INFLUENCES OF NORMAL AND ANOMALOUS DOPLER EFFECTS ON DEVELOPMENT OF BEAM-PLASMA INSTABILITY

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The influences of normal and anomalous Dopler effects on development of a beam-plasma Cherenkov instability in the linear approximation is investigated. It is shown, that normal Dopler effect influences only on an absolute instability, leading to suppression of backward wave. The anomalous Dopler effect influences not only on absolute, but also on convection instabilities and under the certain conditions it may lead to complete suppression of Cherenkov beam-plasma instability.
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The fundamental mechanisms of beam-plasma instability, which is a basic for plasma relativistic microelectronics [1], are the single particle and collective Cherenkov effects, or in other words the Tomson and the Ruman regimes of stimulated Cherenkov radiation. In the limit of small beam density the resonance condition for single particle Cherenkov instability looks as:

$$\omega = K_z u, \qquad (1)$$

where ω -is a frequency, K_z- a longitudinal wave number, u-beam velocity. The only single particle regime of Cherenkov instability was realized in experiments [1]. Moreover, in experiments the external magnetic field is usually strong and Larmour frequency $\Omega_e = \frac{eB_0}{mc}$ is much higher than plasma frequency $\omega_p = \sqrt{\frac{4\pi e^2 n_p}{m}}$, where B_o- is a strength of magnetic field, n_p-is a plasma density. By this reason the most of theoretical investigations were carried out under the assumption, that Bo is infinite. At the same time, in the recent experiments [2] it was shown, that the beam-plasma microwave sources are efficiently working even when the Larmor and Langmuir frequencies are of one order. Theoretically it was predicted [3] and experimentally it was confirmed [2] that the frequency spectrum of Cherenkov radiation in a plasma waveguide practically does not depend on the strength of magnetic field. But in the finite magnetic field the new resonances and new mechanisms of beam-plasma interaction arise. They are known as normal and anomalous Dopler effects and take place when[4]:

$$\omega = K_z u \pm \frac{\Omega_e}{\gamma} \qquad (2)$$

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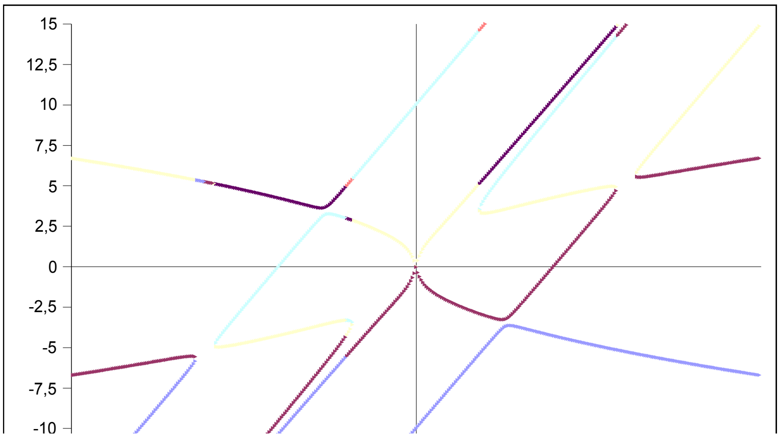

Fig.2.

where $\gamma = \left(1 - \frac{u^2}{c^2}\right)^{-\frac{1}{2}}$ - is the energy relativistic factor of beam electrons. The resonances (1) and (2) are quite different, but in spite of this it is possible of mutual influence of Dopler and Cherenkov instabilities to each other. It will be shown bellow, that normal Dopler effect leads to forbidden of backward wave with $K_z<0$, exited by the stimulated Cherenkov radiation (or when the conditions (1) for $K_z>0$ and (2) for $K_z<0$ are satisfied simultaneously). This may lead to suppression of feedback coupling in a beam-plasma oscillator and even to break its working. As far anomalous Dopler effect it leads to increasing transverse of velocity of beam electrons and decreasing of longitudinal velocity and finally to violation of Cherenkov resonance condition (1) and complete suppression of beam-plasma instability. It is obvious, that the problem of influence of Dopler effects on Cherenkov beam-plasma instability may be settled only in the frame of general nonlinear theory. Nevertheless in this report we restrict ourselves by consideration this problem in linear approximation on the basis of dispersion equation and qualitative analyzes of nonlinear processes.

Let us now discuss the restrictions of linear approximation. According to the conditions of real experiments [2], we consider a cylindrical waveguide with radius R, to be filled up by thin annular cylindrical beam and plasma layers with:

$$\Delta_b, \Delta_p << r_b < r_p < R \qquad (3)$$

Here r_p and r_b –are the mean radiuses of layers, Δ_p and Δ_b –their thicknesses. In fig.1 a principal scheme of beam-plasma Cherenkov microwave source [2] is presented.

One of most important condition, which simplifies the problem, is:

$$\omega << \frac{c}{\Delta_p} \qquad (4)$$

In this limit it was predicted theoretically [3] and confirmed in experiments [2], that the frequency of excited waves does not depend on magnetic field. Therefore the process of Cherenkov radiation may be considered in infinite magnetic field. At the same time for considering of Dopler effects the finite strength of magnetic field must be taken into account. This complicates the problem. But if the beam density is small,

$$\left(\frac{n_b}{2n_p}\right)^{\frac{1}{3}} \frac{1}{\gamma} << 1 \ , \qquad (5)$$

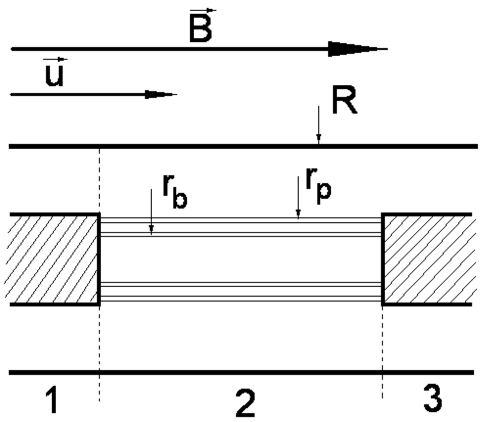

Fig.1. A principal scheme of beam-plasma Cherenkov microwave source:1 - metallic waveguide with radius R; 2 – plasma and beam layers with main radiuses r_p and rb and thicknesses Δ_p and Δ_b; 3 – collector

then for simplification we can the use well known perturbation theory.

Omitt the details of calculations we write here the dispersion equation for cable (symmetric) wave, excited by electron beam in a plasma waveguide:

$$\omega^2-\Omega_p^2=G_{dp}\frac{\omega_p^2\omega_b^2\gamma^{-1}}{\left(\omega-K_zu\right)^2-\dfrac{\Omega_e^2}{\gamma^2}}+G_{ch}\frac{\omega_p^2\omega_b^2\gamma^{-3}}{\left(\omega-K_zu\right)^2} \tag{6}$$

The first term in the right side of this equation describes Dopler effects and the second term-Cherenkov effect. In (6) $\omega_b=\sqrt{\dfrac{4\pi e^2 n_b}{m}}$ is the Langmuir frequency of beam electrons, the quantities

$$\Omega_p^2=\omega_p^2\frac{x_0^2}{K_p^2},\frac{1}{K_p^2}=r_p\Delta_p I_0^2(x_0r_p)\left[\frac{K_0(x_0r_p)}{I_0(x_0r_p)}-\frac{K_0(x_0R)}{I_0(x_0R)}\right] \tag{7}$$

determine the frequency Ω_p and transverse wave number K_p of exited cable wave, $x_0^2=K_z^2-\dfrac{\Omega_p^2}{c^2}$,

$$G_{dp}=x_0^2\frac{r_b\Delta_b}{r_p\Delta_pK_p^2}\frac{I_1^2(x_0r_b)}{I_0^2(x_0r_p)}Q,G_{ch}=x_0^2\frac{r_b\Delta_b}{r_p\Delta_pK_p^2}\frac{I_0^2(x_0r_b)}{I_0^2(x_0r_p)}, \tag{8}$$

where $Q=(1-\dfrac{u^2}{c^2}\dfrac{\Omega_p}{K_zu})$.

The results of numerical solutions of (6)-(7), which will be discussed in the next section, are presented in Fig.2-4. Here we well give the growth rates of Cherenkov and anomalous Dopler instabilities and their analysis. From (6) it follows:

$$\omega\to\omega+\delta\omega=\Omega_p+\delta\omega,$$

$$\delta\omega=\begin{cases}\dfrac{1+i\sqrt{3}}{2}\left(\dfrac{1}{2}\dfrac{G_{ch}\omega_p^2\omega_b^2}{\Omega_e\gamma^3}\right)^{\frac{1}{3}}-for\ Cherenkov,\\ \dfrac{i}{2}\left(\dfrac{G_{dp}\omega_p^2\omega_b^2}{\Omega_p\Omega_e}\right)^{\frac{1}{2}}-for\ anomalous\ Dopler.\end{cases} \tag{9}$$

For normal Dopler effect there is no instability in the beam-plasma system. As we can see from Fig.2-4 suppression of backward cable wave with $K_z<0$ takes place. This means that when

$$\Omega_p=\frac{\Omega_e}{2\gamma} \tag{10}$$

the backward cable wave can't propagate. This cuts off feedback coupling in a beam-plasma oscillator and breaks its work. At the same time normal Dopler effect does not influence the amplifiers work.

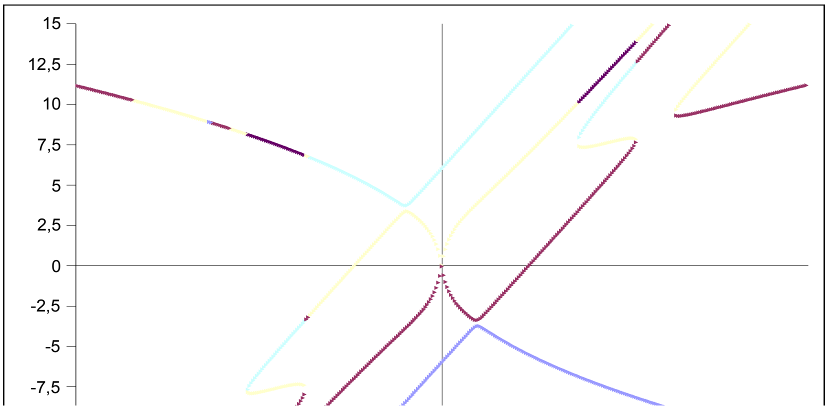

Fig.3

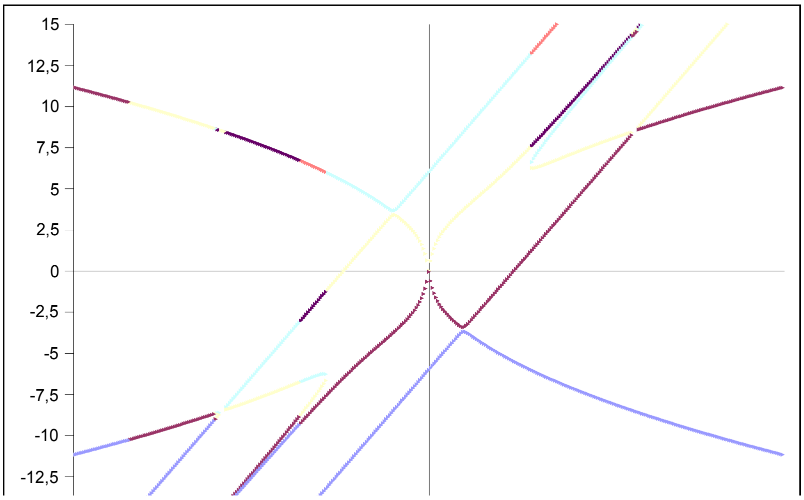

Fig.4.

As for anomalous Dopler effect, when the second condition (2) is satisfied, the beam-plasma system becomes unstable with growth rate (8). It leads to increasing of transverse velocity of beam electrons and decreasing of longitudinal one. As a result the resonance condition (1) may violate and Cherenkov beam-plasma instability stops. In this sense anomalous Dopler effect may turned out to be catastrophic for both beam-plasma oscillator and amplifier. Of course, it is possible only if the external magnetic field is weak and growth rate of Cherenkov instability is less, then Dopler instability.

Let us now demonstrate the above statements by discussing the numerical solutions of equation (6) presented in Fig.2-4. The invariable parameters of the system taken from the experiments [2] are: radius of waveguide R=2cm., main radius of beam r_b=1cm, $\Delta_b = \Delta_p$ =0,1 cm, Langmuir frequency of beam electrons $\omega_b=2\cdot10^{+10}$ c^{-1} and velocity $u=2\cdot6\cdot10^{10}$ cm/c (γ=2). The cyclotron Ω_e and plasma ω_p frequencies and main radius of plasma r_p, were varying. In the figures the dependencies $\omega(K_z)$ in units $10^{10}c^{-1}$ and wave numbers K_z in cm-1 are presented.

In Fig.2 the case of strong magnetic field is shown: $\Omega_e=10\cdot10^{10}c^{-1}$ and $\omega_b=6\cdot10^{10}c^{-1}$. We see 2 regions of instability for $K_z\geq 0$, which are marked by vertical lines «a», «b», «c» and K_z=0 (picture is asymmetric relative to co-ordinates): between the K_z=0 and «a» the instability is stipulated by Cherenkov effect, whereas between «b» and «c»-by anomalous Dopler effect.

In Fig.3 the opposite case of relatively weak magnetic field is shown: $\Omega_e = 6\cdot10^{10}c^{-1}$, and $\omega_p = 10\cdot10^{10}c^{-1}$. Here again we have 2 regions of instability, but now they are wider, that corresponds to expressions (9). In Fig.3 as in Fig.2 the Cherenkov and Dopler instability regions are separate, but now their growth rates become of one order. The further decreasing of magnetic field leads to overlap the instability regions. It is obvious, that in this case only in the frame of nonlinear theory problem may be solved.

At the same time one can easily suppress the anomalous Dopler instability by separation of beam and plasma layers. In the cases, which are presented in Fig.2 and 3 the layers were very close, whereas in the case presented in the Fig.4 they were separated- the mean radius of plasma was r_p =1,2 cm,. or the clearance between the layers was 1mm.We see very essential changes: the instability regions become very narrow, especially for Dopler instability. This means that by separation of beam and plasma layers one can successfully suppress anomalous Dopler instability.

Finally let us discuss very shortly the influence of normal Dopler effect on the beam-plasma instability. As it was noticed above and as it is seen in Fig.2-4 the normal Dopler effect leads to suppression of backward wave with $K_z<0$. In Fig.2-4 the frequency range of normal Dopler effect is marked by arrow. We see that this region is very narrow near the resonance frequency (10). Nevertheless this phenomenon may suppress the undesirable modes in a beam-plasma amplifier.

From the above analysis one can make the following conclusions:

1. Normal and anomalous Dopler effects can essentially influence the character of development of Cherenkov beam-plasma instability, and thus the work of Cherenkov plasma sources of the microwave radiation (generators and amplifiers) only in conditions of moderate magnetic fields when Larmor frequency electrons is comparable with plasma frequency.

2. Normal Dopler effect can suppress the backward cable plasma wave excited by a beam, at performance of a condition (10) and by that to break generation, having suppressed a feedback in the microwave generator. However, action of normal Dopler effect is shown in very narrow range of frequencies of generation near frequency (10). On the forward wave the normal effect of influence does not render and consequently it does not influence work of plasma Cherenkov amplifier.

3. Anomalous Dopler effect is one of dangerous instabilities of beam-plasma system and consequently its influence on Cherenkov instability can appear more dramatic. The anomalous Dopler effect leads to increasing of transverse velocity of beam electrons and consequently to full failure of Cherenkov beam-plasma instability. In this sense anomalous Dopler effect can affect essentially work of plasma microwave sources like generators and amplifiers.

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ВЛИЯНИЕ НОРМАЛЬНОГО И АНОМАЛЬНОГО ЭФФЕКТОВ ДОПЛЕРА НА РАЗВИТИЕ ПУЧКОВО-ПЛАЗМЕННОЙ НЕУСТОЙЧИВОСТИ

М.В. Кузелев, А.А. Рухадзе

В линейном приближении исследуются влияния нормального и аномального эффектов Доплера на развитие пучково-плазменной черенковской неустойчивости в продольно ограниченных системах. Показано, что нормальный эффект Доплера влияет лишь на абсолютную неустойчивость. Он приводит к непропусканию встречной волны в определенной области частот, срывая тем самым абсолютную неустойчивость. Аномальный же эффект влияет не только на абсолютную, но и на конвективную неустойчивость и может в определенных условиях полностью задавить черенковскую пучково-плазменную неустойчивость.

ВПЛИВ НОРМАЛЬНОГО Й АНОМАЛЬНОГО ЕФЕКТІВ ДОПЛЕРА НА РОЗВИТОК ПУЧКОВО-ПЛАЗМОВОЇ НЕСТІЙКОСТІ

М.В. Кузельов, А.А. Рухадзе

У лінійному наближенні досліджуються впливи нормального й аномального ефектів Доплера на розвиток пучково-плазмової нестійкості Черенкова в подовжньо обмежених системах. Показано, що нормальний ефект Доплера впливає лише на абсолютну нестійкість. Він призводить до непропускання зустрічної хвилі у визначеній області частот, зриваючи тим самим абсолютну нестійкість. Аномальний же

ефект впливає не тільки на абсолютну, але і на конвективну нестійкість і може у визначених умовах цілком задавити пучково-плазмову нестійкість Черенкова.