DEFORMATION OF THE PLASMA CONCENTRATION PROFILE DUE TO THE MODULATED ELECTRON BEAM

I.O.Anisimov[1], O.A.Borisov, O.I.Kelnyk[2], S.V.Soroka
Taras Shevchenko Kyiv National University, Radio Physics Faculty,
64, Volodymyrs'ka St., 01033, Kyiv, Ukraine, [1]ioa@rpd.univ.kiev.ua, [2]oles@univ.kiev.ua

Report is devoted to the numerical simulation of the electron beam with the longitudinal modulation moving along the concentration gradient of the planarly stratified plasma with the initially linear profile (one-dimensional model). Nonlinear modification of the plasma concentration profile due to the HF electric field excited by the beam is studied. The stationary case and initial problem were calculated.
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1. INTRODUCTION

The problem of the Langmuir waves excitation in non-uniform plasma due to the modulated electron beam is studied during a long time [1–3] because this phenomenon is typical for plasma electronics. But only the asymptotic solutions for stationary problem of Langmuir waves excitation were obtained previously. However, for the large magnitudes of the beam current the electric field excited in the local plasma resonance region (LPRR) can modify the plasma concentration profile [5-8].

2. MODEL DESCRIPTION AND BASIC EQUATIONS

Isotropic warm ($T_e \neq 0$) weakly inhomogeneous planarly stratified plasma is considered (density depends on z only). Near the plasma resonance region the dependence of plasma density upon z is linear:

$$n_p(z)=n_0(1+z/L), \tag{1}$$

where L is the characteristic length of the plasma non-uniformity, $n_0=n_p(0)$ is the plasma density in the LPRR where the modulation frequency ω of the electron beam coincides with the local Langmuir frequency. Modulated electron beam moves along z-axis. Its alternative current density can be represented as

$$j(z,t) = j_m \exp(i\omega t - i\kappa z)\Theta(t \pm z/\mathrm{v}_o), \tag{2}$$

where v_o is the beam velocity, $\Theta(x)$ is the step function that describes the beam front motion, $\kappa=\omega/\mathrm{v}_o$ is the wave number.

The system under consideration is described by a set of Maxwell equations, continuity equation and linearized equation of plasma electrons' motion:

$$\begin{cases} \dfrac{4\pi}{c} j + \dfrac{1}{c}\dfrac{\partial E}{\partial t} - \dfrac{4\pi e n_o u}{c} = 0; \\ \dfrac{\partial \delta n}{\partial t} + n_o \dfrac{\partial u}{\partial z} = 0; \\ m n_o \dfrac{\partial u}{\partial t} = -e n_o E - 3m\mathrm{v}_{Te}^2 \dfrac{\partial}{\partial z}\delta n - m n_o \nu u. \end{cases} \tag{3}$$

Here n_o and δn are the time averaged plasma density and its deviation, u and ν are the plasma electrons' instantaneous velocity and collision frequency, respectively.

Excluding u and δn from (3) gives

$$\frac{\partial^2 E}{\partial t^2} + \omega_p^2(z)E - 3\mathrm{v}_{Te}^2\frac{\partial^2 E}{\partial z^2} + \nu\frac{\partial E}{\partial t} = -4\pi\left(\nu j + \frac{\partial j}{\partial t} - \frac{3\mathrm{v}_{Te}^2}{\mathrm{v}_o}\frac{\partial j}{\partial z}\right), \tag{4}$$

where $\omega_p^2(z)=4\pi n_0(z)e^2/m$ is the local Langmuir plasma frequency. Inhomogeneous wave equation (4) describes the excitation of Langmuir waves in the warm inhomogeneous plasma by the modulated electron beam. After introducing dimensionless variables

$$\varepsilon = \frac{\omega E}{4\pi j_m}, \quad \tilde{j} = \frac{j}{j_m}, \quad \tau = \omega t, \quad \varsigma = \frac{\omega z}{\mathrm{v}_o}, \quad \tilde{T} = \frac{3v_{Te}^2}{\mathrm{v}_o^2},$$
$$\tilde{\nu} = \frac{\nu}{\omega}, \quad \lambda = L\frac{\omega_{po}}{\mathrm{v}_o}, \quad n(\varsigma) = (\omega_p/\omega)^2,$$

equation (4) takes the form

$$\frac{\partial^2 \varepsilon}{\partial \tau^2} + n(\varsigma)\varepsilon - \tilde{T}\frac{\partial^2 \varepsilon}{\partial \varsigma^2} + \tilde{\nu}\frac{\partial \varepsilon}{\partial \tau} = -\left(\frac{\partial \tilde{j}}{\partial \tau}(1 \pm \tilde{T}) + \tilde{\nu}\tilde{j}\right), \tag{5}$$

where upper and lower signs correspond to the positive and negative beam velocity respectively. The deformation of the plasma concentration profile can be described by the inhomogeneous equation for ion-acoustic waves:

$$\tilde{M}\frac{\partial^2 \delta n}{\partial \tau^2} - \frac{\partial^2 \delta n}{\partial \varsigma^2} = \Lambda\frac{\partial^2}{\partial \varsigma^2}\left[n_0(\varsigma)|\varepsilon|^2\right], \tag{6}$$

When transient processes caused by the electron beam front are finished and only oscillations on the modulation frequency remain than equations set (5-6) can be reduced to the single equation

$$\tilde{T}\frac{d^2\varepsilon}{d\varsigma^2} + [1 - i\tilde{\nu} - n_0(\varsigma)(1 - \Lambda\,|\varepsilon|^2)]\varepsilon = \\ = (i(1 \pm \tilde{T}) + \tilde{\nu})\exp(\mp i\varsigma). \qquad (7)$$

In the next section we examine this stationary regime.

3. LINEAR STATIONARY EXCITATION OF LANGMUIR WAVES

For $\delta n_p = 0$, $t \to \infty$ equation (5) describes the linear stationary excitation of Langmuir waves by the alternative current (2). Outside LPRR the solution can be presented as a superposition of the field of current (2) and Langmuir wave excited by this current (fig. 1). Conversion of the modulated electron beam field into Langmuir waves is most efficient in the vicinity of Cherenkov resonance point where the Langmuir wave phase velocity is equal to the electron beam velocity. Predominantly the accompanying wave is excited. This phenomenon determines the dependence of the LPRR field magnitude and magnitude of the Langmuir wave that propagates to subcritical plasma on the direction of the electron beam velocity (see fig. 1*a*,*b*). The dependence on the beam velocity direction vanishes when $\tilde{\nu} >> \tilde{T}$.

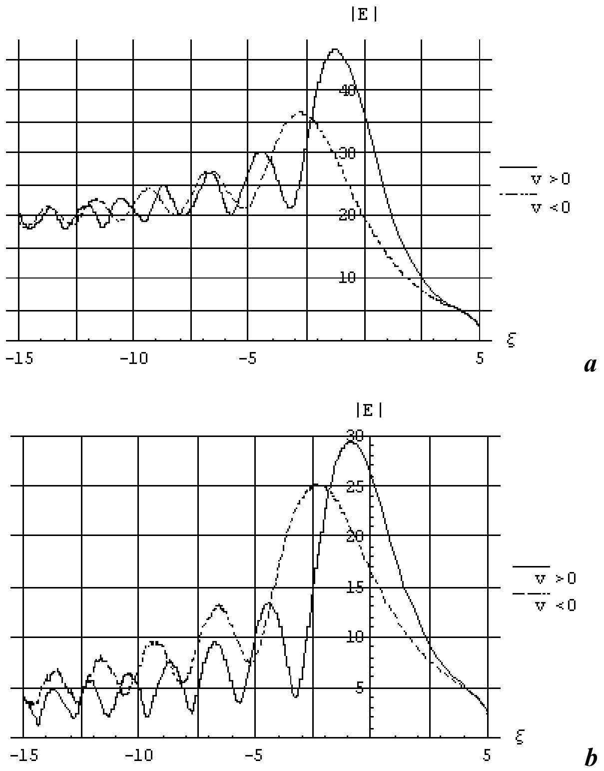


Fig. 1. Spatial distribution of electric field in the warm plasma caused by the modulated electron beam for $\tilde{T} = 0.01$, *ωL/v₀=10: a –* $\tilde{\nu} = 0$ *; b –* $\tilde{\nu} = 0.02$

4. TRANSIENT PROCESSES CAUSED BY THE FOREFRONT OF ELECTRON BEAM

The forefront of electron beam excites the Langmuir oscillations in plasma at the local electron plasma frequencies. For the small collisions' frequency ($\tilde{\nu} << \tilde{T}$) they leak to the subcritical plasma (fig. 2). The motion of the forefront of the Langmuir wave with the modulation frequency that is excited in the resonance region can be observed for this case. As a result the stationary distribution of electric field is formed (see fig.1).

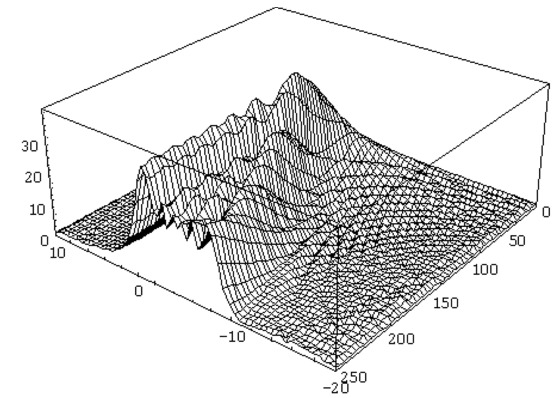


Fig.2. Electric field excited by electron beam moving into plasma for $\omega L/v_o=10$, $\tilde{T} = 0.01$, $\tilde{\nu} = 0.001$

5. NONLINEAR DEFORMATION OF THE CONCENTRATION PROFILE

It was already noticed that for the large magnitudes of the beam current the electric field excited in the LPRR could modify the plasma concentration profile.

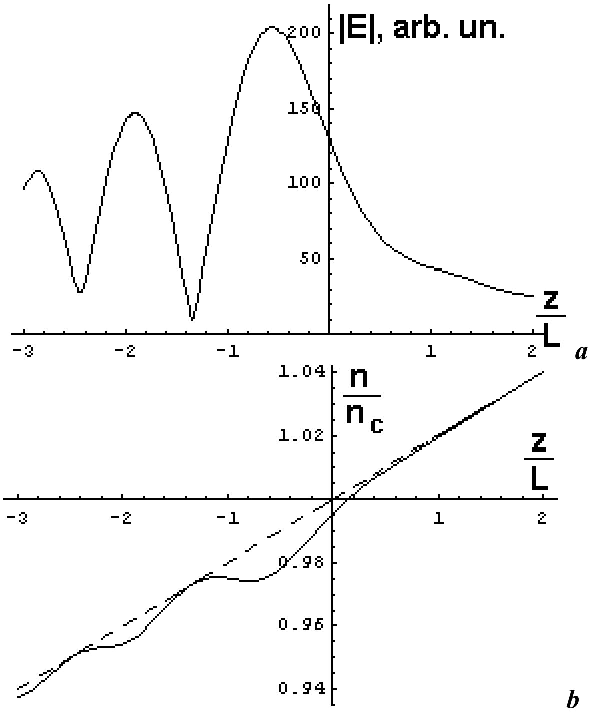


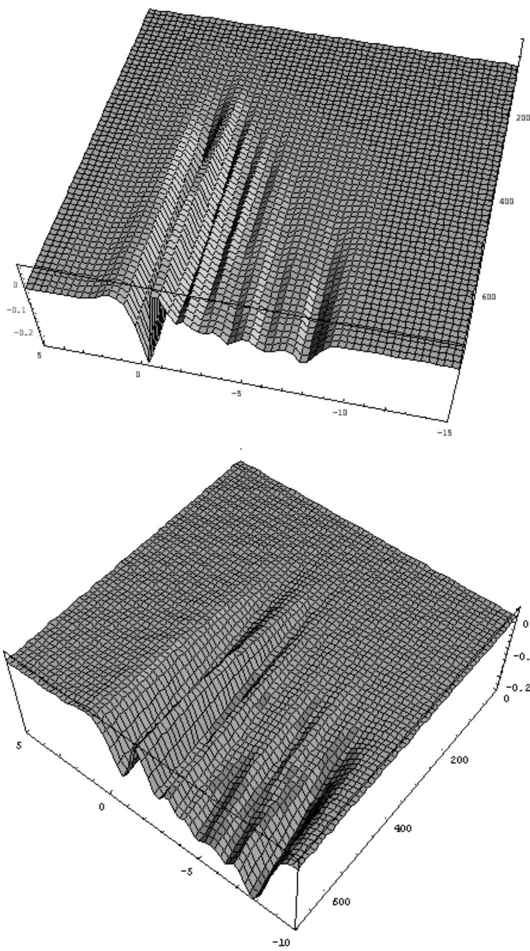
Fig. 3. Electric field (a) and nonlinear deformation of the concentration profile near the LPRR (b); dashed line – initial profile, solid line – disturbed profile; $\tilde{T} = 0.005$, $\Lambda=3\cdot10^{-7}$, $\omega L/v_o=50$

The concentration disturbance in the stationary case can be presented in a form:

$$\delta\tilde{n} = -\Lambda\, n_0(\varsigma)\left|\varepsilon^2\right|. \qquad (8)$$

Fig.3 shows the disturbance of the plasma concentration profile in the LPRR in the stationary case that is obtained from the numerical solution of equation (7).

Deformation of the plasma concentration profile strongly depends on the direction of the electron beam velocity. It is more significant for the beams moving into plasma because the electric field in the LPRR is stronger for this case (see fig.1).



a

b

Fig.4. Spatial and temporal dependence of the plasma density variation for positive (a) and negative (b) beam velocity direction; $\Lambda=3\cdot10^{-4}$, $M=6\cdot10^{5}$, $L=10$, $\nu=0.02$, $T=0.01$

6. EVOLUTION OF THE PLASMA DENSITY PROFILE DEFORMATION

The nonlinear deformation caused by the electron beam's eigen field and Langmuir waves is evolving in time. Fig. 4 shows the spatial and temporal dependence of the concentration disturbance. That result was obtained by the numerical integration of the equations' set (5-6)

For the case of the positive beam velocity plasma concentration profile deformation is mostly localized near the LPRR and the local concentration minimum is formed. If the beam velocity is negative, main concentration minimum is much less relatively to the previous case, and also this minimum is shifted toward the subcritical plasma.

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