EXCITATION OF EIGEN AZIMTHAL WAVES BY ANNULAR CHARGED PARTICLE BEAM

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Investigations of an interaction between charged-particle beams and hybrid waveguide structures are important for radio engineering and plasma electronics. It is the distinctive feature of the interaction between charged particle beams and such structures of finite dimensions that motivated the choice of the subject for our study. Here we investigate the excitation of an extraordinarily polarized electromagnetic azimuthal waves (AW), that are eigen modes of a cylindrical metal waveguide partially filled with a cold magnetized plasma. These modes propagate in azimuthal direction strictly transverse to a constant external axial magnetic field. A two-dimensional self-consistent set of differential equations for interaction between AW and cold low-density charged-particle beam moving above the plasma surface is constructed in the single-mode approximation and is solved numerically. Constructed nonlinear theory can be applied for modeling an operation of short-scale plasma filled devices capable of generating continuously tunable radiation over a broad frequency band (eigenfrequency of the plasma structure can be continuously tuned by varying the plasma density). Influence of nonuniformity of charged particle beam distributions in co-ordinate and phase spaces on evolution of beam-driven instability of the AW are examined.

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Various aspects of the problem of the interaction between electron beams and the eigenmodes of hybrid waveguides are investigated in [1]. Interest in studying the beam–plasma interaction stems primarily from the potential importance of the results obtained in this field, because they are expected to have a broad range of applications: from the beam heating of plasmas in controlled fusion devices [2] and geophysical experiments in space [3, 4] to solving the problems in plasma electronics. Our theoretical paper is devoted to investigating one of the problems of plasma electronics [5, 6], namely, the problem of the interaction between annual charged particle beam and the eigenwaves of the plasma waveguide. It is sequel of study wich is carried out in [7] where we have applied homogeneous distribution of the beam particles in order to examine excitation of AW.

We propose to excite AW in a cylindrical metal waveguide of radius R_2 with a coaxial plasma column of radius R_1. The gap between the plasma and the metal waveguide wall is assumed to be small, $R_2 - R_1 << R_1$. The density of an annular electron beam rotating in the gap region around the plasma column is much lower than the plasma density. The desired set of differential equations describing the nonlinear stage of AW excitation by an electron beam can be obtained from the hydrodynamic equations for the plasma, Maxwell's equations, and the equation of motion for the beam electrons. Fields of AW depend on time and angle co-ordinate by the following way: $E_r, E_\varphi, H_z \propto \exp(im\varphi - it\omega)$. Space is assumed to be uniform in axial direction. In the plasma region, the magnetic component of AW field is expressed in terms of the modified Bessel functions and the solutions for the electric field components are represented as linear combinations of the modified Bessel functions and their derivatives with respect to the argument. In the beam region, the AW fields are described by Bessel functions of the first kind, the Neumann functions, their derivatives and radial and azimuth components of the beam current density. Solving these equations for tangential electric component of the AW one can find differential equations for real amplitude and phase of the waves. The equation of motion for the beam electrons can be conveniently written in terms of the electron momentum in relativistic approach. Substituting the AW field components calculated in the gap region yields the following set of equations for motion of anyone electron. As far as we take into account influence of AW field on the electrons motion then it allows us to consider nonlinear effect of the beam on the AW. If we consider that electron beam is consisted of N macro-particles then we obtain 4N differential equations for description of the particles motion in two-dimensional spaces, thus we apply radial and angular co-ordinates and azimuthal and radial impulses in phase space.

The obtained set of differential equations has been studied by Runge-Cutta method. It is one of the best standard methods for numerical integration of differential equations and which makes it possible to reduce the number of computational operations required to calculate their right-hand sides. This circumstance is especially important for the solution of equations whose right-hand sides are very complicated. Fourth-order methods provide high accuracy of the numerical integration of differential equations and are traditionally used to solve the problems of the beam–plasma interaction. The time integration step was varied depending on the rate at which the functions changed during the process of numerical integration. The number of macro-particles used to model an electron beam was $N = 500$. The interaction of the beam electrons with the plasma boundary and metal waveguide wall was simulated using the mirror reflection model, which implies that the electrons do not disappear in interactions, but rather their radial impulses are reversed by mirror reflection and they are reflected back into the gap region.

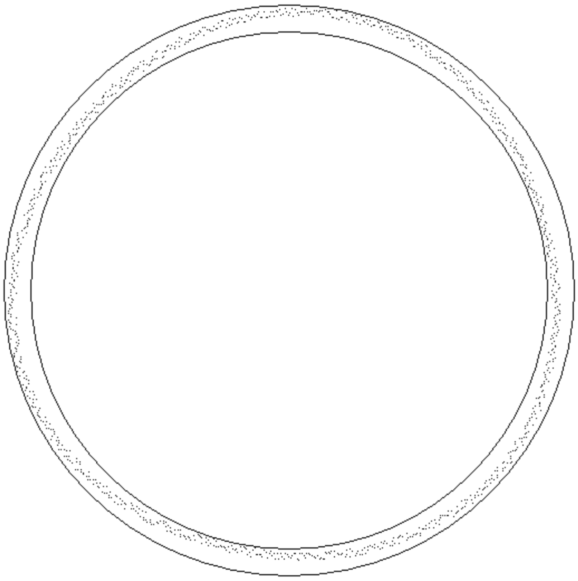

Fig.1. Distribution of particles in coordinate space for mode m=3 at the moment τ=0.21 (intermediate stage of beam driven instability)

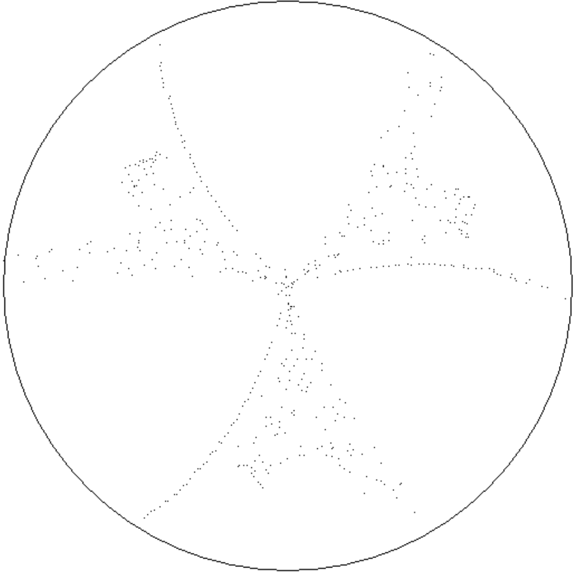

Fig. 3. Distribution of transverse impulses of particles vs. azimutal angle for mode m=3 at the moment τ=0.27 (intermediate stage of beam driven instability)

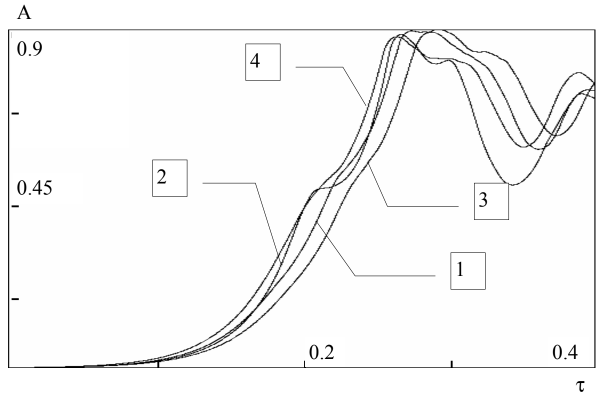

Fig.5. Temporal evolution of the AW amplitude (m=2) for following starting conditions: particles fill third part of gap witch are closed to plasma (1), particles fill middle part of gap (2), particles fill outer third part of gap (3) and particles fill the gap homogeneously (4)

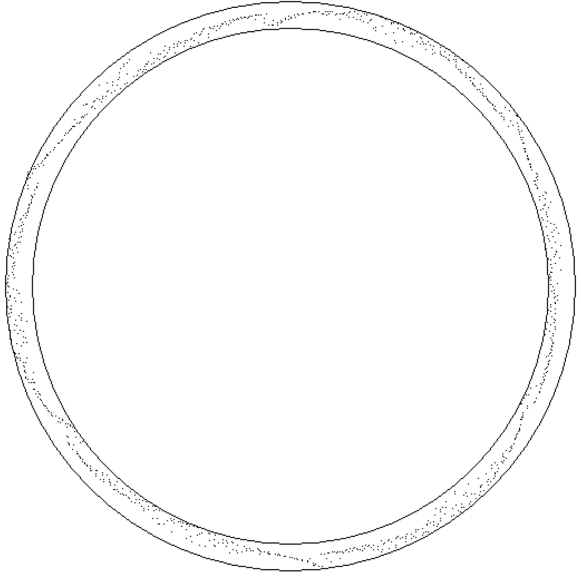

Fig. 2. Distribution of particles in coordinate space for mode m=3 at the moment τ=0.31 (final stage of beam driven instability)

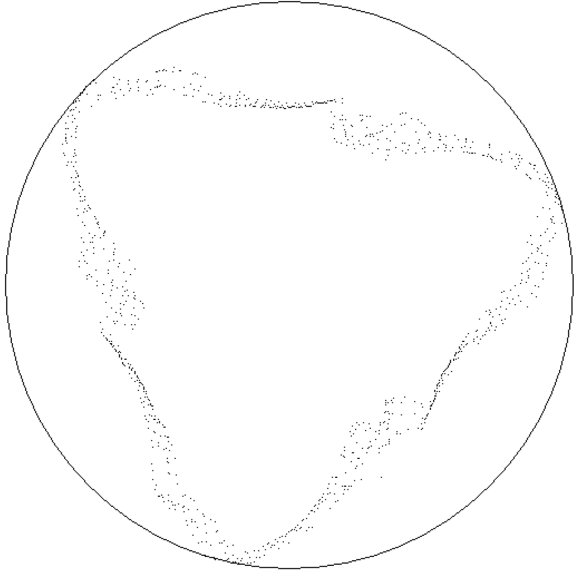

Fig. 4. Distribution of azimuthal impulses of particles vs. azimutal angle for mode m=3 at the moment τ=0.27 (intermediate stage of beam driven instability)

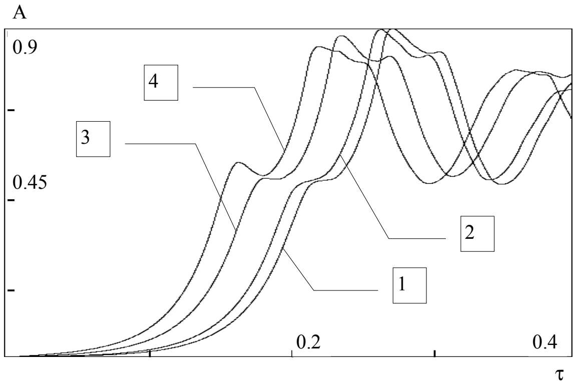

Fig. 6. Temporal evolution of AW amplitude (m=2) in the cases of different starting distribution of the beam particles along azimuthal angle: (1) – homogenous, (2) randomized homogeneous, (3) – particles are weakly bunched near

definite value of azimuthal angle,(4) – particles are strongly bunched near definite value of azimuthal angle

This model is frequently used to investigate the interaction of charged particle beams with finite-size plasmas and is best suited for the description of a beam–solid body boundary. The results of numerical simulations of the development of the resonant beam-driven instability of an ASW are illustrated in Figs. 1–6. The simulations were carried out for the following starting values of the waveguide and beam parameters: the wave amplitude was $A = 10^{-3}$, value of the AW phase and radial impulses were equal to zero. The geometric parameters of the waveguide were chosen to satisfy the condition $R_2 - R_1 = 0.1R_1$. Results of numerical analysis are represents in the form of temporal evolution of the AW amplitude, distributions of the beam particles in co-ordinate and phase spaces, respectively.

We have studied the excitation of AW by an annular electron beam rotating around the plasma column that partially fills a cylindrical metal waveguide operating with steady-state axial magnetic field. The resonant beam-driven instability of an ASW has been investigated in the single-mode approximation. We have derived a two-dimensional model set of equations describing the evolution of the envelope of the wave field, the phases of AW, and the coordinates and impulses of the electrons of a low-density beam. We have numerically analyzed the effect of the waveguide and beam parameters on the development of beam-driven instability. Unlike the case considered in [7] we have studied influence of the initial distribution of the beam particles in coordinate and phase spaces. It is shown that changing the initial radial distribution of the beam particles leads to the following changes of the AW temporal evolution: growth rate of the instability is maximum if beam fills the gap completely and its value decreases if it fills a part of the gap. Aggregating of the beam particles along azimuthal angle affects on the AW amplitude evolution strongly than in the previous case. Growth rate of the beam-driven instability in the case of the beam particles bunched near definite value of the azimuthal angle is larger than in the case of homogeneous distribution. Temporal evolution of the AW amplitude is practically independent on changing of the beam particles distribution on azimuthal impulse. Obtained results could be interesting for elaboration of plasma electronical devices.

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ВОЗБУЖДЕНИЕ СОБСТВЕННЫХ АЗИМУТАЛЬНЫХ ВОЛН КОЛЬЦЕВЫМ ПОТОКОМ ЗАРЯЖЕННЫХ ЧАСТИЦ

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Исследование взаимодействия потока заряженных частиц с гибридными волноводными структурами представляется важным для радиофизики и плазменной электроники. В данной статье исследовано возбуждение необыкновенно поляризованных электромагнитных азимутальных волн, которые являются собственными модами цилиндрического металлического волновода, частично заполненного холодной магнитоактивной плазмой. Двумерная самосогласованная система дифференциальных уравнений была выведена для описания нелинейного взаимодействия между азимутальными волнами и потоком заряженных частиц малой плотности, двигающимся над поверхностью плазмы, и исследована числовыми методами в одномодовом приближении. В работе исследовано влияние неоднородности распределения потока заряженных частиц в координатном и фазовом пространствах на эволюцию пучковой неустойчивости азимутальных волн.

ЗБУДЖЕННЯ ВЛАСНИХ АЗИМУТАЛЬНИХ ХВИЛЬ КІЛЬЦЕВИМ ПОТОКОМ ЗАРЯДЖЕНИХ ЧАСТИНОК

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Дослідження взаємодії між потоками заряджених частинок та гібридними хвилеводними структурами є важливим для радіофізики та плазмової електроніки. В даній статті досліджено збудження незвичайно поляризованих електромагнітних азимутальних хвиль, які є власними модами циліндричного металевого хвилеводу, який частково заповнено холодною магнітоактивною плазмою. Двовимірна самоузгоджена система диференціальних рівнянь була виведена для опису нелінійної взаємодії між азимутальними хвилями та потоком заряджених частинок малої густини, який рухається над поверхнею плазми, та досліджена числовими методами в одномодовому наближенні. В роботі досліджено вплив неоднорідності розподілу потоку заряджених частинок в координатному та фазовому просторах на еволюцію пучкової нестійкості азимутальних хвиль.