Iq/Mc^2 v b^2 \quad (15)$$

In Eq. (15) I is the current in plasma, q and M are the charge and mass of the ion, c is the light velocity, v is the ion velocity, b is the initial radius of the current channel. Under condition $B_z(z) = \text{const}$ (or 0) from Eq. (16) we have: $r = r_0 \cos kz$, and the focusing distance in the plasma: $L_f = \pi/2k$. For a lens of length $l < L_f$: $L_f = l + k^{-1} ctg(kl)$, whence at $kl << 1$, (i.e., for a thin lens): $L_f = (k^2 l)^{-1}$.

To put together all ions in the focus, it is needed an optimization of the external magnetic field distribution. The magnetic surface that limits the current channel is determined from the condition that its radius (R) coincides with the current channel radius (b) and the radius of the focused beam. The functions $R(z)$ and $B_z(z)$ are determined as it follows:

$$R'' + \kappa / R = 0 , \quad (16)$$

where $\kappa = 2qI/Mvc^2 \propto P_{EM}/Mv$.

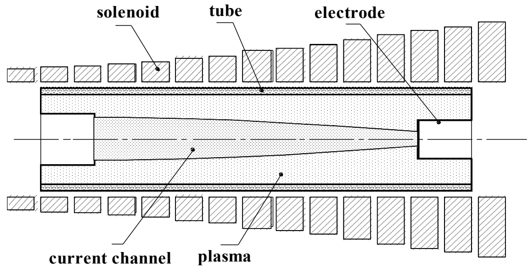

Fig.4. Non-uniform plasma magnetic lens

In the case of the parallel ion beam injection, the solution of Eq. (17) takes the form:

$$z = b\sqrt{\pi/2\kappa}\, \Phi_0\left(\sqrt{2\ln b/R}\right) . \quad (17)$$

In the real experiment the current channel compression leads to the certain value R_g (not equal to zero) that corresponds to the lens end coordinate z_g. So, the focus coordinate is defined as follows:

$$z_f = b\sqrt{\frac{\pi}{2\kappa}} \Phi_0\left(\sqrt{2\ln\frac{b}{R_g}}\right) + \frac{R_g}{\sqrt{2\kappa \ln(b/R_g)}} . \quad (18)$$

In conclusion of this part, we add the following remark. For vacuum magnetic lenses the focusing length $L_f \propto \kappa^{-2}$, but for short magnetic plasma lenses $L_f \propto \kappa^{-1}$, i.e., it is much less. In this work it is shown that for long plasma magnetic lenses $L_f \propto (\kappa^{-1})^{1/2}$, i.e., it is more less. The compression of the focusing channel gives additional gaining of several times over.

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